On Riesz's Theorem on Intuitionistic Fuzzy Measure

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Abstract

In this paper, the concepts of the property(S), property(PS) and the converse autocontinuity from below of an intuitionistic fuzzy measure on an intuitionistic fuzzy $\sigma -$ algebra of an intuitionistic fuzzy sets will be introduced, and we proved Riesz's Theorem and three forms of Riesz'sTheorem for a sequence of measurable functions on an intuitionistic fuzzy $\sigma -$algebra.

Keywords: Riesz's Theorem, Intuitionistic Fuzzy Measure, intuitionistic fuzzy $\sigma -$algebra., Intuitionistic fuzzy sets.

1-Introduction

The concept of fuzzy measure defined on a classical $\sigma -$algebra, were first proposed by Sugenoin[1]. Some structural characteristics of fuzzy measure were introduced and discussed by Wang[2]. Ageneralization of fuzzy measure were established on fuzzy sets by Qiao[3], and the Lebesgu's theorem and Riesz's theorem for a sequence of measurable functions had been proved on fuzzy σ-algebra of fuzzy set. In 1996 ,L.Jun and M. Yasuda[4] show that the Egoroff's theorem for a sequence of fuzzy measurable functions also holds on fuzzy σ-algebra. Also they[5]introduced the concept of converse autocontinuity of set function and they discussed the relationship between the convergence in measure and the convergence pseudo in measure.

Many authors defined new types of measures ,Adrain I. Ban[6] one of the authors who defined an intuitionistic fuzzy measure on an intuitionistic fuzzy $\sigma -$ algebra $\tilde{\mathcal{A}}$on an intuitionistic fuzzy sets. The notion of intuitionistic fuzzy sets introduced by Atanassov[7] in 1983, as a generalization of the notion of fuzzy sets which introduced by Zadeh[8] in 1965.

In this paper , we will prove Riesz's theorem and three forms of this theorem for a sequence of intuitionistic fuzzy measurable functions on an intuitionistic fuzzy σ-algebra by using the concepts of property(S) , property(PS) and the converse autocontinuity from below of an intuitionistic fuzzy measure.

2- Intuitionistic fuzzy measure

In this section , we recall some definitions which will be used for this work .

Definition(2.1)[8]:

Let X be a non-empty set and let I be the closed interval [0,1] of the real line . A fuzzy set μ in X is characterized by membership function $\mu: X \longrightarrow I$, which associates with each point $x \in X$ its grade or degree of membership $\mu(x) \in [0,1]$.

Definition(2.2)[7]:

Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A is an object having the form:

A=$\{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$,where the functions $\mu_A: X \longrightarrow I$ and $v_A: X \longrightarrow I$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A, respectively, and

$0 \leq \mu_A(x) + v_A(x) \leq 1$for each $x \in X$.

Definition(2.3)[9]:

=$\{\langle x, 0,1\rangle, x \in X\}\tilde{0}$

=$\{\langle x, 1,0\rangle, x \in X\}\tilde{1}$

are the intuitionistic fuzzy sets corresponding to empty set and the entire universe respectively .

Note : Every fuzzy set A on a non-empty set X is obviously an IFS having the form $\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle, x \in X\}$.

Definition(2.4):

Let A be a subset of a set, we define the intuitionistic characteristic function of A as follows:

$$I\chi_A = \begin{cases} \tilde{1}, & if\ x \in A \\ \tilde{0}\,, & if\ x \notin A \end{cases}$$

Definition(2.5)[7,10]:

Let X be a non-empty set and let A and B are IFSs in the form

A=$\{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$,

B=$\{\langle x, \mu_B(x), v_B(x)\rangle, x \in X\}$.

Then:

1) $A \subseteq B$if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$.

2) $A = B$if and only if $A \subseteq B$ and $B \subseteq A$.

3)$A^c = \{\langle x, v_A(x), \mu_A(x)\rangle, x \in X\}$.

4)$A \cap B = \{min\{\mu_A(x), \mu_B(x)\}, .max\{v_A(x), v_B(x)\}, x \in X\}$

5) $A \cup B = \{max\{\mu_A(x), \mu_B(x)\}, .min\{v_A(x), v_B(x)\}, x \in X\}$

6)$A/B = A \cap B^c$.

Definition(2.6)[9]:

Let $\{A_i, i \in J\}$ be an arbitrary family of IFSs in X, then

1)$\cap_i A_i = \{\langle x, \wedge_i \mu_{A_i}(x)\ , \vee_i v_{A_i}(x)\rangle, x \in X\}$

2) $\bigcup_i A_i =$
$\{\langle x, \bigvee_i \mu_{A_i}(x), \bigwedge_i v_{A_i}(x)\rangle, x \in X\}$.

Definition(2.7)[(6)]:

An intuitionistic fuzzy σ-algebra (σ-field) on $X \neq \emptyset$ is a family $\tilde{\mathcal{A}}$ of IFSs in X satisfying the properties :

1) $\tilde{1} \in \tilde{\mathcal{A}}$;

2) If $A \in \tilde{\mathcal{A}}$ this implies that $A^c \in \tilde{\mathcal{A}}$;

3) If $(A_n)_{n\in N} \subseteq \tilde{\mathcal{A}}$, then $.\bigcup_{n\in N} A_n \in \tilde{\mathcal{A}}$

The pair $(X, \tilde{\mathcal{A}})$ is called an intuitionistic fuzzy measurable space.

Example(2.8)[(6)]:

LetA=$\{<x, \mu_A(x), v_A(x)>, x \in X\} \in$ IFSs . Let
$\Omega_A = \{x \in X;\ \mu_A(x) > 0\}$,
$\Lambda_A = \{x \in X;\ v_A(x) > 0\}$and
$N = \{A \in IFS(X); \Omega_A or\ \Lambda_A is\ a\ finite\ or\ countable\}$, then the family N of IFSs is an intuitionistic fuzzy σ-algebra.

Definition(2.9)[(6)]:

Let $\tilde{\mathcal{A}}$ be an IF σ-algebra in X. A function $\tilde{m}: \tilde{\mathcal{A}} \longrightarrow [0, \infty]$ is said to be an intuitionistic fuzzy measure if it satisfies the following conditions:

1) $\tilde{m}(\tilde{0}) = 0$;

2) For any $A, B \in \tilde{\mathcal{A}}$ and $A \subseteq B$ this implies that$\tilde{m}(A) \leq \tilde{m}(B)$.

The intuitionistic fuzzy measure $\tilde{m}$ is called σ-additive if $\tilde{m}(\bigcup_{n\in N} A_n) = \sum_{n\in N} \tilde{m}(A_n)$ for every sequence $(A_n)_{n\in N}$ of pairwise disjoint IFSs in $\tilde{\mathcal{A}}$.

The triple $(X, \tilde{\mathcal{A}}, \tilde{m})$ is called intuitionistic fuzzy measure space.

Definition(2.10):

The intuitionistic fuzzy measure $\tilde{m}$ is called :

1) Finite if $\tilde{m}(\tilde{1}) < \infty$, and infinite if $\tilde{m}(\tilde{1}) = \infty$.

2) Finitely additive if $\tilde{m}(A \cup B) = \tilde{m}(A) + \tilde{m}(B)$.

Example(2.11) [(6)]:

The function $\tilde{m}: \tilde{\mathcal{A}} \rightarrow [0, \infty]$ defined by

$$\tilde{m}(A) = \frac{1}{2}\sum_{x\in X}(\mu_A(x) + 1 - v_A(x))$$

for A=$\{<x, \mu_A(x), v_A(x)>, x \in X\} \in \tilde{\mathcal{A}}$, is a σ-additive intuitionistic fuzzy measure.

Definition(2.12):

Let $(X, \tilde{\mathcal{A}})$ be an intuitionistic fuzzy measurable space. An intuitionistic fuzzy measure $\tilde{m}: \tilde{\mathcal{A}} \rightarrow [0, \infty]$ is said to be:

1) Double asymptotic null- additive if $\tilde{m}(A_n \cup B_m) \longrightarrow 0$ $(n, m \rightarrow 0)$ whenever $\{A_n\} \subset \tilde{\mathcal{A}}$, $\{B_m\} \subset \tilde{\mathcal{A}}$, $\tilde{m}(A_n) \longrightarrow 0$ and $\tilde{m}(B_m) \longrightarrow 0$.

2) Have property (S) if for any $(A_n)_{n\in N} \subseteq \tilde{\mathcal{A}}$, $\lim_{n\to\infty} \tilde{m}(A_n) = 0$, there exists a subsequence $\{A_{n_k}\}$ of $\{A_n\}$ such that $\tilde{m}(\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_{n_k}) = 0$.

3) Have property (PS) if for any $(A_n)_{n\in N} \subseteq \tilde{\mathcal{A}}$ with $\lim_{n\to\infty} \tilde{m}(A_n) = 0$, there exists a subsequence $\{A_{n_k}\}$ of $\{A_n\}$ such that $.\tilde{m}(A / \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_{n_k}) = \tilde{m}(A)$

4) Converse-autocontinuous from below , if for any $A \in \tilde{\mathcal{A}}$, $\{B_n\} \subset A \cap \tilde{\mathcal{A}}$ and $\tilde{m}(B_n) \longrightarrow \tilde{m}(A)$, then $\tilde{m}(A/B_n) \longrightarrow 0$.

5) Weakly-null-countable additive if $\tilde{m}(\bigcup_{i=1}^{\infty} A_i) = 0$, whenever $A_i \in \tilde{\mathcal{A}}$ with $\tilde{m}(A_i) = 0$.

3-Main result

In this section , we introduced the definitions of the convergence almost everywhere and the convergence in measure and we proved Riesz's theorem and three forms of this theorem .

Definition(3.1):

Let $(X, \tilde{\mathcal{A}}, \tilde{m})$ be an intuitionistic fuzzy measure space and $f : X \to [0, \infty]$ be a function, we say that f is an intuitionistic fuzzy real-valued measurable function on an IF σ-algebra $\tilde{\mathcal{A}}$ if $I\chi_{F_\alpha} \in \tilde{\mathcal{A}}$, where $F_\alpha = \{x\colon f(x) \geq \alpha\}$ and

$$I\chi_{F_\alpha} = \begin{cases} \tilde{1}, & if\ x \in F_\alpha \\ \tilde{0}, & if\ x \in F_\alpha \end{cases}$$

Let $\mathcal{M}$ denoted the collection of all intuitionistic fuzzy real-valued measurable functions on $(X, \tilde{\mathcal{A}}, \tilde{m})$.

Definition(3.2):

Let $f \in \mathcal{M}$, $A \in \tilde{\mathcal{A}}$ and $\{f_n, n \geq 1\} \subseteq \mathcal{M}$ we say that:

1) $\{f_n\}$ converges to f everywhere on A and denote it by $f_n \overset{e.}{\to} f$ on A if there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ such that $\{f_n\}$ converges to f on D and $A \subseteq I\chi_D$.

2) $\{f_n\}$ converges to f almost everywhere on A and denote it by $f_n \overset{a.e.}{\longrightarrow} f$ on A if there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_D) = 0$ such that $\{f_n\}$ converges to f everywhere on $A \cap I\chi_D^c$.

3) $\{f_n\}$ converges to f pseudo-almost everywhere on A and denote it by $f_n \overset{p.a.e.}{\longrightarrow} f$ on A if there exists $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(A \cap I\chi_D^c) = \tilde{m}(A)$ and $\{f_n\}$ converges to f everywhere on $A \cap I\chi_D^c$.

Definition(3.3):

Let $f \in \mathcal{M}$, $\{f_n, n \geq 1\} \subseteq \mathcal{M}$ and $\in \tilde{\mathcal{A}}$, we say that:

1) $\{f_n\}$ converges to f in measure $\tilde{m}$ on A and denote it by $f_n \overset{\tilde{m}}{\to} f$ on A, if for any $\epsilon > 0$, when $n \longrightarrow \infty$, we have $\tilde{m}(A \cap I\chi_{\{x:|f_n(x) - f(x)| \geq \epsilon\}}) \longrightarrow 0$.

2) $\{f_n\}$ converges to f pseudo in measure $\tilde{m}$ on A and denote it by

$f_n \overset{p.\widetilde{m}}{\longrightarrow} f$ on A, if for any $\epsilon > 0$, when $n \longrightarrow \infty$, we have $\widetilde{m}\left(A \cap I\chi_{\{x:|f_n(x)-f(x)|<\epsilon\}}\right) \longrightarrow \widetilde{m}(A)$.

Theorem(3.4):

Let $\{f_n, f, n \geq 1\} \subset \mathcal{M}$; $f, g \in \mathcal{M}, A \in \tilde{\mathcal{A}}$ and $\widetilde{m}$ is weakly-null-countable additive:

1- If $f_n \overset{a.e.}{\longrightarrow} f$ and $f_n \overset{a.e.}{\longrightarrow} g$ on A, then $f = g\ a.e.$ on A.

2- If $f_n \overset{a.e.}{\longrightarrow} f$ on A and g intuitionistic fuzzy real-valued measurable function such that $f = g\ a.e.$ on A, then $f_n \overset{a.e.}{\longrightarrow} g$ on A.

3- If $f_n \overset{a.e.}{\longrightarrow} f$ on A and g_n is a sequence of an intuitionistic fuzzy real-valued measurable functions such that $f_n = g_n\ a.e.$ on A, then $g_n \overset{a.e.}{\longrightarrow} f$ on A.

Proof:

1- Since $f_n \overset{a.e.}{\longrightarrow} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\widetilde{m}(I\chi_D) = 0$ such that $f_n \overset{e.}{\to} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$ there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on H and $A \cap I\chi_D^c \subset I\chi_H$.

Since $f_n \overset{a.e.}{\longrightarrow} g$ on A, then there exists a subset $N \subseteq X$ with $I\chi_N \in \tilde{\mathcal{A}}$ and $\widetilde{m}(I\chi_N) = 0$ such that $f_n \overset{e.}{\to} g$ on $A \cap I\chi_N^c$.

$\Longrightarrow$ there exists $M \subseteq X$ with $I\chi_M \in \tilde{\mathcal{A}}$ such that f_n converges to g on M and $A \cap I\chi_N^c \subset I\chi_M$.

Let $E = I\chi_D \cup I\chi_N = I\chi_{D\cup N}$

$\Longrightarrow E \in \tilde{\mathcal{A}}$.

Since $\widetilde{m}$ is weakly-null-countable additive $\Longrightarrow \widetilde{m}(E) = 0$.

Since f_n converges to f on H

$\Longrightarrow f_n(x) \longrightarrow f(x), \forall x \in H$

and f_n converges to g on M

$\Longrightarrow f_n(x) \longrightarrow g(x), \forall x \in M$

$\Longrightarrow \forall x \in H \cap M,\ f(x) = g(x)$

$\Longrightarrow f = g$ on $H \cap M$

Since $A \cap I\chi_D^c \subset I\chi_H$ and $A \cap I\chi_N^c \subset I\chi_M$

$\Longrightarrow (A \cap I\chi_D^c) \cap (A \cap I\chi_N^c) \subset I\chi_H \cap I\chi_M$

$\Longrightarrow A \cap (I\chi_D \cup I\chi_N)^c$

$= A \cap E^c \subset I\chi_{H\cap M}$

Therefore, $f = g\ e.$ on $A \cap E^c$.

Since $D \cup N \subset X$ and $\widetilde{m}(I\chi_{D\cup N}) = 0$.

So, $f = g\ a.e.$ on A.

2- Since $f_n \overset{a.e.}{\longrightarrow} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\widetilde{m}(I\chi_D) = 0$ such that $f_n \overset{e.}{\to} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$ there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on H and $A \cap I\chi_D^c \subset I\chi_H$.

Since $f = g\ a.e.$ on A, then there exists a subset $N \subseteq X$ with $I\chi_N \in$

$\tilde{\mathcal{A}}$and $\tilde{m}(I\chi_N) = 0$ such that $f = g$ e. on$A \cap I\chi_N^c$.

$\Longrightarrow$there exists $M \subseteq X$ with $I\chi_M \in \tilde{\mathcal{A}}$ such that $f = g$ on $M$and $A \cap I\chi_N^c \subset I\chi_M$.

Let $E= I\chi_D \cup I\chi_N = I\chi_{D\cup N}$

$\Longrightarrow E \in \tilde{\mathcal{A}}$.

Since $\tilde{m}$ is weakly-null-countable additive $\Longrightarrow \tilde{m}(E) = 0$.

Since f_n converges to f on H

$\Longrightarrow f_n(x) \longrightarrow f(x), \forall x \in H$

and$f = g$ on $M \Longrightarrow f(x) = g(x), \forall x \in M$

$\Longrightarrow \forall x \in H \cap M, f_n(x) \longrightarrow g(x)$

$\Longrightarrow f_n \longrightarrow g$on $H \cap M$.

Since $A \cap I\chi_D^c \subset I\chi_H$ and

$$A \cap I\chi_N^c \subset I\chi_M$$

$\Longrightarrow (A \cap I\chi_D^c) \cap (A \cap I\chi_N^c) \subset I\chi_H \cap I\chi_M$

$\Longrightarrow A \cap E^c \subset I\chi_{H\cap M}$

Therefore ,$f_n \overset{e.}{\rightarrow} g$ on $A \cap E^c$.

Since$\tilde{m}(E) = 0$, so $f_n \overset{a.e.}{\longrightarrow} g$ on A.

3-Since $f_n \overset{a.e.}{\longrightarrow} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_D) = 0$ such that $f_n \overset{e.}{\rightarrow} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on $H$and $A \cap I\chi_D^c \subset I\chi_H$.

Since $f_n = g_n$ $a.e.$on A, then there exist a sequence $\{E_n\}$⊆X with$I\chi_{E_n} \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_{E_n}) = 0$ for all $n \geq 1$ such that $f_n = g_n$ $e.$ on $A \cap I\chi_{\cup E_n}^c$.

$\Longrightarrow$there exist $M_n \subseteq X$ with $I\chi_{M_n} \in \tilde{\mathcal{A}}$ for all $n \geq 1$ such that $f_n = g_n$ on $\bigcap_{n=1}^{\infty} M_n$and $A \cap I\chi_{\bigcup_{n=1}^{\infty} E_n}^c \subset I\chi_{\bigcap_{n=1}^{\infty} M_n}$.

Let $C= I\chi_D \cup I\chi_{\bigcup_{n=1}^{\infty} E_n}$

$$= I\chi_D \cup \left(\bigcup_{n=1}^{\infty} \chi_{E_n}\right)$$

$\Longrightarrow C \in \tilde{\mathcal{A}}$and $\quad = I\chi_{\bigcup_{n=1}^{\infty}(D\cup E_n)}$

since $\tilde{m}$ is weakly-null-countable additive $\Longrightarrow \tilde{m}(C) = 0$.

Since f_n converges to f on H

$\Longrightarrow f_n(x) \longrightarrow f(x) \forall x \in Handf_n = g_n$ on $\bigcap_{n=1}^{\infty} M_n$

$\Longrightarrow f_n(x) = g_n(x) \forall x \in \bigcap_{n=1}^{\infty} M_n$

$\Longrightarrow g_n(x) \longrightarrow f(x) \forall x \in H \cap (\bigcap_{n=1}^{\infty} M_n) = \bigcap_{n=1}^{\infty}(H \cap M_n)$

Thus, $g_n \longrightarrow f$on $\bigcap_{n=1}^{\infty}(H \cap M_n)$.

Since $A \cap I\chi_D^c \subset I\chi_H$ and

$$A \cap I\chi_{\bigcup_{n=1}^{\infty} E_n}^c \subset I\chi_{\bigcap_{n=1}^{\infty} M_n}$$

$\Longrightarrow (A \cap I\chi_D^c) \cap \left(A \cap I\chi_{\bigcup_{n=1}^{\infty} E_n}^c\right) \subset I\chi_H \cap I\chi_{\bigcap_{n=1}^{\infty} M_n}$

$\Longrightarrow A \cap C^c \subset I\chi_{\bigcap_{n=1}^{\infty}(H\cap M_n)}$.

Therefore, $g_n \overset{e.}{\rightarrow} f$on $A \cap C^c$.

Since, $\tilde{m}(C) = 0$.

So,$g_n \overset{a.e.}{\longrightarrow} f$on A.

Theorem(3.5):

Let$\{f_n, g_n, f, g, n \geq 1\} \subset \mathcal{M}$; $f, g \in \mathcal{M}$,$A \in \tilde{\mathcal{A}}$ and $\tilde{m}$ is weakly-null-countable additive, $f_n \xrightarrow{a.e.} f$ on A and $g_n \xrightarrow{a.e.} g$ on A, $c \in R$,then

1) $c \cdot f_n \xrightarrow{a.e.} c \cdot f$.

2) $f_n + g_n \xrightarrow{a.e.} f + g$.

3)$|f_n| \xrightarrow{a.e.} |f|$.

Proof:

1) Since $f_n \xrightarrow{a.e.} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_D) = 0$ such that $f_n \xrightarrow{e.} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$ there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on $H$and $A \cap I\chi_D^c \subset I\chi_H$.

Since f_n converges to f on H

$\Longrightarrow f_n(x) \longrightarrow f(x)$,$\forall x \in H$

$\Longrightarrow c \cdot f_n(x) \longrightarrow c \cdot f(x)$,$\forall x \in H$

$\Longrightarrow c \cdot f_n \longrightarrow c \cdot f$on H.

Therefore,$c \cdot f_n \xrightarrow{a.e.} c \cdot f$.

2) Since $f_n \xrightarrow{a.e.} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_D) = 0$ such that $f_n \xrightarrow{e.} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on $H$and $A \cap I\chi_D^c \subset I\chi_H$.

Since $g_n \xrightarrow{a.e.} g$on A, there exists a subset $N \subseteq X$ with $I\chi_N \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_N) = 0$ such that $g_n \xrightarrow{e.} g$ on $A \cap I\chi_N^c$.

$\Longrightarrow$ there exists $M \subseteq X$ with $I\chi_M \in \tilde{\mathcal{A}}$ such that g_n converges to g on $M$and $A \cap I\chi_N^c \subset I\chi_M$.

Let E=$I\chi_D \cup I\chi_N$, since$\tilde{m}$ is weakly-null-countable additive $\Longrightarrow \tilde{m}(E) = 0$.

Since f_n converge to f on H

$\Longrightarrow \forall x \in H$, $f_n(x) \longrightarrow f(x)andg_n$ converge to g on M

$\Longrightarrow \forall x \in M$, $g_n(x) \longrightarrow g(x)$

$\Longrightarrow \forall x \in H \cap M$, $f_n(x) + g_n(x) \longrightarrow f(x) + g(x)$.

Thus ,$f_n + g_n$ converge to $f + g$ on $H \cap M$.

Since $A \cap I\chi_D^c \subset I\chi_H$ and $A \cap I\chi_N^c \subset I\chi_M$

$\Longrightarrow (A \cap I\chi_D^c) \cap (A \cap I\chi_N^c\ ) \subset I\chi_H \cap I\chi_M = I\chi_{D \cap M}$

$\Longrightarrow A \cap E^c \subset I\chi_{H \cap M}$.

Therefore, $f_n + g_n \xrightarrow{e.} f + g$on $A \cap E^c$.

Since $\tilde{m}(E) = 0$, so on A. $f_n + g_n \xrightarrow{a.e.} f + g$

3)Since $f_n \xrightarrow{a.e.} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_D) = 0$ such that $f_n \xrightarrow{e.} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$ there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on $H$and $A \cap I\chi_D^c \subset I\chi_H$.

Since f_n converges to f on H

$\Longrightarrow f_n(x) \longrightarrow f(x)$,$\forall x \in H$

$\Longrightarrow |f_n(x)| \longrightarrow |f(x)|$,$\forall x \in H$

$\Longrightarrow |f_n| \longrightarrow |f|$on A.

Therefore, $|f_n| \overset{a.e.}{\longrightarrow} |f|$.

Theorem(3.6):

Let $\{f_n, f, n \geq 1\} \subset \mathcal{M}; f, g \in \mathcal{M} A \in \tilde{\mathcal{A}}$ and $\widetilde{m}$ is weakly-null-countable additive, then:

1) If $f_n \overset{a.e.}{\longrightarrow} f$ on $A$and $f_n \geq 0$ $a.e.$on A, then $f \geq 0$ $a.e.$ on A.

2) If $f_n \overset{a.e.}{\longrightarrow} f$ on $A$and $f_n \leq g$ $a.e.$ on A for each$n$, then $f \leq g$ $a.e.$ on A.

3) If $f_n \overset{a.e.}{\longrightarrow} f$ on $A$and $|f_n| \leq |g|$ on$A$ for each n, then $|f| \leq a.e.$ $|g|$ $a.e.$ on A.

4)If $f_n \overset{a.e.}{\longrightarrow} f$ on $A$and $f_n \leq f_{n+1}$ $a.e.$ on$A$ for each n, then $f_n \uparrow f$ $a.e.$ on A.

Proof:

1) Since $f_n \overset{a.e.}{\longrightarrow} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\widetilde{m}(I\chi_D) = 0$ such that $f_n \overset{e.}{\to} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on H and $A \cap I\chi_D^c \subset I\chi_H$.

Since $f_n \geq 0$ $a.e.$on A, then there exists a sequence $\{E_n\} \subset X$ with $I\chi_{E_n} \in \tilde{\mathcal{A}}$ and $\widetilde{m}(I\chi_{E_n}) = 0$ for all $n \geq 1$ such that $f_n \geq 0$ $e.$ on $A \cap I\chi^c_{\cup_{n=1}^{\infty} E_n}$.

$\Longrightarrow$ there exist $F_n \subseteq$X with$I\chi_{F_n} \in \tilde{\mathcal{A}}$ such that $f_n \geq 0$ on $\cap_{n=1}^{\infty} F_n$ and $A \cap I\chi^c_{\cup_{n=1}^{\infty} E_n} \subset I\chi_{\cap_{n=1}^{\infty} F_n}$.

Let $M = I\chi_D \cup I\chi_{\cup_{n=1}^{\infty} E_n}$

$= I\chi_D \cup \left(\cup_{n=1}^{\infty} I\chi_{E_n}\right)$

Since $\widetilde{m}$ is weakly-null-countable additive $\Longrightarrow \widetilde{m}(M) = 0$

Since f_n converges to f on H

$\Longrightarrow \forall x \in H, f_n(x) \longrightarrow f(x)$

and$f_n \geq 0$ on $\cap_{n=1}^{\infty} F_n$

$\Longrightarrow f_n(x) \geq 0 \forall x \in \cap_{n=1}^{\infty} F_n$

for all $x \in H \cap (\cap_{n=1}^{\infty} F_n) f(x) \geq 0$

$$= \bigcap_{n=1}^{\infty} (H \cap F_n)$$

Thus, $f \geq 0$ on $\cap_{n=1}^{\infty}(H \cap F_n)$

Since $A \cap I\chi_D^c \subset I\chi_H$ and

$$A \cap I\chi^c_{\cup_{n=1}^{\infty} E_n} \subset I\chi_{\cap_{n=1}^{\infty} F_n}$$

$\Longrightarrow (A \cap I\chi_D^c) \cap \left(A \cap I\chi^c_{\cup_{n=1}^{\infty} E_n}\right) \subset I\chi_H \cap I\chi_{\cap_{n=1}^{\infty} F_n}$

$\Longrightarrow A \cap M^c \subset I\chi_{\cap_{n=1}^{\infty}(H \cap F_n)}$

Therefore, $f \geq 0$ $e.$ on $A \cap M^c$.

Since$\widetilde{m}(M) = 0$, so $f \geq 0$ $a.e.$ on A.

2) since $f_n \leq g$ $a.e.$ on A

$\Longrightarrow g - f_n \geq 0$ $a.e.$

Since $f_n \overset{a.e.}{\longrightarrow} f$on A

$\Longrightarrow g - f_n \overset{a.e.}{\longrightarrow} g - f$onA.

From (1) $g - f \geq 0$ a. e. on A

$\Longrightarrow f \leq g$ $a.e.$on A.

3) Since$f_n \overset{a.e.}{\longrightarrow} f$on A $\Longrightarrow |f_n| \overset{a.e.}{\longrightarrow} |f|$ on.

Since $|f_n| \leq |g|$ $a.e.$ on A

$\Longrightarrow$from (2) $|f| \leq |g|$ $a.e.$ on A.

4) Since $f_n \overset{a.e.}{\longrightarrow} f$ on A, then there exists a subset $D \subseteq X$ with $I\chi_D \in$

$\tilde{\mathcal{A}}$and $\tilde{m}(I\chi_D) = 0$ such that $f_n \overset{e.}{\to} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$ there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on $H$and $A \cap I\chi_D^c \subset I\chi_H$.

Since $f_n \leq f_{n+1}$ $a.e.$ on A, then there exist a sequence $\{M_n\} \subset X$ with $I\chi_{M_n} \in \tilde{\mathcal{A}}$and $\tilde{m}\left(I\chi_{M_n}\right) = 0$ $\forall n \geq 1$ and $f_n \leq f_{n+1}$ $e.$ on $A \cap I\chi_{\cup M_n}^c$.

$\Longrightarrow$ there exists a sequence $\{F_n\} \subset X$ with $I\chi_{F_n} \in \tilde{\mathcal{A}}$ for all $n \geq 1$ and $f_n \leq f_{n+1}$ on $\bigcap_{n=1}^{\infty} F_n$ and $A \cap I\chi_{\bigcup_{n=1}^{\infty} M_n}^c \subset I\chi_{\bigcap_{n=1}^{\infty} F_n}$.

Let $C = I\chi_D \cup \left(\cup I\chi_{M_n}\right)$

. $= I\chi_{\bigcup_{n=1}^{\infty}(D \cup M_n)}$

Since $\tilde{m}$ is weakly-null-countable additive $\Longrightarrow \tilde{m}(C) = 0$.

Since f_n converges to f on H

$\Longrightarrow f_n(x) \to f(x) \forall x \in H$and since $f_n \leq f_{n+1}$ $\forall n$ on $\cap F_n$

$\Longrightarrow f_n(x) \leq f_{n+1}(x) \forall x \in \cap F_n$,

This implies that $\forall x \in H \cap (\bigcap_{n=1}^{\infty} F_n)$, $f_n(x) \uparrow f(x)$.

Therefore,$f_n \uparrow f$ on $H \cap (\bigcap_{n=1}^{\infty} F_n)$ $= \bigcap_{n=1}^{\infty}(H \cap F_n)$.

Since $A \cap I\chi_D^c \subset I\chi_H$ and

$$A \cap I\chi_{\cup M_n}^c \subset I\chi_{\bigcap_{n=1}^{\infty} F_n}$$

This implies that $A \cap C^c \subset I\chi_{H \cap (\bigcap_{n=1}^{\infty} F_n)}$ and therefore, on $A \cap C^c . f_n \uparrow f$ $e.$

Since,$\tilde{m}(C) = 0$,

so$f_n \uparrow f$ $a.e.$ on A.

Theorem(3.7):

Let$\{f_n, g_n, f, g, n \geq 1\} \subset \mathcal{M}$, $A \in \tilde{\mathcal{A}}$ and $\tilde{m}$ is weakly-null-countable additive, then:

1) If $f_n \overset{a.e.}{\longrightarrow} f$ on A, $g_n \overset{a.e.}{\longrightarrow} g$ on A and $f_n = g_n$ $a.e.$ for all n, then $.f = g$ $a.e.$

2) If $f_n \overset{a.e.}{\longrightarrow} f$on A, $f_n = g_n$ $a.e.$ for all n and $f = g$ $a.e.$, then onA. $g_n \overset{a.e.}{\longrightarrow} g$

Proof:

1) Since $f_n \overset{a.e.}{\longrightarrow} f$ on A, there exists a subset $D \subseteq X$ with $I\chi_D \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_D) = 0$ such that $f_n \overset{e.}{\to} f$ on $A \cap I\chi_D^c$.

$\Longrightarrow$ there exists $H \subseteq X$ with $I\chi_H \in \tilde{\mathcal{A}}$ such that f_n converges to f on H and $A \cap I\chi_D^c \subset I\chi_H$.

Since $g_n \overset{a.e.}{\longrightarrow} g$ on A, there exists a subset $N \subseteq X$ with $I\chi_N \in \tilde{\mathcal{A}}$ and $\tilde{m}(I\chi_N) = 0$ such that $g_n \overset{e.}{\to} g$ on $A \cap I\chi_N^c$.

$\Longrightarrow$ there exists $M \subseteq X$ with $I\chi_M \in \tilde{\mathcal{A}}$ such that g_n converges to g on $M$and $A \cap I\chi_N^c \subset I\chi_M$.

Since $f_n = g_n$ $a.e.$on A, then there exists a sequence $\{E_n\}$⊆X with $I\chi_{E_n} \in \tilde{\mathcal{A}}$ and $\tilde{m}\left(I\chi_{E_n}\right) = 0$ for all $n \geq 1$ such that $f_n = g_n$ $e.$ on $A \cap I\chi_{\bigcup_{n=1}^{\infty} E_n}^c$.

$\Rightarrow$there exist $F_n \subseteq X$ with $I\chi_{F_n} \in \tilde{\mathcal{A}}$ for all $n \geq 1$ such that $f_n = g_n$ on $\bigcap_{n=1}^{\infty} F_n$and $A \cap I\chi^c_{\bigcup_{n=1}^{\infty} E_n} \subset I\chi_{\bigcap_{n=1}^{\infty} F_n}$.

Let$B = I\chi_D \cup I\chi_N \cup \left(\bigcup_{n=1}^{\infty} I\chi_{E_n}\right)$.

Since $\tilde{m}$ is weakly-null-countable additive $\Rightarrow \tilde{m}(B) = 0$.

Since f_n converges to f on H

$\Rightarrow f_n(x) \to f(x) \forall x \in H$,

converges to g on M g_n

$\Rightarrow \forall x \in M, g_n(x) \longrightarrow g(x)$and since $f_n = g_n$ on $\bigcap_{n=1}^{\infty} F_n$

$\Rightarrow \forall x \in \bigcap_{n=1}^{\infty} F_n$, $f_n(x) = g_n(x)$.

This implies that $\forall x \in H \cap M \cap (\bigcap_{n=1}^{\infty} F_n)$, $f(x) = g(x)$.

So $f = g$ on$H \cap M \cap (\bigcap_{n=1}^{\infty} F_n)$.

Since $A \cap I\chi_D^c \subset I\chi_H$, $A \cap I\chi_N^c \subset I\chi_M$ and $A \cap I\chi^c_{\bigcup_{n=1}^{\infty} E_n} \subset I\chi_{\bigcap_{n=1}^{\infty} F_n}$

$\Rightarrow A \cap B^c \subset I\chi_{H \cap M \cap (\bigcap_{n=1}^{\infty} F_n)}$and since $\tilde{m}(B) = 0$.

Therefore, $f = g$ $a.e.$.

2)It is similar to proof (1).

Theorem(3.8):

Let $\{f_n, g_n, f, g, n \geq 1\} \subset \mathcal{M}$, $A \in \tilde{\mathcal{A}}$ and $\tilde{m}$ isa double asymptotic null-additive , and $c \in R$, $f_n \xrightarrow{\tilde{m}} f$, $g_n \xrightarrow{\tilde{m}} g$, on A then:

1)$c \cdot f_n \xrightarrow{\tilde{m}} c \cdot f$on A.

2)$f_n + g_n \xrightarrow{\tilde{m}} f + g$on A.

3)$|f_n| \xrightarrow{\tilde{m}} |f|$on A.

Proof:

1) If $c = 0$, the proof is trivial.

If $c \neq 0$,let $c > 0$.

Since $f_n \xrightarrow{\tilde{m}} f$ on A

$\Rightarrow \tilde{m}\left(A \cap I\chi_{\{x:|f_n(x) - f(x)| \geq \epsilon\}}\right) \longrightarrow 0$and since

$$\{x: |c \cdot f_n(x) - c \cdot f(x)| \geq \epsilon\} = \left\{x: |f_n(x) - f(x)| \geq \frac{\epsilon}{|c|}\right\}$$

$\Rightarrow A \cap I\chi_{\{x:|c \cdot f_n(x) - c \cdot f(x)| \geq \epsilon\}}$

$= A \cap I\chi_{\left\{x:|f_n(x) - f(x)| \geq \frac{\epsilon}{|c|}\right\}}$

This implies that $\tilde{m}(A \cap I\chi_{\{x:|c \cdot f_n(x) - c \cdot f(x)| \geq \epsilon\}}) \longrightarrow 0$

So, $c \cdot f_n \xrightarrow{\tilde{m}} c \cdot f$ on A.

2) Since$\left|\left(f_n(x) + g_n(x)\right) - \left(f(x) + g(x)\right)\right| \leq |f_n(x) - f(x)| + |g_n(x) - g(x)|$

This implies that

$\left\{x: \left|\left(f_n(x) + g_n(x)\right) - \left(f(x) + \right.\right.\right.$

$\subseteq \left\{x: |f_n(x) - f(x)| \geq g(x)\right)\Big| \geq \epsilon\Big\}$

$\left.\frac{\epsilon}{2}\right\} \cup \left\{x: |g_n(x) - g(x)| \geq \frac{\epsilon}{2}\right\}$

This implies that

$$A \cap I\chi_{\{x:|(f_n(x) + g_n(x)) - (f(x) + g(x))| \geq \epsilon\}}$$

$$\subseteq A \cap I\chi_{\left\{x:|f_n(x) - f(x)| \geq \frac{\epsilon}{2}\right\}} \cup A \cap I\chi_{\left\{x:|g_n(x) - g(x)| \geq \frac{\epsilon}{2}\right\}}$$

Since $f_n \xrightarrow{\tilde{m}} f$, $g_n \xrightarrow{\tilde{m}} g$ on A and$\tilde{m}$ is a double asymptotic null- additive.

Thus, $\widetilde{m}\left(A \cap I\chi_{\left\{x:\left|\begin{matrix}(f_n(x)+g_n(x))- \\ (f(x)+g(x))\end{matrix}\right|\geq\epsilon\right\}}\right) \to 0$

Therefore, $f_n + g_n \overset{\widetilde{m}}{\to} f + g$ on A.

3) since $\big||f_n(x)| - |f(x)|\big| \leq |f_n(x) - f(x)|$ this implies that

$\{x: \big||f_n(x)| - |f(x)|\big| \geq$

$\subseteq \{x: |f_n(x) - f(x)| \geq \epsilon\}\epsilon\}$

$\subseteq A \cap A \cap I\chi_{\{x:||f_n(x)|-|f(x)||\geq\epsilon\}}$

$I\chi_{\{x:|f_n(x)-f(x)|\geq\epsilon\}}$

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{\{x:||f_n(x)|-|f(x)||\geq\epsilon\}}\right) \leq$

$\widetilde{m}(A \cap I\chi_{\{x:|f_n(x)-f(x)|\geq\epsilon\}})$

Since $f_n \overset{\widetilde{m}}{\to} f$ on A, so $|f_n| \overset{\widetilde{m}}{\to} |f|$ on A.

Theorem(3.9):

Let $\{f_n\}_{n\geq1} \subset \mathcal{M}, f \in \mathcal{M}$, $A \in \tilde{\mathcal{A}}$ and $A \cap A^c = \tilde{0}$ for every $A \in \tilde{\mathcal{A}}$, then:

1) If $\widetilde{m}$ has property(S) and $f_n \overset{\widetilde{m}}{\to} f$ on A, then there exists a subsequence $\{f_{n_i}\}$ of $\{f_n\}$ such that $f_{n_i} \overset{a.e.}{\longrightarrow} f$ on A.

2) If $\widetilde{m}$ has property(PS) and $f_n \overset{\widetilde{m}}{\to} f$ on A, then there exists a subsequence $\{f_{n_i}\}$ of $\{f_n\}$ such that $f_{n_i} \overset{p.a.e.}{\longrightarrow} f$ on A.

3) If $\widetilde{m}$ is a converse-autocontinuous from below, has property(S) and $f_n \overset{p.\widetilde{m}}{\longrightarrow} f$ on A, then there exists a subsequence $\{f_{n_i}\}$ of $\{f_n\}$ such that $f_{n_i} \overset{a.e.}{\longrightarrow} f$ on A.

4) If $\widetilde{m}$ is a converse-autocontinuous from below, has property(PS) and $f_n \overset{p.\widetilde{m}}{\longrightarrow} f$ on A, then there exists a subsequence $\{f_{n_i}\}$ of $\{f_n\}$ such that $f_{n_i} \overset{p.a.e.}{\longrightarrow} f$ on A.

Proof :

1) Since $f_n \overset{\widetilde{m}}{\to} f$ on A

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{\{x:|f_n(x)-f(x)|\geq\frac{1}{k}\}}\right) \longrightarrow 0$ for any $k \geq 1$.

Let $E_n^k = \left\{x : |f_n(x) - f(x)| \geq \frac{1}{k}\right\}$

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{E_n^k}\right) \longrightarrow 0.$

Then there exists a subsequence $\{n_k\}$ such that

for any $k \geq 1$, then $\widetilde{m}\left(A \cap I\chi_{E_{n_k}^k}\right) \leq \frac{1}{k}$

$\widetilde{m}\left(A \cap I\chi_{E_{n_k}^k}\right) \longrightarrow 0$ as $k \to 0$.

By using property(S), there exists a subsequence $\left\{A \cap I\chi_{E_{n_{k_i}}^{k_i}}\right\}$ of a sequence $\left\{A \cap I\chi_{E_{n_k}^k}\right\}$ such that

$$\widetilde{m}\left(\cap\cup\left(A \cap I\chi_{E_{n_{k_i}}^{k_i}}\right)\right) = 0$$

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{\cap\cup E_{n_{k_i}}^{k_i}}\right) = 0.$

Let $D = \cap\cup\cap\left\{x : \left|f_{n_{k_i}}(x) - f(x)\right| \leq \frac{1}{k_i}\right\}$.

Since $I\chi^c_{\cap\cup E_{n_{k_i}}^{k_i}} \subseteq I\chi_D$ and $\left\{f_{n_{k_i}}\right\}$ converges to f on D.

$\Rightarrow A \cap I\chi^{c}_{\cap\cup E^{k_i}_{n_{k_i}}} \subseteq I\chi_D$.

Therefore, $f_{n_{k_i}} \xrightarrow{a.e.} f$ on A.

2) Since $f_n \overset{\widetilde{m}}{\rightarrow} f$ on A

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{\{x:|f_n(x)-f(x)|\geq\frac{1}{k}\}}\right) \longrightarrow 0$for any $k \geq 1$.

Let $E^k_n = \left\{x : |f_n(x) - f(x)| \geq \frac{1}{k}\right\}$

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{E^k_n}\right) \longrightarrow 0$.

Then there exists a subsequence $\{n_k\}$ such that

for any $k \geq 1$, then $\widetilde{m}\left(A \cap I\chi_{E^k_{n_k}}\right) \leq \frac{1}{k}$

$\widetilde{m}\left(A \cap I\chi_{E^k_{n_k}}\right) \longrightarrow 0$ as $k \rightarrow 0$.

By using property(PS), there exists a subsequence $\left\{A \cap I\chi_{E^{k_i}_{n_{k_i}}}\right\}$ of a sequence $\left\{A \cap I\chi_{E^k_{n_k}}\right\}$ such that

$$\widetilde{m}\left(A/\cap\cup\left(A \cap I\chi_{E^{k_i}_{n_{k_i}}}\right)\right) = \widetilde{m}(A)$$

$\Rightarrow \widetilde{m}\left(A \cap I\chi^{c}_{\cap\cup E^{k_i}_{n_{k_i}}}\right) = \widetilde{m}(A)$

Let $= \cap\cup\cap\left\{x : \left|f_{n_{k_i}}(x) - f(x)\right| \leq \frac{1}{k_i}\right\}$.

$\Rightarrow A \cap I\chi^{c}_{\cap\cup E^{k_i}_{n_{k_i}}} \subseteq I\chi_D$and since $\left\{f_{n_{k_i}}\right\}$ converges to f on D.

Therefore, $f_{n_{k_i}} \xrightarrow{p.a.e.} f$ on A.

3) Since $f_n \xrightarrow{p.\widetilde{m}} f$ on A

$\Rightarrow \widetilde{m}\left(A \cap I\chi_{\{x:|f_n(x)-f(x)|<\epsilon\}}\right)$

for $\geq 1.\longrightarrow \widetilde{m}(A)$

By using the converse-autocontinuity from below of$\widetilde{m}$, we have $\widetilde{m}\left(A/\left(A \cap I\chi_{\{x:|f_n(x)-f(x)|<\epsilon\}}\right)\right) \longrightarrow$, this implies that0

$$\widetilde{m}\left(A \cap I\chi_{\{x:|f_n(x)-f(x)|\geq\epsilon\}}\right) \longrightarrow 0$$

This shows that $f_n \overset{\widetilde{m}}{\rightarrow} f$ on.

From (1) there exists a subsequence $\{f_{n_i}\}$ of $\{f_n\}$ such that $f_{n_i} \xrightarrow{a.e.} f$ on A.

4) Since $f_n \xrightarrow{p.\widetilde{m}} f$ on A and as in proof (3), by using the converse-autocontinuity from below, we have onA. $f_n \overset{\widetilde{m}}{\rightarrow} f$

From (2), there exists a subsequence $\{f_{n_i}\}$ of $\{f_n\}$ such that $f_{n_i} \xrightarrow{p.a.e.} f$on A.

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حول مبرهنة رايز في القياس الضبابي الحدسي

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الخلاصة

في هذا البحث، قُدّمت الخصائص الاتية: الخاصية (S) ، الخاصية (PS)، converse autocontinuityمن الاسفل للقياس الضبابي الحدسي على المجوعات الضبابية الحدسية وبرهنا مبرهنة رايز وثلاثة صيغ لهذه المبرهنة لمتتابعة الدوال الضبابية الحدسية القابلة للقياس.