Different Techniques for Calculating Apparent and Incremental Inductances using Finite Element Method

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Abstract
This work focuses on using two-dimensional finite element method to calculate apparent and incremental inductances for 40W fluorescent lamp ballast with different loading conditions based on its design documents. Seven inductance calculation techniques are adopted by calculating ballast stored magnetic energy and flux linkage using ANSYS software. The calculated results for apparent inductances show a good agreement with the design and average measured values, while calculated incremental inductances have been verified by noting the behavior of core hysteresis loop experimentally. These seven techniques establish a good base for researchers and designers to obtain an accurate inductance profile for any iron-core inductor.

Index Terms— ANSYS, Apparent inductance ,Ballast ,Incremental inductance , Inductance profile , Iron-core inductor , Finite Element Method.

I. INTRODUCTION

Inductance (L) , which is the characteristic of an inductor, is the change in magnetic flux with respect to a change in coil current. An inductor is known to resist any change in the rate of flow of current in the circuit , similar to the presence of a mass in a mechanical system, having the property of inertia that resists any kind of change in its state of motion . An alternative definition of inductance states that it is a measure of the ability of an inductor to store energy in a magnetic field. The inductor stored energy (W) , is directly proportional to the square of the inductor current, and the factor of proportionality is (L/2). Inductances play an important role in the dynamic performance and characteristics of electrical apparatus, and it is calculated for any coil based on the calculation of flux linkage (λ), or stored magnetic energy, and current (I). The incremental values of such inductances ($\partial \lambda/\partial$ I) rather than the apparent values (λ/I) are the most needed parameters, especially when one is more interested in the transient

performance of such devices operating under conditions of (on/off) switching operations [1].
Apparent and incremental inductances don't differ while inductor is working below saturation point of iron core (linear operating region) and incremental inductance decreased with respect to apparent value when the device is working after the saturation point (nonlinear operation region). This work adopts seven techniques to calculate apparent and incremental inductances for the test iron- core inductor (fluorescent lamp ballast) using two-dimensional finite element method (2DFEM) based on ANSYS14 software, which uses Maxwell´s equations as a basis for electromagnetic field analysis. The significance of this work is to highlight the fact that, the inductance (and so the equivalent circuit) of any iron- core device is changing under variable operating conditions.

II. MODELLING

This work considers the ANSYS14 to build a 2DFEM model (coil end effects are ignored) for the test ballast shown in figure (1) based on the design

documents on state company of electrical industries - Baghdad [2]. Table-1 and figure (2) show the operating characteristics and dimensions of the test ballast.

Table-1 Test ballast data

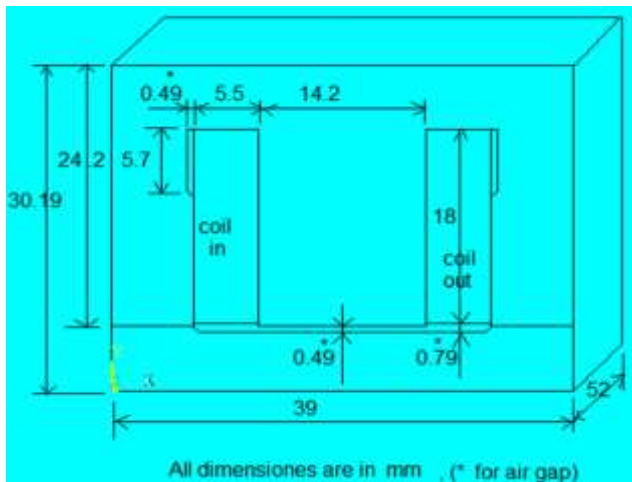| Power	40W
Voltage	220V AC
Current	0.43A
Frequency	50Hz
Cos Φ	0.52
Coil resistance	31.5 Ω at 20° C
Coil impedance	(390 - 415) Ω
Coil inductance	(1.238 - 1.317) H

Fig-1 Photograph of the test ballast

Fig-2 Test ballast dimensions

The magnetic material of ballast core is made from non-oriented silicon steel type V600-50A according to German standard DIN 46400.The BH curve of this material is illustrated in figure 3. Ballast coil consists of 825 turns of 0.3 mm diameter copper wire.
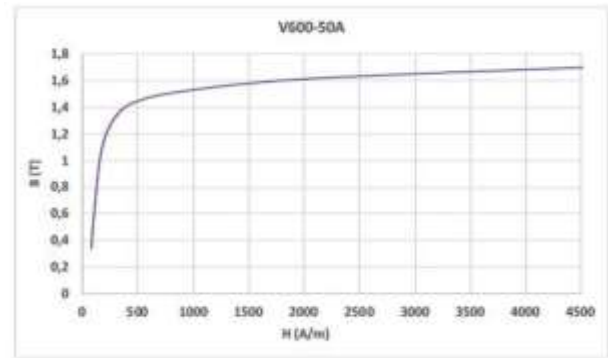

Fig-3 B-H curve for the ballast core

The symmetry of the core and winding allows analyzing one-half of the model (symmetry factor=2) to reduce the solution time. Generally, building and activating of the 2DFEM ballast model goes through the following steps in ANSYS:-

1- Building the geometry (key points, lines, areas) according to design drawings.

2- Meshing (subdivision of whole 2D model for triangular element type PLANE53) as in figure (4).

3-Assigning material properties (air gap, insulation, copper wire, silicon steel).

4- Applying Dirichlet boundary condition (make magnetic vector potential A = 0 at the entire boundary of half model), so we assume no magnetic flux lines can cross this boundary.
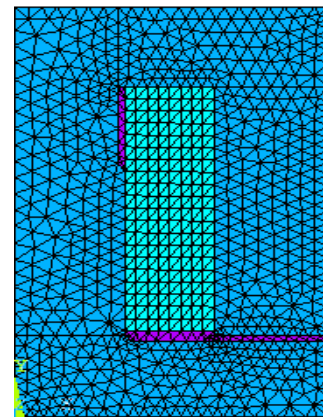
5- Applying load (current density on coil area).

Fig-4 Half FEM model of the test ballast

III. INDUCTANCE CALCULATION TECHNIQUES

a. Inductance from Magnetic Energy

The finite element method is a variational procedure, minimizing or maximizing the energy stored in the studied domain. Thus, energy is the

most accurate single number characterizing a solution [3]. The total magnetic field energy for electrical apparatus can be calculated via the integral:

$$W = \frac{1}{2}\int A\,J\,dV \qquad (1)$$

Where (A) is the magnetic vector potential, (J) is the current density, and (V) is the volume.

For 2DFEM, the field has no dependence on the into-the-page depth (h), so the integral can be simplified to [4]:

$$W = \frac{1}{2}h\int A\,J\,da \qquad (2)$$

Since the energy stored in a magnetic system excited by a coil carrying current is:

$$L = \frac{2W}{i^2} \qquad (4)$$

Where (i) is the inductor current.
Solving (2) and (4) for (L) results in:

$$L = \frac{h\int A\,J\,da}{i^2} \qquad (5)$$

An alternative approach to obtain magnetic energy via the integral is:

$$W = \frac{1}{2}\int B\,H\,dV \qquad (6)$$

And in the same manner adopted in eq.(5) ,the inductance can be calculated as:

$$L = \frac{h\int B\,H\,da}{i^2} \qquad (7)$$

Where (B) is the magnetic flux density, and (H) is the magnetizing force [4].
This work adopts four techniques to calculate inductance based on equation 4 by ANSYS as follows:-
1- Harmonic nonlinear solution (direct technique).
2- Static nonlinear solution (direct technique).
3- Static nonlinear solution (modified technique-1).
4-Static nonlinear solution (modified technique -2).

a1) Harmonic nonlinear solution (direct technique)
This technique uses the facility of ANSYS program to calculate the inductance of coil elements after adopting harmonic (steady-state sinusoidal loading at 50Hz frequency) nonlinear solution. The circuit of this analysis is illustrated in figure 5. Figure 6 shows the flux plot (real only) of the harmonic nonlinear solution at rated current of 0.43A.
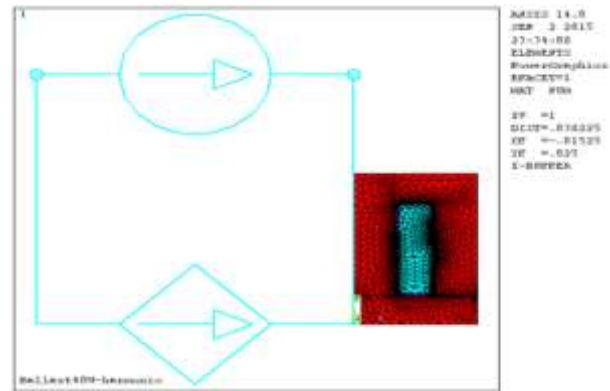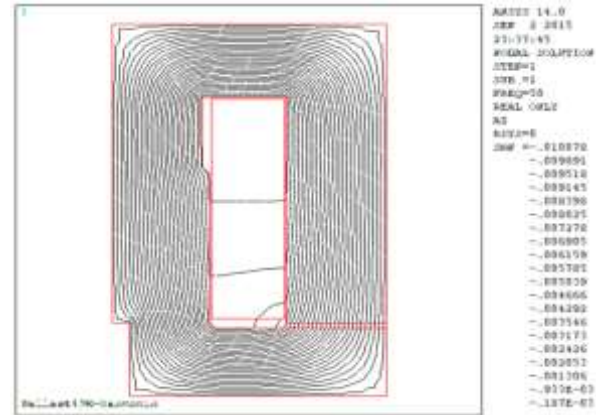

Fig-5 Circuit of harmonic analysis

Fig-6 Flux plot at rated current

The flow chart of this technique is shown in figure7. Note: All values of inductances calculated by all techniques in this work are multiplied by a symmetry factor of 2 and z-axis depth (=0.052 m core depth × 0.97 iron filling factor).
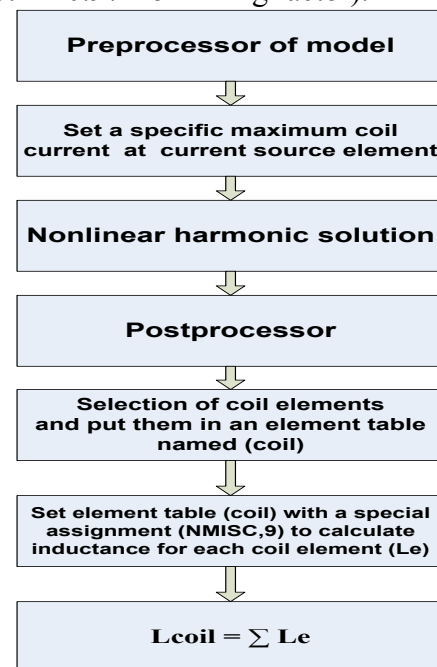

Fig-7 Flowchart of harmonic analysis technique

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a2) Static nonlinear solution (direct technique)
This technique is similar to the previous one except, that it, uses a static nonlinear solution by applying different values of current density at the coil area, instead of AC current source. Figure 8 shows the values of the magnetic flux density at different model elements with rated coil current , which show that the finite element method is an accurate method, since its solution is obtained from numerical manipulation of different electric and magnetic values at different model finite elements.

Fig-8 Flux density distribution at rated current

a3)Static nonlinear solution (modified technique-1)

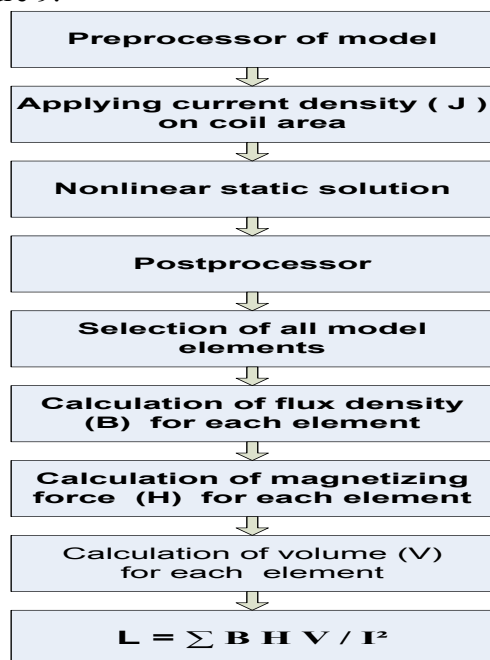This technique is illustrated in the flowchart shown in figure 9.

Fig-9 Flowchart of modified technique-1

a4)Static nonlinear solution (modified technique-2)
This technique is illustrated in the flowchart shown in figure 10. The value of current density (J) applied on coil area is assigned by multiplying the value of RMS coil current by the number of coil turns (825 turns) divided by the coil area ($92.82 \times 10^{-6}\ m^2$).
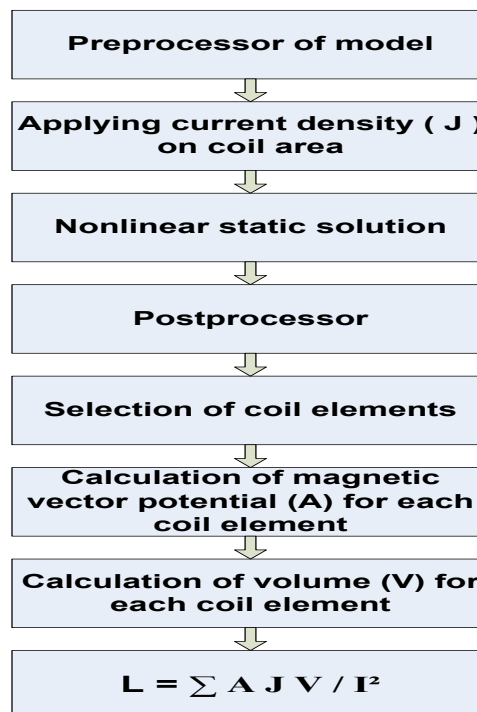
Fig-10 Flowchart of modified technique-2

b. Flux Linkage Technique

The flux linkage in the ith turn of a coil (λi) can be calculated as follows:

$$\lambda_i = \frac{1}{S[(\int A_i^+)dS - (\int A_i^-)dS]} \qquad (8)$$

Where (S) is the cross-sectional area of a single turn and $(A_i^{\mp})$ are the magnetic vector potential values on the surface (S) for plus and minus current on each coil side, the coil flux linkage (λt) can be calculated as follows:

$$\lambda t = \sum_{i=1}^{N} \lambda i \qquad (9)$$

Where N is the number of coil turns.
The total flux linkage can be calculated by summation of flux linkage for each element in the coil area in the FE model. Figure 11 shows the

flowchart of adopting flux linkage technique by ANSYS.

Fig-11 Flowchart of flux linkage technique

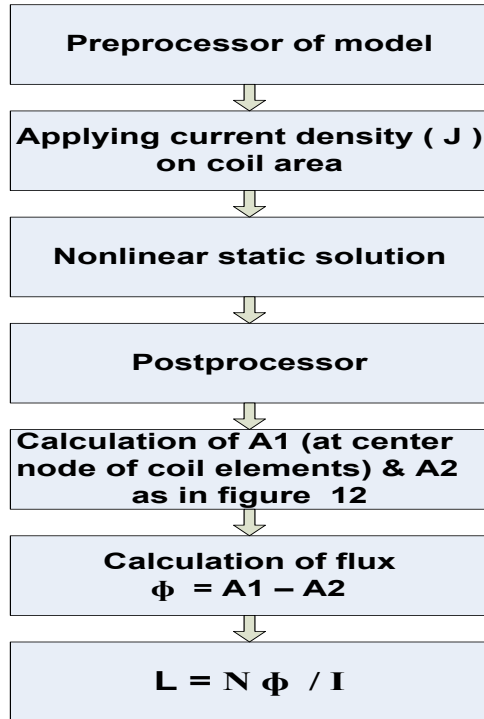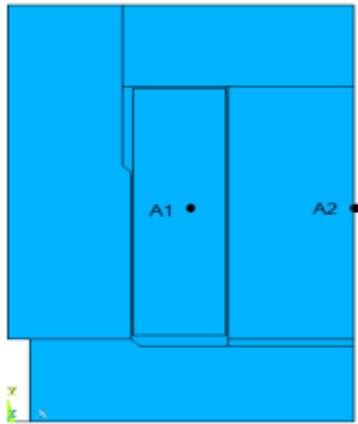Fig-12 Positions of MVP to calculate flux

c- Energy Perturbation Technique
This technique was first proposed by Demerdash in 1982 [1] .It is mainly used for inductance calculation for multiple-coil system based on calculation of stored magnetic energy in it , as in the following equation:

$$L = \partial^2 W / \partial^2 i \qquad (10)$$

Where L is the incremental inductance or the apparent inductance depending on how the energy perturbation is made, if the perturbation is along the tangent to the B-H curve at the operating point (nonlinear case), then the calculated inductance is incremental, and if the perturbation is a long straight line between the operating point and the origin (linear case), then the calculated inductance would be apparent as in figure 13 [1].
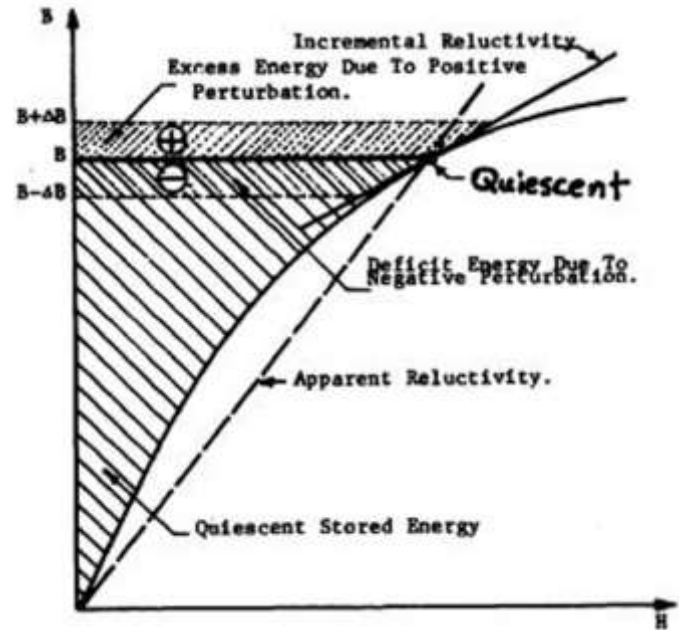
Fig-13 Assigning of $W^+, W, and\ W^-$

In one coil system (as in our case) the following formula is used to determine the inductance:

$$L = \frac{[W^+ - 2W + W^-]}{(\Delta i)^2} \qquad (11)$$

Where W is the global energy, calculated at an operating point (nominal i),
W^+ is the global energy, calculated at positive perturbation $(i + \Delta i)$,
 W^- is the global energy, calculated at negative perturbation $(i - \Delta i)$.
The above energies are calculated by using special macro-programs written by ANSYS Parametric Design Language (APDL).The procedure of this technique can be summarized in the flowchart of figure 14. The energy-perturbation method requires several solutions and can be numerically unstable in some situations; therefore current perturbation must be carefully chosen to achieve accurate results [5]. In this work different values of current perturbation were investigated, the perturbation of 50% of current is found to produce accurate results of inductances with respect to other techniques.

$$L = [(+W) - 2(W) + (-W)] / (\Delta I)^2$$

Preprocessor of model

Calculation of magnetic energy (-W)

Applying current density (J) on coil area

Postprocessor

Static nonlinear solution

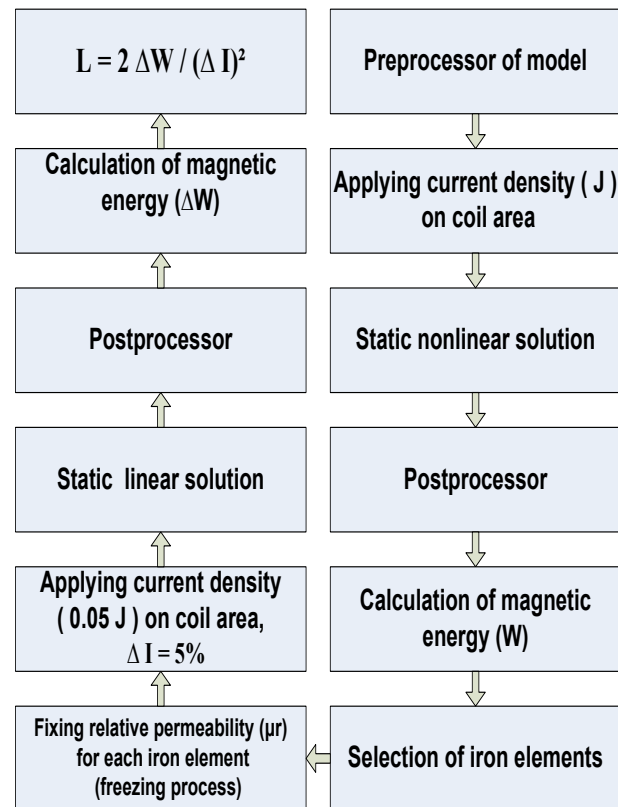**Static linear solution**

Postprocessor

Applying current density (0.5 J) on coil area

Calculation of magnetic energy (W)

Calculation of magnetic energy (+W)

Selection of iron elements

Postprocessor

Fixing relative permeability (µr) for each iron element (freezing process)

Static linear solution

Applying current density (1.5J) on coil area, ΔI = 50%

Fig-14   Flowchart of Demerdash technique

d. Enhanced Incremental Energy Technique
This approach was first proposed by Gyimesi in 1999 [6],and it has only one incremental analysis as in the following simple equation:

$$L = \frac{2\Delta W}{(\Delta i)^2} \qquad (12)$$

It is clear from equation (12) that, there is no need to evaluate the nonlinear magnetic energy (W), instead an incremental energy (ΔW) is only calculated, which improved the accuracy and computational efficiency of inductance with respect to Demerdash technique .Figure 15 shows the flowchart of this technique.

$$L = 2 \Delta W / (\Delta I)^2$$

Preprocessor of model

Calculation of magnetic energy (ΔW)

Applying current density (J) on coil area

Postprocessor

Static nonlinear solution

Static linear solution

Postprocessor

Applying current density (0.05 J) on coil area, ΔI = 5%

Calculation of magnetic energy (W)

Fixing relative permeability (µr) for each iron element (freezing process)

Selection of iron elements

Fig-15   Flowchart of Gyimesi technique

IV. EXPERIMENTAL VERIFICATION

The test is carried out on ballast as in the circuit illustrated in figures 16 and 17. This test achieves two goals, the first is to measure the ballast average inductance in the linear operation region (up to rated current) in order to compare it with the calculated one for each of the seven techniques at this region (apparent inductance). Since ammeter is designed to work with sinusoidal waveform only, so it is not correct to use its readings in measuring incremental inductance in the nonlinear region, at which the shape of current waveform is non sinusoidal as in figures 19 and 20.
The average inductance is calculated as follows:

$$L_{av} = \frac{\sqrt{Z_{av}^2 - R^2}}{\omega} \qquad (13)$$

Where , Z_{av} is the average value of impedances calculated by the ratio of voltmeter reading to the ammeter reading (as in circuit diagram of figure 17)for eight values of ballast current in linear operation region (up to the rated current) , R is the total effective resistance of ballast resistance and 3Ω external resistance, $\omega = 2\pi f = 314$.
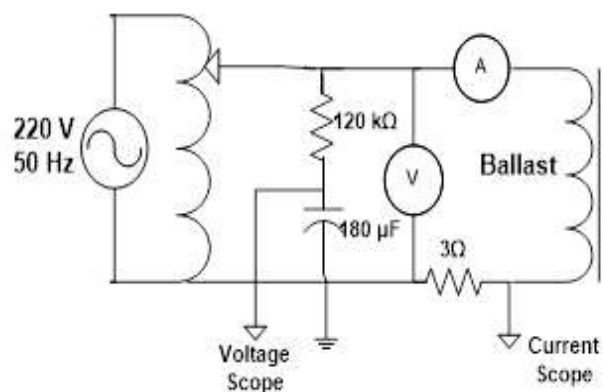
Fig-16   Photograph of ballast under test
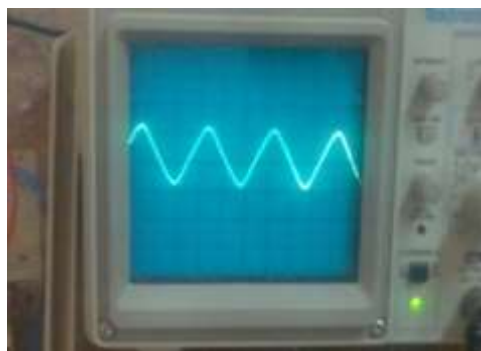

Fig-17   Test circuit diagram

Fig-18 Ballast current waveform and core hysteresis loop at rated current (0.43 A)
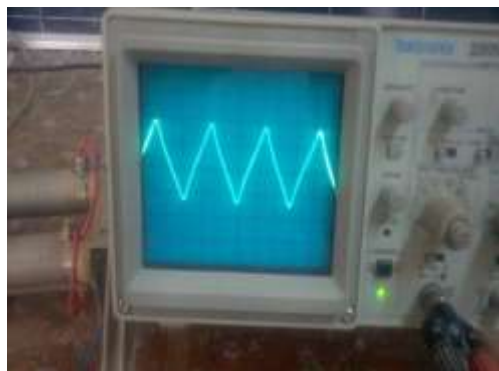

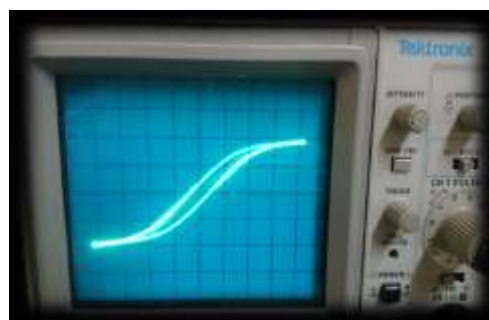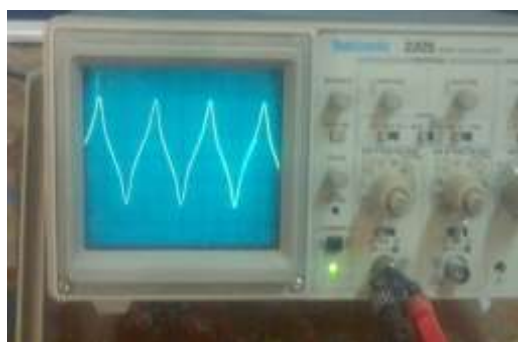Fig-19   Ballast current waveform and core hysteresis loop at 0.6 A

Fig-20   Ballast current waveform and core hysteresis loop at 0.85A

The measured value of Z_{av} is 415 Ω , and so L_{av} is 1.313 H which is very slightly less than the maximum design value of the inductance mentioned in table 1.The second goal of this test is to show the shape of the hysteresis loop for ballast iron core which lead us to assign the current at which the saturation occurs experimentally (it happens here about ballast rated current). The assignment of the core saturation point from its hysteresis loop shape gave sureness to the calculated incremental inductances by the seven techniques, because any inductance calculated in nonlinear region is incremental one. The hysteresis loop photograph presented in figure 18 shows the approximate linear shape of this loop at rated current (start of nonlinearity region) ,which enhance the validity of the calculated inductances for currents up to rated values to be apparent inductance, while figures 19 and 20 show the increment in nonlinearity of this loop as the ballast current increases to values higher than its rated value and its waveform shape is outside sinusoidal shape, hence the values of calculated inductances associated with these currents are incremental.

V. RESULTS AND DISCUSSION

Table 2 and figure 21 show the results of test ballast inductances calculated by the seven techniques previously mentioned, at different loading currents. The calculated results of inductances for currents up to rated (linear region) show good agreement with the average inductance measured by test, and ballast design inductance. These calculated inductances are nearly constant and identical for this linear region and they are called apparent inductances. The values of inductances start to decrease after rated current due to saturation effect (nonlinear region) and they are called incremental inductances. The proposed procedures in this work present good methodology to the other researchers and designers to distinguish between the apparent, and the incremental inductance. Figure 21 show that, the inductance profile obtained by technique 1 (nonlinear harmonic solution) appears the highest difference for incremental inductances when compared to the other techniques due to the fact that, this technique manipulates the maximum currents instead of the RMS currents as in the other techniques, as a result, the saturation level (decreases in incremental inductance)is high in technique 1.Other comparison is conducted between the applied seven techniques with the way at which they are adopted in ANSYS, either directly adopted like techniques 1 and 2, or need special APDL programs for other five techniques, and this work introduces a wide range of options to use of ANSYS in calculating inductance with clear steps. Table 3 shows the detailed calculation of the stored magnetic energy in different model areas (air gap, copper, and iron) for technique 1 at rated current. The highest value (70%) of magnetic energy lies in the air gap since the magnetic energy is directly proportional to the magnetic reluctance, the value of inductance can be classified for each model region, and the participation of different device material in this inductance can be assigned precisely. Defining this participation assists the work of researchers and designers, in the field of controlling the value of inductance for any iron-core device, by changing the length of the air gap, or by changing the properties of device materials. It is noted from the results obtained in this paper, that, the use of finite element method presents a precise research and design tool in calculating the inductance profile, due to the consideration of the variation in electric and magnetic quantities at different positions in the device under analysis, similar to the analysis of the actual working state.

VI. CONCLUSIONS

Seven techniques based on ANSYS finite element method software and real design dimensions and specifications were presented in this work to calculate precisely the apparent and incremental inductances for iron- core inductor at different loading conditions including saturation with clear steps. Straight-forward FEM methodologies for estimating detailed stored magnetic energy and coil flux linkage were displayed. The apparent calculated inductances for all techniques showed good agreement with the measured and designed values in linear operation. The validity of calculating the incremental inductances is verified by noting the behavior of core hysteresis loop in nonlinear operation. This work suggests a vaible template for researchers and designers to obtain a precise inductance profile for any iron- core inductor.

Table-2 The calculated results obtained

| I_{rms} (A)	J (A/m2)	L (H) Tech.1 (a1)	L (H) Tech.2 (a2)	L (H) Tech.3 (a3)	L (H) Tech.4 (a4)	L (H) Tech.5 (b)	L (H) Tech.6 (c)	L (H) Tech.7 (d)
0.1	888 800	1.28266	1.28022	1.28074	1.27449	1.29073	1.270	1.2744
0.2	1 777 500	1.280794	1.281618	1.28125	1.27502	1.29135	1.273	1.2750
0.3	2 666 400	1.272862	1.27593	1.27822	1.27201	1.28823	1.272	1.2721
0.4	3 555 300	1.253592	1.26746	1.27291	1.26691	1.28287	1.267	1.2670
0.43	3 821 900	1.241832	1.26423	1.27084	1.26493	1.28081	1.265	1.2650
0.5	4 444 100	1.195796	1.25326	1.26408	1.25813	1.27402	1.260	1.2585
0.6	5 332 900	1.073608	1.21882	1.24380	1.23777	1.25371	1.245	1.2393
0.7	6 221 800	0.94129	1.14348	1.20019	1.19459	1.21003	1.208	1.1974
0.8	7 110 600	0.83509	1.007838	1.12135	1.11606	1.13117	1.150	1.1229
0.9	7 999 400	0.750698	0.84406	1.02224	1.01749	1.03208	1.073	1.0281
1	8 888 300	0.68256	0.707616	0.93267	0.92852	0.94254	0.995	0.9419
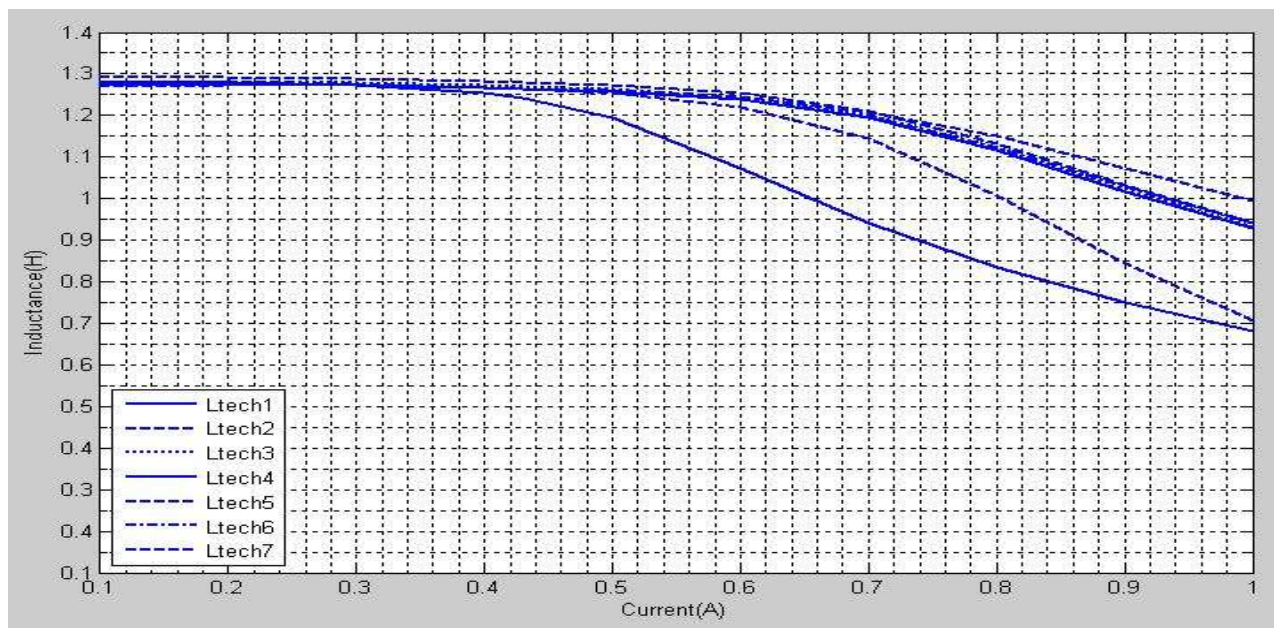
Fig-21   Ballast inductance profile calculated by the seven techniques

Table-3 Detailed calculated energy at rated current

Material	rms stored energy (J/m)	percentage
Copper (coil)	0.044052	3%
Iron (core)	0.39725	27%
Air gap	1.03150	70%
Total	1.47280	100%

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