Nonpolynomial Spline Method of Singularly Perturbed Boundary Value Problems

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Abstract

This paperis concerned with the approximated solution of linear two-points boundary value problem using nonpolynomial spline method. The result of this method is compared with the exact solution. Three numerical examples are given for conciliated the results of this method.

Keywords: nonpolynomial spline method, boundary value problem,, truncation error, exact solution.

1. Introduction

Singular perturbation problems containing a small perturbation parameter , arise very frequently in many branches of applied mathematics such as,fluid dynamics, quantum mechanics, chemical reactor theory, elasticity, aerodynamics, and the other domain of the great world of fluid motion[1,2,3].

A well known fact is that the solution of such problems has a multiscale character ,i.e. there are thin transition layers where the solution varies very rapidly, while away from the layer the solution behaves regularly and varies slowly. Numerically,the presence of the perturbation parameter leads to difficulties when classical numerical techniques are used to solve such problems, this is due to the presence of the boundary layers in these problems.We consider a second order singularly perturbed boundary problem[4,5].

$$\varepsilon y'' + p(x)y' + q(x)y = r(x) \ , x \in [a,b] \quad (1)$$

with the boundary conditions

$$y(a) = l_1 \ \ and \ \ y(b) = l_2 \, , \quad (2)$$

whereϵ is a small positive parameter$0 < \varepsilon \ll 1$, α and β are given constants, $p(x)$, $q(x)$ and $r(x)$ are assumed to be sufficiently continuously differentiable functions.
Thenonpolynomial spline method[6,7,8,9,10,11]developed in this paper has lower computational cost and its only requires solving $n + 1$ linear or non-linear equations.

2. Derivation of the Method

We divide the interval $[a, b]$ into $n + 1$ equal subintervals using the point

$$x_i = a + ih, \qquad i = 0,1,2, \dots , n, n + 1,$$

with

$$a = x_0, \ b = x_{n+1} \ and \ \ h = \frac{b - a}{n + 1}$$

Where arbitrary positive integer.

Let $y(x)$ be the exact solution and $y_i$be an approximation to$y(x_i)$ obtained by the non polynomial $P_i(x)$ passing through the points (x_i, y_i) and (x_{i+1}, y_{i+1}), we do not only require that $P_i(x)$ satisfies interpolatoryconditions at x_i and x_{i+1}but also the continuity of first derivative at the common nodes (x_i, y_i) are fulfilled. We write $P_i(x)$ in the form[7,8]:

$$P_i(x) = a_i \sin\tau(x - x_i) + b_i \cos\tau(x - x_i) + c_i(x - x_i) + d_i \,, i = 0,1,2, \dots, n+1 \qquad (3)$$

where a_i, b_i, c_i and d_i are constants and τ is free parameter to be determined later .

A non-polynomial function $P(x)$ of class $C^2[a,b]$ interpolates $y(x)$ at the grid points $x_i, i = 0,1,2, \dots, n+1$ depends on a parameter τ , and reduces to ordinary spline $P(x)$ in $[a,b]$ as $\tau \to 0$.

To derive expression for the coefficient of Eq. (3) in term $y_i, y_{i+1}, D_i, D_{i+1}, S_i$ and S_{i+1}, we first define:

$$P_i(x_i) = y_i\,, \quad P_i(x_{i+1}) = y_{i+1}\,, \quad P'_i(x_i) = D_i\,, \quad P'_i(x_{i+1}) = D_{i+1}\,,$$

$$P''_i(x_i) = S_i, \; P''_i(x_{i+1}) = S_{i+1}. \qquad (4)$$

From algebraic manipulation, we get the following expression:

$$a_i = h^2\,\frac{-S_{i+1} + S_i\cos(\theta)}{\theta^2\sin(\theta)}, \quad b_i = -h^2\,\frac{S_i}{\theta^2},$$

$$c_i = \frac{y_{i+1} - y_i}{h} - \frac{h(S_{i+1} - S_i)}{\theta^2}, \qquad d_i = y_i + \frac{h^2 S_i}{\theta^2},$$

(5) where $\theta = \tau h$ and $i = 0,1,2, \dots, n$.

We applying the first derivative at (x_i, y_i), that is $P'_{i-1}(x_i) = P'_i(x_i)$, gives the following consistency relation for $i = 1, \dots n$:

$$y_{i-1} - h^2 S_{i-1}\left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right) - 2y_i - 2h^2 S_i\left(\frac{1}{\theta^2} - \frac{\cos\theta}{\theta\sin\theta}\right) + y_{i+1} - h^2 S_{i+1}\left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right) = 0 \qquad (6)$$

which can further be written as ,

$$y_{i-1} - 2y_i + y_{i+1} = h^2[\alpha(S_{i-1} + S_{i+1}) + 2\beta S_i] \qquad (7)$$

where

$$\alpha = \left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right),$$

$$\beta = \left(\frac{1}{\theta^2} - \frac{\cos\theta}{\theta\sin\theta}\right)$$

3. Truncation Error

Now the corresponding truncation error associated with (7)

$$T = y_{i-1} - 2y_i + y_{i+1} - h^2(\alpha y''_{i-1} + 2\beta y''_i + \alpha y''_{i+1})$$

Applying Taylors theorem and simplify we get

$$T = h^2(1 - 2\alpha - 2\beta)y''_i + \frac{h^2}{12}(1 - 12\alpha)y_i^{iv} + \frac{h^6}{360}(1 - 30\alpha)y_i^{vi} + \cdots \qquad , i = 1,2, \dots n$$

(8)

4. Non-Polynomial Spline Solutions

In this section, anonpolynomial spline approximation to equation(1), use $P_i(x)$ and $P_{i-1}(x)$ at the node x_i implies,

$$P_{i-1}(x_i) = \frac{y_i - y_{i-1}}{h} + hS_i\left(\frac{1}{\theta^2} - \frac{\cos\theta}{\theta\sin\theta}\right) + hS_{i-1}\left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right) \quad (9)$$

$$P_i(x_i) = \frac{y_{i+1} - y_i}{h} + h\,S_i\left(-\frac{1}{\theta} + \frac{\cos\theta}{\theta\sin\theta}\right) + hS_{i+1}\left(-\frac{1}{\theta\sin\theta} + \frac{1}{\theta^2}\right) \qquad (10)$$

where $i = 1,2, \dots, n$.

Using equation (9) and (10) in equation (1) we get

$$S_i\left(\varepsilon + h\frac{p_i \cos\theta}{\theta \sin\theta} - h\frac{p_i}{\theta^2}\right) + S_{i+1}h\left(\frac{p_i}{\theta^2} - \frac{p_i}{\theta \sin\theta}\right) = r_i - q_i y_i - \frac{p_i}{h}(y_{i+1} - y_i) \qquad (11)$$

$$S_i\left(\varepsilon + h\frac{p_i}{\theta^2} - h\frac{p_i \cos\theta}{\theta \sin\theta}\right) + S_{i-1}h\left(\frac{p_i}{\theta \sin\theta} - \frac{p_i}{\theta^2}\right) = r_i - q_i y_i - \frac{p_i}{h}(y_i - y_{i-1}) \qquad (12)$$

For $i = 1,2,\dots,n$. Addition of equation (11)and (12)we get the following equation:

$$S_{i-1}h\left(\frac{p_i}{\theta \sin\theta} - \frac{p_i}{\theta^2}\right) + 2\varepsilon S_i + S_{i+1}h\left(\frac{p_i}{\theta^2} - \frac{p_i}{\theta \sin\theta}\right) = 2(r_i - g_i y_i) - \frac{p_i}{h}(y_{i+1} - y_{i-1}) \qquad (13)$$

Elimination of S_i between equation (13)and equation(6) yields the following equation,

$$\left(hp_i - \frac{\varepsilon\theta^2 \sin\theta}{\sin\theta - \theta\cos\theta}\right)y_{i+1} + \left(-hp_i + \frac{\varepsilon\theta^2 \sin\theta}{\sin\theta - \theta\cos\theta}\right)y_{i-1} + \left(2h^2 p_i - \frac{2\theta^2 \sin\theta}{\sin\theta - \theta\cos\theta}\right)y_i + S_{i+1}\left(p_i h^3\left(\frac{1}{\theta^2} - \frac{1}{\theta \sin\theta}\right) + \varepsilon h^2\left(\frac{\sin\theta - \theta}{\sin\theta - \theta\cos\theta}\right)\right) + S_{i-1}\left(p_i h^3\left(\frac{1}{\theta \sin\theta} - \frac{1}{\theta^2}\right) + \varepsilon h^2\left(\frac{\sin\theta - \theta}{\sin\theta - \theta\cos\theta}\right)\right) - 2h^2 r_i = 0 \quad , \quad i = 1,2,\dots n \qquad (14)$$

An explicit expression can be obtained for S_{i-1}in terms y_{i-1} and y_i by eliminating S_i between equation (11) with i replaced by$i-1$ and (12) manly,

$$A_i S_{i-1} = \left(r_{i-1} - q_{i-1}y_{i-1} - \frac{p_{i-1}}{h}(y_i - y_{i-1})\right)\left(1 + p_i h\left(\frac{1}{\theta} - \frac{\cos\theta}{\theta \sin\theta}\right)\right) - \left(r_i - q_i y_i - \frac{p_i}{h}(y_i - y_{i-1})\right)\left(p_{i-1}h\left(\frac{1}{\theta} - \frac{1}{\theta \sin\theta}\right)\right) \qquad (15)$$

where

$$A_i = \left(\varepsilon + p_i h\left(\frac{1}{\theta^2} - \frac{\cos\theta}{\theta \sin\theta}\right)\right)\left(1 + p_{i-1} h\left(\frac{\cos\theta}{\theta\sin\theta} - \frac{1}{\theta^2}\right)\right) + p_i p_{i-1} h^2 \left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right)^2 \quad (16)$$

Similarly S_{i+1} can be obtain in term of y_{i+1} and y_i from equation (11) and equation (12) with replaced by $i+1$ the resulting expression being

$$B_i S_i = \left(r_i - q_{i+1}y_{i+1} - \frac{p_{i+1}}{h}(y_{i+1} - y_i)\right)\left(1 + p_i h\left(\frac{\cos\theta}{\theta\sin\theta} - \frac{1}{\theta}\right)\right) - (r_i - q_i y_i - \frac{p_i}{h}(y_{i+1} - y_i)(p_{i+1} h\left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right)) \quad (17)$$

for $i = 1,2,\dots,n$ where

$$B_i = \left(\varepsilon + p_{i+1} h\left(\frac{1}{\theta^2} - \frac{\cos\theta}{\theta\sin\theta}\right)\right)\left(1 + p_i h\left(\frac{\cos\theta}{\theta\sin\theta} - \frac{1}{\theta^2}\right)\right) + p_i p_{i+1} h^2 (\left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right)^2 \quad (18)$$

$$y_{i+1}[\left(hp_i + \frac{\varepsilon\theta^2\sin\theta}{(\sin\theta - \theta\cos\theta)}\right)A_i B_i + (C_i\left(-q_{i+1} - \frac{p_{i+1}}{h}\right) + E_i \frac{p_i p_{i+1}}{h})A_i] + y_i\left[h^2 q_i - \frac{2\varepsilon\theta^2\sin\theta}{(\sin\theta - \theta\cos\theta)}\right)A_i B_i + \left(D_i \frac{-p_{i-1}}{h} + F_i p_{i-1}\left(q_i + \frac{p_i}{h}\right)\right)B_i + \left(C_i\frac{p_{i+1}}{h} + E_i q_{i+1}\left(q_i - \frac{p_i}{h}\right)\right)A_i] + y_{i-1}\left[-p_i + \frac{\varepsilon\theta^2\sin\theta}{(\sin\theta - \theta\cos\theta)}\right)A_i B_i + (D_i\left(-q_{i-1} + \frac{p_{i-1}}{h}\right) - F_i\frac{p_i p_{i+1}}{h})B_i] + r_{i+1}(A_i C_i) + r_i[-2A_i B_i h^2 - B_i F_i p_{i+1} - A_i E_i p_{i+1}] + r_{i-1} B_i D_i = 0 \quad (19)$$

Where A_i and B_i are given by equation (16)and (17) and C_i, D_i, E_i and $F_i$are given the following:

$$C_i = \left[p_i h^3\left(\frac{1}{\theta^2} - \frac{1}{\theta\sin\theta}\right) + \varepsilon h^2\left(\frac{\sin\theta - \theta}{\sin\theta - \theta\cos\theta}\right)\right][1 + p_i h\left(\frac{\cos\theta}{\theta\sin\theta} - \frac{1}{\theta^2}\right)]$$

$$D_i = \left[p_i h^3\left(\frac{1}{\theta^2} - \frac{1}{\theta\sin\theta}\right) + \varepsilon h^2\left(\frac{\sin\theta - \theta}{\sin\theta - \theta\cos\theta}\right)\right][1 + p_i h\left(\frac{1}{\theta^2} - \frac{\cos\theta}{\theta\sin\theta}\right)]$$

$$E_i = \left[[p_i h^3\left(\frac{1}{\theta^2} - \frac{1}{\theta\sin\theta}\right) + \varepsilon h^2\left(\frac{\sin\theta - \theta}{\sin\theta - \theta\cos\theta}\right)\right][h\left(\frac{1}{\theta\sin\theta} - \frac{1}{\theta^2}\right)]$$

$$F_i = \left[p_i h^3\left(\frac{1}{\theta^2} - \frac{1}{\theta\sin\theta}\right) + \varepsilon h^2\left(\frac{\sin\theta - \theta}{\sin\theta - \theta\cos\theta}\right)\right][h\left(\frac{1}{\theta^2} - \frac{1}{\theta\sin\theta}\right)] \quad (20)$$

for $i = 1,2,\dots,n$

Formula (19) is counterpart of formula(6) when the first derivativeterm is present .

5. Numerical Results

We solve three singular perturbed problems using different values of h and ε. The numerical solutions are computed and compared with the exact solutions at grade points. All calculations are implemented by Maple 13.

Example 1[12] : Consider the following equation with constant coefficients

$$-\varepsilon y'' + y = -((cos^2(\pi x) + 2\varepsilon\pi^2 \cos(2\pi x)) , \qquad 0 \le x \le 1$$

$$y(0) = y(1) = 0,$$

the exact solution is given by

$$y(x) = \frac{exp(-\frac{1-x}{\sqrt{\varepsilon}} + exp(-x/\sqrt{\varepsilon}}{1 + exp\left(-\frac{1}{\sqrt{\varepsilon}}\right)} - cos^2(\pi x)$$

the numerical result of the example are presented in table 1and figure1for different values subinterval N and $\varepsilon = 1/64$ .Figure 2show the physical behavior of the numerical solutions for different valuesofε.

Example 2[1]:Consider the following equation with variable coefficients:

$$\varepsilon y'' + \big(1 + x(1-x)\big)y = 1 + x(1-x) + \left(2\sqrt{\varepsilon} - x^2(1-x)\right) exp\left(-\frac{1-x}{\sqrt{\varepsilon}}\right) + (2\sqrt{\varepsilon} - x(1-x)^2) exp\left(-\frac{x}{\sqrt{\varepsilon}}\right)$$

The exact solution is given by:

$$y(x) = 1 + (x-1)\, exp\left(-\frac{1}{\sqrt{\varepsilon}}\right) - x\, exp(-\frac{1-x}{\sqrt{\varepsilon}})$$

The numerical result of the example are presented in table 2 and figure 3 for different values subinterval N and $\varepsilon = 1/32$. Figure 4 show the physical behavior of the numerical solutions for different values of ε.

Example 3[1]: Consider the following equation with variable coefficients

$$\varepsilon y'' + xy' - y = -(1 + \varepsilon\pi^2)\cos(\pi x) - (\pi x)sin(\pi x)$$

$$y(-1) = -1, \qquad y(1) = 1,$$

The exact solution is given by:

$$y(x) = \cos(\pi x) + x + \frac{x\, erf\left(\frac{x}{\sqrt{2\varepsilon}}\right) + \sqrt{2\varepsilon/\pi}\, exp(-\frac{x^2}{2\varepsilon})}{erf\left(\frac{1}{\sqrt{2\varepsilon}}\right) + \sqrt{2\varepsilon/\pi}\, exp(-\frac{1}{2\varepsilon})}.$$

The numerical result of the example are presented in table 3 and figure 5 for different values subinterval $N$and $\varepsilon = 1/64$. Figure 6 show the physical behavior of the numerical solutions for different values of ε.

Table 1: Numerical solution of Example 1 at different value of subintervals.

x	Numerical Sol.				Exact Sol.
	N=16	N=32	N=64	N=128	
1/16	-0.1740613181	-0.1740535992	-0.1740522915	-0.1740517442	-0.1740519068
2/16	-0.228908954	-0.2284334136	-0.2283165751	-0.2282870599	-0.2282776718
3/16	-0.1911307967	-0.1898286086	-0.1895029921	-0.1894212971	-0.1893944408
4/16	-0.0929919425	-0.09063332439	-0.09004269872	-0.08989483097	-0.0898457280
5/16	0.03086117788	0.03431676118	0.03518261484	0.03539918112	0.0354714190
6/16	0.1474006708	0.1518063028	0.1529105397	0.1531865969	0.1532788852

7/16	0.2293560424	0.2344040967	0.2356695326	0.2359858163	0.2360916672
8/16	0.2587638902	0.2640387549	0.2653611037	0.2656915887	0.2658022288
9/16	0.2293560426	0.2344040966	0.2356695325	0.2359858164	0.2360916672
10/16	0.1474006708	0.1518063027	0.1529105395	0.1531865969	0.1532788852
11/16	0.03086117786	0.03431676129	0.03518261487	0.03539918123	0.0354714190
12/16	-0.09299194243	-0.09063332432	-0.09004269853	-0.08989483104	-0.0898457280
13/16	-0.1911307968	-0.1898286086	-0.1895029920	-0.1894212969	-0.1893944408
14/16	-0.2289028956	-0.2284334135	-0.2283165754	-0.2282870598	-0.2282776718
15/16	-0.1740613181	-0.1740535993	-0.1740522913	-0.1740517437	-0.1740519068

Table 2: Numerical solution of Example 2 at different value of subintervals.

x	*Numerical Sol.*				Exact Sol.
	N=16	**N=32**	**N=64**	**N=128**	
1/16	0.2694507465	0.2686645346	0.2684693382	0.2684203316	0.2684044067
2/16	0.4671405863	0.4659161991	0.4656121409	0.4655357779	0.4655109999
3/16	0.6108517734	0.6094094686	0.6090511884	0.6089611707	0.6089320119
4/16	0.7136795557	0.7121501159	0.7117700682	0.7116745389	0.7116436520
5/16	0.7851097856	0.7835628984	0.7831783980	0.7830816983	0.7830504955

6/16	0.8318053349	0.8302703669	0.8298887324	0.8297927183	0.8297617754
7/16	0.8581620967	0.8566433164	0.8562656400	0.8561705976	0.8561399960
8/16	0.8666777544	0.8651658059	0.8647898043	0.8646951755	0.8646647168
9/16	0.8581620967	0.8566433163	0.8562656399	0.8561705979	0.8561399960
10/16	0.8318053350	0.8302703669	0.8298887327	0.8297927184	0.8297617754
11/16	0.7851097854	0.7835628984	0.7831783978	0.7830816986	0.7830504955
12/16	0.7136795558	0.7121501157	0.7117700683	0.7116745385	0.7116436520
13/16	0.6108517732	0.6094094688	0.6090511883	0.6089611705	0.6089320119
14/16	0.4671405862	0.4659161995	0.4656121412	0.4655357776	0.4655109999
15/16	0.2694507467	0.2686645349	0.2684693378	0.2684203316	0.2684044067

Table 3: Numerical solution of Example 3 at different value of subintervals.

x	*Numerical Sol.*				Exact Sol.
	N=64	**N=128**	**N=512**	**N=1024**	
-7/8	-0.9190258015	-0.9219199438	-0.9237709378	-0.9240415171	-.9243033330
-6/8	-0.6984560039	-0.7035617854	-0.7068294729	-0.7073106180	-.7077785740
-5/8	-0.3722043319	-0.3783607642	-0.3822747958	--0.3828522232	-.3834150957
-	0.009537	0.003855	0.000364	-	-

4/8	748283	7192113	1358348	0.0001416429488	0.6318936e-3
-3/8	0.3881155740	0.3845924696	0.3827696718	0.3825442896	0.3823396261
-2/8	0.7079360848	0.7077704103	0.7085372990	0.7087543527	.7090026623
-1/8	0.9384379783	0.9408096810	0.9435055183	0.9440534564	0.9446355666
0	1.092840722	1.095498023	1.098469883	1.099075370	1.099715459
1/8	1.188481264	1.190838603	1.193508694	1.194053107	1.194635567
2/8	1.204935832	1.206883138	1.208472633	1.208735746	1.209002662
3/8	1.131158356	1.132521790	1.132618238	1.132502994	1.132339626
4/8	0.9998408258	1.000964472	1.000152356	0.9998016554	.9993681064
5/8	0.8674062236	0.8685359797	0.8674978662	0.8670873438	0.8665849039
6/8	0.7927591290	0.7938125624	0.7929781403	0.7926384601	.7922214264
7/8	0.8258581569	0.8265503836	0.8261169714	0.8259288994	0.8256966672

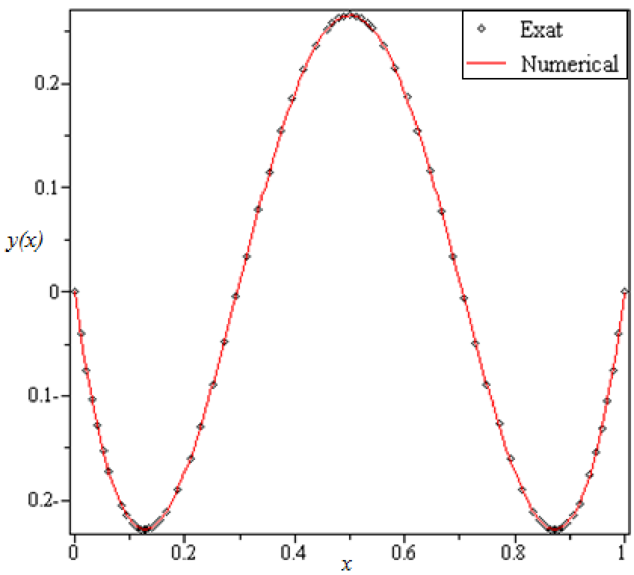

Figure 1: Comparison of exact and numerical solutions of Example 1 for $N = \varepsilon = 64$.

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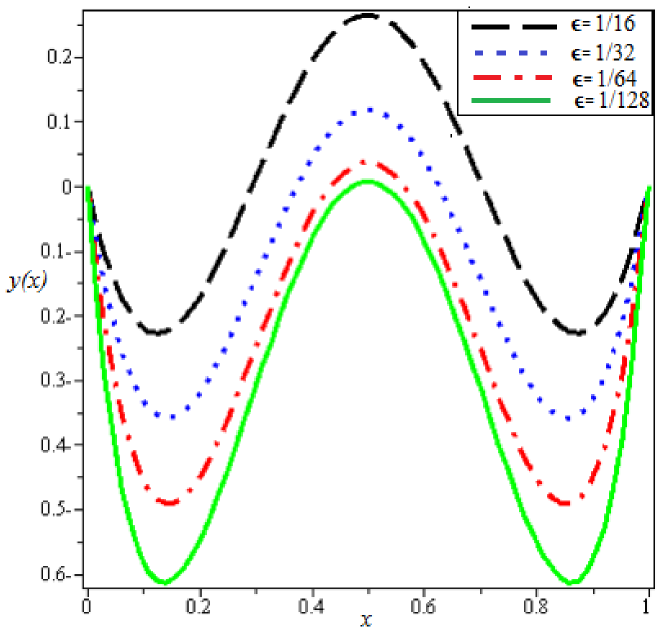

Figure 2: Numerical behavior of numerical solutions of Example 1 at different values ofε.

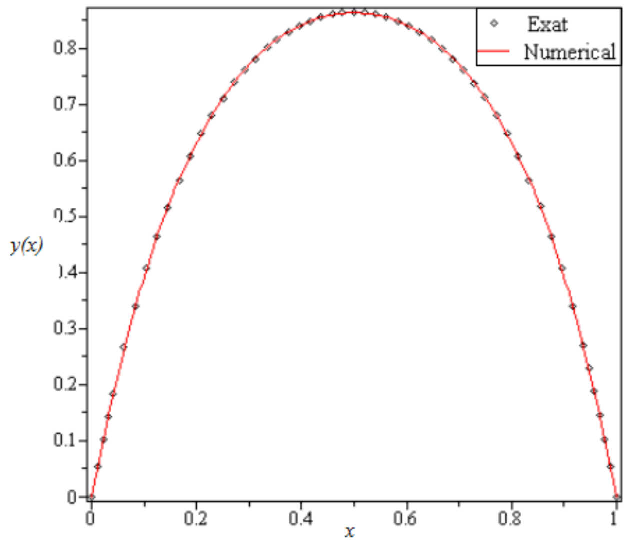

Figure 3: Comparison of exact and numerical solutions of example 2 for $N = 32$ *and* $\varepsilon = 64$.

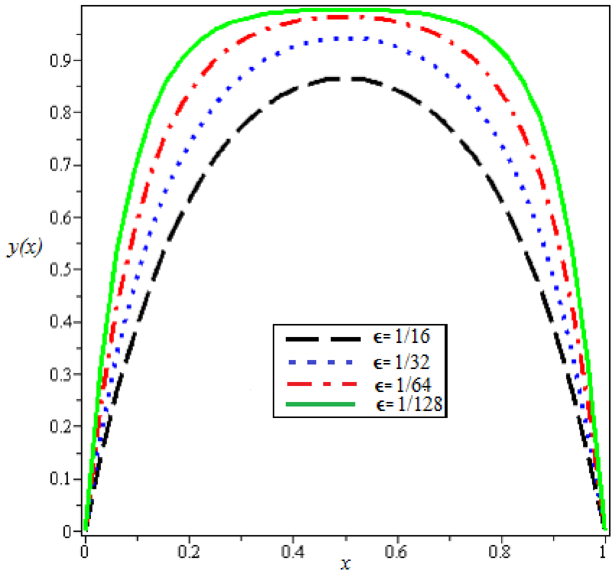

Figure 4: Numerical behavior of numerical solutions of Example 2 at different values of ε.

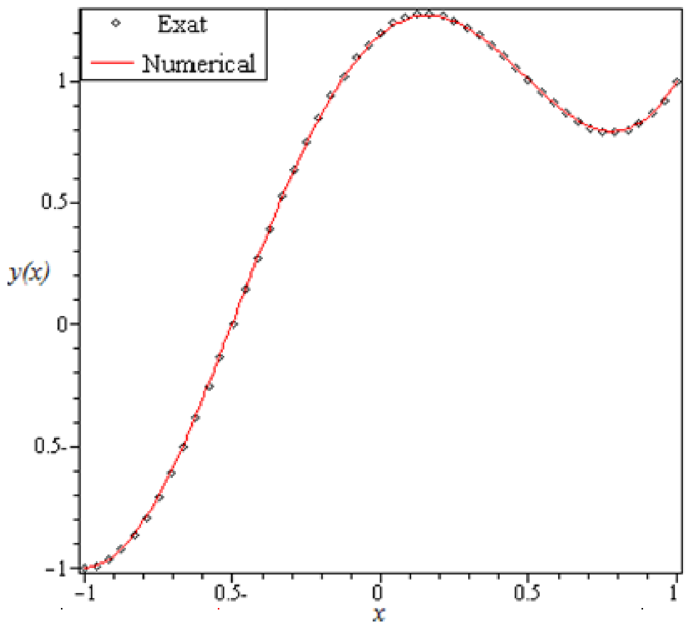

Figure 5: Comparison of exact and numerical solutions of example 3 for $N = \varepsilon = 64$.

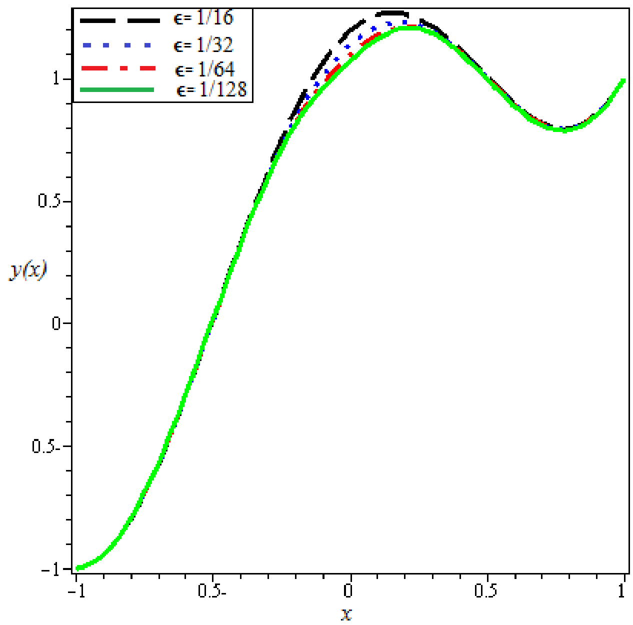

Figure 6: Numerical behavior of numerical solutions of Example 3 at different values of ε.

6. Conclusion

In this paper , a numerical technique for singularly perturbed boundary value problems using Nonpolynomial Spline functions is derived. Simplicity of the adaptation of Nonpolynomial Spline and obtaining acceptable solutions can be noted as advantages of given numerical methods. The method is tested on three problems and the results obtained are very encouraging. The method is simple and easy to apply.

7. References

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طريقة الشرائح لغير متعددات الحدود لمسائل قيم حدودية ذات الاضطراب المنفرد

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الخلاصة

في هذا البحث نقدم تطبيق طريقة الشرائح(السبلاين) لغير متعددات الحدود على مسائل قيم حدودية ذات الاضطراب المنفرد. ثم قمنا بحل ثلاث امثلة لنرى فائدة الطريقة ومدى تقاربها مع الحل الحقيقي.