Decomposition Matrix of the Projective Characters of S_{19} modulo 7

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Abstract

The main purpose of this paper is to compute the decomposition matrix of the projective characters of the symmetric group S_{19} modulo 7, by using the method of $(r,\bar{r})$ −inducing.

1. Introduction

A group representationof a group$G$over a field $K$is a homomorphism $T: G \to GL(n,K)$, if the characteristic of$K$is zero then the representation is called ordinary representation, and if Khas finite characteristic then the representation is called modular representation[1].The ordinary character of a representation $T$is the trace of $T(g)$, $g \in G$. The irreducible ordinary characters of the symmetric group S_n are labeled by partitions of n [2].The covering group of a group G is a group$\bar{G}$ if and only if there is a homomorphism from $\bar{G}$ to Gsuch that the kernel of this homomorphism is contained in the center of $\bar{G}$[3]. Schur[4] showed that the symmetric group S_n has a representation group $\overline{S_n}$ and showed that$\overline{S_n}$ has a central subgroup $Z = \{-1,1\}$ such that $\overline{S_n}/Z \cong S_n$[4],the representations of $\overline{S_n}$fall into two classes:

1) The representations which have Zin their kernel; these are called the representations of S_n.
2) The representations which do not have Z in their kernel; these representations are called the projective (spin) representations ofS_n.

The projective (spin) characters of the projective (spin) representations of S_n are labeled by the distinct parts of the partitions of n [5] .The spin characters of the symmetric group S_n are distributed into blocks [5].The decomposition matrix for the spin characters of the symmetric group S_n is the relationships between the irreducible spin characters and the irreducible modular spin characters of S_n. There is a method called $(r,\bar{r})$ −inducing [5] to generate principle spin characters of the symmetric group S_n from the principle spin characters of the symmetric group S_{n-1} . It is more difficult to find the decomposition matrix of the spin characters for S_n than finding the decomposition matrix for the ordinary characters of S_n; because of the little known information about the spin characters and coefficients of their decomposition matrix as compared with theorems about ordinary characters of S_n and the coefficients of their decomposition matrix, for example the form of the decomposition matrix of ordinary characters of S_n is known for all primes [2],but the form of the decomposition matrix for spin characters is not known in general.There is no general method for finding the decomposition matrix for spin characters of S_n, and for $n$increase the problem is complicated.The decomposition matrices to spin characters of S_n for a prime number $p = 7, \forall\ 7 \le n \le 13$ are found by Yaseen[6], for $n = 14$ is found by Yaseen and Taban[7], and for n= 16, 17, 18 are found by Taban[8,9,10] .

The main object of this paper is to determine the decomposition matrices for the spin characters of S_{19} modulo 7.

2. Spin Blocks of S_{19}

The group $\overline{S_{19}}$ has 59 (7, α)-regular classes,so there are fifty nine irreducible modular spin characters (i.m.s.) to S_{19} [3]. There are eleven 7-blocks of S_{19} [11] five of these blocks of defect 0are <13,6> , <13,6>' , <10,6,3>* , <10,5,3,1> , <10,5,3,1>'and these characters are i.m.s.have the degrees 2558976, 2558976, 891376646, 72930816, 72930816 respectively[5].The blocks B_6, B_5, B_4 of defect 1[12] and the other spin blocks of S_{19} are B_3, B_2, B_1 of defect 2[12].

The block B_6 has 7-bar core <6,4,2>and contains the characters<13,4,2>*,<11,6,2>* , <9,6,4>* , <7,6,4,2> , <7,6,4,2>'.The block B_5 has 7-bar core <8,3,1> and contains the characters <15,3,1>*, <10,8,1>* , <8,7,3,1> , <8,7,3,1>' , <8,5,3,2,1>*.The block B_4 contains the characters <16,2,1>* , <9,8,2>*, <9,7,2,1> , <9,7,2,1>' , <9,4,3,2,1>* with 7-bar core <9,2,1>.The block B_3 contains the characters <17,2> , <17,2>' , <16,3> , <16,3>' , <10,7,2>* , <9,7,3>* , <10,6,2,1> , <10,6,2,1>' , <9,6,3,1> , <9,6,3,1>' , <10,4,3,2> , <10,4,3,2>' , <9,5,3,2> , <9,5,3,2>' , <14,3,2>* , <13,3,2,1> , <13,3,2,1>' , <8,6,3,2> , <8,6,3,2>' , <7,6,3,2,1>* , <10,9> , <10,9>' with 7-bar core <3,2> .

And the block B_2 contains the characters<9,5,4,1> , <9,5,4,1>' , <8,5,4,2> , <8,5,4,2>' , <12,4,2,1> , <12,4,2,1>' , <11,5,2,1> , <11,5,2,1>', <11,4,3,1> , <11,4,3,1>' , <8,6,4,1> , <8,6,4,1>' , <18,1> , <18,1>' , <15,4>, <15,4>' , <14,4,1>* , <11,8> , <11,8>' , <11,7,1>* , <8,7,4>* , <7,5,4,2,1>* with 7-bar core <4,1> .Finallythe characters <19>*, <14,5> , <14,5>' , <13,5,1>* , <12,7> , <12,7>' , <12,6,1>* , <12,5,2>* , <12,4,3>* , <11,5,3>* , <10,5,4>* , <8,6,5>* , <7,6,5,1> , <7,6,5,1>' , <7,5,4,3> , <7,5,4,3>' , <6,5,4,3,1>* are in the principal spinblock B_1 with 7-bar core <5> .

3. Spin Blocks (of defect 1) B_4, B_5, B_6

All the irreducible modular spin characters in the decomposition matrices for the spin blocks B_4 , B_5 , B_6 are self-associate[11], $<\lambda> = <\lambda>'$ on (7, α)-regular classes.Then by[12] on a blocks of defect 1and the 7-decomposition matrix for S_{18} we can find the decomposition matrices for the blocks B_4, B_5, B_6.

<u>Lemma (3.1)</u>

The Brauer tree for the block B_4 is :

<16,2,1>* ——<9,8,2>* ——<9,7,2,1> = <9,7,2,1>' ——<9,4,3,2,1>*

<u>Proof</u> :Sincedeg<16,2,1>* ≡deg{<9,7,2,1> + <9,7,2,1>'} ≡ 14

deg<9,8,2>* ≡deg<9,4,3,2,1>* ≡ –14

By (2,6)- inducing of p.i.s. D_{31}, D_{33}, D_{35} of S_{18} (see the Appendix) to S_{19} gives

<16,2,1>* + <9,8,2>* = C_1

<9,8,2>* + <9,7,2,1> + <9,7,2,1>' = C_2

<9,7,2,1> + <9,7,2,1>' + <9,4,3,2,1>* = C_3

respectively

And since on (7, α)-regular classes:

1. <9,7,2,1> = <9,7,2,1>'
2. <9,7,2,1> = <9,8,2>* + <9,4,3,2,1>* – <16,2,1>*

which are given the Brauer tree of the block B_4

<u>Lemma (3.2)</u>

The Brauer tree for the block B_5 is :

<15,3,1>*——<10,8,1>*——<8,7,3,1> = <8,7,3,1>'——<8,5,3,2,1>*

Proof *:* Sincedeg<15,3,1>* ≡deg{<8,7,3,1> + <8,7,3,1>'} ≡ 7

deg<10,8,1>* ≡deg<8,5,3,2,1>* ≡ –7

By (3,5)-inducing of p.i.s. D_{31}, D_{33}, D_{35} of S_{18} (see the Appendix) to S_{19} gives

<15,3,1>* + <10,8,1>* = C_1

<10,8,1>* + <8,7,3,1> + <8,7,3,1>' = C_2

<8,7,3,1> + <8,7,3,1>' + <8,5,3,2,1>* = C_3

respectively

And since on (7, α)-regular classes :

1. <8,7,3,1> = <8,7,3,1>'
2. <8,7,3,1> = <10,8,1>* + <8,5,3,2,1>* – <15,3,1>*

which are given the Brauer tree of the block B_5

Lemma (3.3)

The Brauer tree for the block B_6 is :

<13,4,2>*——<11,6,2>*——<9,6,4>*——<7, 6,4,2> = <7,6,4,2>'

Proof *:* Since deg<13,4,2>* ≡deg<9,6,4>*≡ 21

deg<11,6,2>* ≡deg{<7,6,4,2> + <7,6,4,2>'} ≡ –21

By (4,4)-inducing of p.i.s. D_{40} & (6,2)-inducing of p.i.s. D_{13}, D_{15} of S_{18} (see the Appendix) to S_{19} gives

1. $D_1\uparrow^{(5,3)} S_{19} = C_1$
2. $D_{37}\uparrow^{(0,1)} S_{19} = C_2$
3. $D_{46}\uparrow^{(6,2)} S_{19} = C_3$
4. $D_{38}\uparrow^{(0,1)} S_{19} = C_4$
5. $D_{11}\uparrow^{(5,3)} S_{19} = C_5$
6. $D_7\uparrow^{(5,3)} S_{19} = 2C_6$
7. $D_{49}\uparrow^{(4,4)} S_{19} = C_7$
8. $D_{15}\uparrow^{(5,3)} S_{19} = C_8$
9. $D_{17}\uparrow^{(5,3)} S_{19} = C_9$

From [(2)],we can divide $2C_6$ by 2 to get C_6 .

Now we will discuss the case*:* $C_7 \not\subset C_5$

Suppose $C_7 \subset C_5$

But $(C_5 - C_7)\downarrow_{(5,3)} S_{18} =$

$D_7 + D_8 + D_{11} + D_{12} - D_{13} - D_{14}$

(see the Appendix)

is not p.s. for S_{18}

$\therefore C_7 \not\subset C_5$

<13,4,2>* + <11,6,2>* = C_1

<11,6,2>* + <9,6,4>* = C_2

2<9,6,4>* + 2<7,6,4,2> + 2<7,6,4,2>' = $2C_3$

respectively .

From [(2)]we can divide $2C_3$ by 2 to get C_3

And since on (7,α)-regular classes :

1. <7,6,4,2> = <7,6,4,2>'
2. <7,6,4,2> = <9,6,4>* + <13,4,2>* – <11,6,2>*

which are given the Brauer tree of the block B_6

4. Principal SpinBlock B_1

All the irreducible modular spin characters in the decomposition matrix forthe principal spin block B_1 are self-associate , $<\lambda>=<\lambda>'$ on (7, α)-regular classes. Then by using the 7-decomposition matrix for S_{18} we can find the decomposition matrix for the block B_1 .

Lemma (4.1)

The decomposition matrix for the spin block B_1 is $D^{(1)}_{19,7}$.

Proof *:* Using $(r,\bar{r})$ -inducing of p.i.s. for S_{18} (see the Appendix) to S_{19} gives

Nowon (7, α)-regular classes :

1. <14,5> = <14,5>'
2. <12,7> = <12,7>'
3. <7,6,5,1> = <7,6,5,1>'
4. <7,5,4,3> = <7,5,4,3>'
5. <19>* = <12,6,1>* + <12,4,3>* – <12,5,2>* – <12,7>
6. <14,5> = <12,7> + <7,5,4,3> – <7,6,5,1>
7. <10,5,4>* = <11,5,3>* + <7,5,4,3> – <6,5,4,3,1>* – <12,4,3>*
8. <13,5,1>* = <12,6,1>* + <6,5,4,3,1>* + <7,6,5,1> – <8,6,5>*

The approximation matrix $R^{(1)}_{19,7}$ contains at most 17 columns since the number of the i.m.s. is equal or less than the numberof the spin characters.

$\therefore D^{(1)}_{19,7} = R^{(1)}_{19,7}$

1. $D_{28}\uparrow^{(5,3)} S_{19} = K_1$
2. $D_{20}\uparrow^{(2,6)} S_{19} = K_2$
3. $D_{40}\uparrow^{(0,1)} S_{19} = C_5$
4. $D_{41}\uparrow^{(0,1)} S_{19} = C_6$
5. $D_{22}\uparrow^{(2,6)} S_{19} = K_3$
6. $D_{23}\uparrow^{(2,6)} S_{19} = K_4$

From [2]we can divide $2K_6$ by 2to get K_6.

Now on (7, α)-regular classes :

1. <10,7,2>* =<17,2> + <17,2>' + <10,6,2,1> + <10,6,2,1>' + <10,9> + <10,9>' – <10,4,3,2> – <10,4,3,2>'
2. <9,7,3>* =<16,3> + <16,3>' + <9,6,3,1> + <9,6,3,1>' – <9,5,3,2> – <9,5,3,2>' – <10,9> – <10,9>'
3. <14,3,2>* = <10,7,2>* + <7,6,3,2,1>* – <9,7,3>*

5. SpinBlock(of defect2) B_3

All the irreducible modular spin characters in the decomposition matrix forthe block B_3 are non self-associate (non double)[11] $<\lambda>\neq<\lambda>'$ on (7, α)-regular classes.

Notes :

1. The symbol • refers to main topicand the symbol ○refers to inclusive topic.
2. If we want to emphasize the number of the irreducible modular spin characters say n in a character $<\lambda>$, we shall write $<\overset{n}{\lambda}>$.

Theorem (5.1)

The decomposition matrix for the spin block B_3 is $D^{(3)}_{19,7}$.

Proof :Using $(r,\bar{r})$ -inducing of p.i.s. for S_{18} (see the appendix) to S_{19} gives

7. $D_{42}\uparrow^{(0,1)} S_{19} = C_{11}$
8. $D_{43}\uparrow^{(0,1)} S_{19} = C_{12}$
9. $D_{30}\uparrow^{(5,3)} S_{19} = K_5$
10. $D_{44}\uparrow^{(0,1)} S_{19} = C_{15}$
11. $D_{45}\uparrow^{(0,1)} S_{19} = C_{16}$
12. $D_{26}\uparrow^{(2,6)} S_{19} = 2K_6$

4. <7,6,3,2,1>* = <9,6,3,1> + <9,6,3,1>' + <16,3> + <16,3> ' +<14,3,2>* – <10,7,2>* – <9,5,3,2> – <9,5,3,2>'–<10,9> – <10,9>'

The approximation matrix contains at most 22 columns since the number of the i.m.s. is equal or less than the number of the spin characters .

Butcontains at most 18 columns since there are4 equations corresponding the spin characters of S_{19} in B_3.

• Since <17,2>≠<17,2>' on (7, α)-regular classes.

Then either K_1 splits or there are another two columns[(6)]. Suppose that there are another two columns, we will try to describe these columns.

Now since :

$$\langle 17,2\rangle\downarrow S_{18} = \overset{1}{\langle 16,2\rangle^*} + \overset{1}{\langle 17,1\rangle^*}$$

$$\langle 16,3\rangle\downarrow S_{18} = \overset{1}{\langle 16,2\rangle^*} + \overset{2}{\langle 15,3\rangle^*}$$

$$\langle 14,3,2\rangle^*\downarrow S_{18} = \overset{1}{\langle 13,3,2\rangle} + \overset{1}{\langle 13,3,2\rangle'} + \overset{2}{\langle 14,3,1\rangle} + \overset{2}{\langle 14,3,1\rangle'}$$

$$\langle 10,7,2\rangle^*\downarrow S_{18} = \overset{2}{\langle 9,7,2\rangle} + \overset{2}{\langle 9,7,2\rangle'} + \overset{2}{\langle 10,6,2\rangle} + \overset{2}{\langle 10,6,2\rangle'} + \overset{5}{\langle 10,7,1\rangle} + \overset{5}{\langle 10,7,1\rangle'}$$

$$\langle 9,7,3\rangle^*\downarrow S_{18} = \overset{4}{\langle 8,7,3\rangle} + \overset{4}{\langle 8,7,3\rangle'} + \overset{2}{\langle 9,6,3\rangle} + \overset{2}{\langle 9,6,3\rangle'} + \overset{2}{\langle 9,7,2\rangle} + \overset{2}{\langle 9,7,2\rangle'}$$

Then we can take the columns Y_1, Y_2 (see table $1(R^{(3)}_{19,7})$)

$x \neq 0$ (when $x = 0$ we get contradiction with assumption)

$\therefore x = 1$, then when $y = 0 \Rightarrow$<16,3> = <16,3>'

Then K_2 must split to get C_3, C_4

(<15,3>* – <17,1>*) ism.s.for S_{18} (see the Appendix)

But (<15,3>* – <17,1>*) $\uparrow^{(2,6)} S_{19}$ is not m.s. for S_{19}

$\therefore y \neq 0 \Rightarrow y = 1$

But <16,3> – <17,2>≠<16,3>' – <17,2>'on (7, α)-regular classes

Then K_2 must split to get C_3, C_4

(see table $1(R^{(3)}_{19,7})$)

But <10,9> + <17,2> – <16,3>≠<10,9>' + <17,2>' – <16,3>'on (7, α)-regular classes.

Then the existence of the two columns lead to contradiction.

$\therefore K_1$ must split to get C_1, C_2

Therefore C_1, C_2 correspond withi.m.s. $\varphi\langle 17,2\rangle, \varphi\langle 17,2\rangle'$ respectively.

• Since <16,3>≠<16,3>' on (7, α)-regular classes

Then either K_2 splits or there are another two columns.

Suppose that there are another two columns,we will take the previous two columnswhen $x = u = 0$ (see table $1(R^{(3)}_{19,7})$)

$y \neq 0$ (when $y = 0$ we get contradiction with assumption)

$\therefore y = 1$,then when $z = 0$

(<14,3,1> + <17,1>* – <15,3>*) is m.s. for S_{18} (see the Appendix)

But (<14,3,1> + <17,1>*– <15,3>*) $\uparrow^{(2,6)} S_{19}$ is not m.s. for S_{19}

$\therefore z \neq 0 \Rightarrow z = 1$

Since (<8,7,3> + <14,3,1> – <10,7,1>) is m.s. for S_{18}

$\therefore$ (<8,7,3> + <14,3,1> – <10,7,1>) $\uparrow^{(2,6)} S_{19}$ is m.s. for S_{19}

Then w must be such that $w \in \{0,1\}$ (see table $1(R^{(3)}_{19,7})$)

And since $\langle 10,7,2\rangle^* \downarrow S_{18} \cap \langle 14,3,2\rangle^* \downarrow S_{18} = 6$ ofi.m.s. (see the Appendix)

$\therefore w = 1 \Rightarrow w = z = y = 1$

Then the existence of the two columns lead to split K_2 .

$\therefore K_2$ must split to get C_3, C_4

Therefore C_3, C_4 correspond with i.m.s. $\varphi\langle 16,3\rangle, \varphi\langle 16,3\rangle'$ respectively.

- Since $\langle 10,9\rangle \neq \langle 10,9\rangle'$ on (7, α)-regular classesand

$\langle 10,9\rangle \downarrow S_{18} = 2\langle 10,8\rangle^*$

$\therefore K_4$ must split to get C_9, C_{10}.Therefore C_9, C_{10} correspond with i.m.s. $\varphi\langle 10,9\rangle, \varphi\langle 10,9\rangle'$ respectively.

$1(R^{(3)}_{19,7})$														
<17,2>	1												x	
<17,2>'	1													x
<16,3>	1	1											y	
<16,3>'	1	1												y
<14,3,2>*		2	1	1									z	z
<13,3,2,1>			1		1									
<13,3,2,1>'				1	1									
<10,9>		1				1								
<10,9>'		1				1								
<10,7,2>*	2	2	1	1		2	1	1					w	w
<10,6,2,1>			1		1		1		1					
<10,6,2,1>'				1	1			1	1					
<10,4,3,2>					1				1					
<10,4,3,2>'					1				1					
<9,7,3>*	2					2	1	1		1	1		u	u
<9,6,3,1>						2	1		1	1		1		
<9,6,3,1>'						2		1	1		1	1		
<9,5,3,2>									1			1		
<9,5,3,2>'									1			1		
<8,6,3,2>						2				1	1	1		
<8,6,3,2>'						2				1	1	1		
<7,6,3,2,1>*										1	1			
	K_1	K_2	C_5	C_6	K_3	K_4	C_{11}	C_{12}	K_5	C_{15}	C_{16}	K_6	Y_1	Y_2

$x \in \{0,1\}$, $y \in \{0,1\}$, $z \in \{0,1\}$,

Now when C_5, C_6 correspond withi.m.s. $\varphi\langle 13,3,2,1\rangle, \varphi\langle 13,3,2,1\rangle'$ respectively , C_{11}, C_{12} correspond withi.m.s. $\varphi\langle 10,6,2,1\rangle, \varphi\langle 10,6,2,1\rangle'$ respectively, C_{15}, C_{16} correspond withi.m.s. $\varphi\langle 9,6,3,1\rangle, \varphi\langle 9,6,3,1\rangle'$ respectively .

- Since $\langle 10,4,3,2\rangle \neq \langle 10,4,3,2\rangle'$ on (7, α)-regular classes.

Then either (K_3 or K_5) splits or there are another two columns.Suppose that there are another two columns,we will try to describe these columns.

Now since :

		2		1			
<13,3,2,1>	=	<12,3,2,1>*	+	<13,3,2>			

$\downarrow S_{18}$

		2		4		2	
<10,6,2,1>	=	<9,6,2,1>*	+	<10,5,2,1>*	+	<10,6,2>	

$\downarrow S_{18}$

		1		2	
<10,4,3,2>	=	<9,4,3,2>*	+	<10,4,3,1>*	

$\downarrow S_{18}$

Then we can take the columns Y_1, Y_2

(see table $2(R^{(3)}_{19,7})$)

$z \neq 0$ (when $z = 0$ we get contradiction with assumption)

$\therefore z = 1$

Since (<10,7,1> + <10,4,3,1>* – <10,5,2,1>*) is m.s. for S_{18} (see the Appendix)

$\therefore$ (<10,7,1> + <10,4,3,1>* – <10,5,2,1>*)

$\uparrow^{(2,6)} S_{19}$ is m.s. for S_{19}

$\therefore y \in \{0,1\}$

And since <13,3,2,1> $\downarrow S_{18} \cap$ <10,6,2,1>

$\downarrow S_{18}$ = 3 of i.m.s.

$\Rightarrow x = y = 1 \Rightarrow x = y = z = 1$

$\Rightarrow K_3$ issplits

$\Rightarrow$The second probability lead to the first probability.Then either K_3 or K_5 splits (see table $2(R^{(3)}_{19,7})$)

Suppose K_5 splits and get C_{13}, C_{14}

Therefore C_{13}, C_{14} correspond withi.m.s. φ<10,4,3,2>,φ<10,4,3,2>' respectively.

And since <9,5,3,2>≠<9,5,3,2>' on (7, α)-regular classes. Then either K_6 splits or there are another two columns .If there are another two columns X_1, X_2

Then we get

$C_1,\ldots,C_6,K_3,C_9,\ldots,C_{16},K_6,X_1,X_2$ (18 columns)

But <8,6,3,2>≠<8,6,3,2>' on (7, α)-regular classes.

Then either K_6 splits or there are another two columns.

But with the two cases we get contradiction with the number of columns.

If K_6 split and get C_{17}, C_{18} then we get $C_1,\ldots,C_6,K_3,C_9,\ldots,C_{18}$ (17 columns)

But <8,6,3,2>≠<8,6,3,2>' on (7, α)-regular classes.

Then we must find another two columns, But the number of columns becomes19,which leads tocontradiction

$\therefore K_3$ must split and get C_7, C_8

Therefore C_7, C_8 correspond withi.m.s. φ<10,4,3,2>,φ<10,4,3,2>' respectively.

$2(R^{(3)}_{19,7})$																
<17,2>	1															

<17,2>'		1															
<16,3>	1		1														
<16,3>'		1		1													
<14,3,2>*			1	1	1	1											
<13,3,2,1>					1		1									x	
<13,3,2,1>'						1	1										x
<10,9>			1					1									
<10,9>'				1					1								
<10,7,2>*	1	1	1	1	1	1		1	1	1	1						
<10,6,2,1>					1		1			1		1				y	
<10,6,2,1>'						1	1				1	1					y
<10,4,3,2>							1					1				z	
<10,4,3,2>'							1					1					z
<9,7,3>*	1	1						1	1	1	1		1	1			
<9,6,3,1>								1	1	1		1	1		1		
<9,6,3,1>'								1	1		1	1		1	1		
<9,5,3,2>												1			1		
<9,5,3,2>'												1			1		
<8,6,3,2>								1	1				1	1	1		
<8,6,3,2>'								1	1				1	1	1		
<7,6,3,2,1>*													1	1			
	C_1	C_2	C_3	C_4	C_5	C_6	K_3	C_9	C_{10}	C_{11}	C_{12}	K_5	C_{15}	C_{16}	K_6	Y_1	Y_2

$x \in \{0,1\}$, $z \in \{0,1\}$,

$y \in \{0,1,2,3,4\}$

- Since <9,5,3,2>≠<9,5,3,2>' on (7, α)-regular classes

Then either (K_5 or K_6) split or there are another two columns [6]. Suppose that there are another two columns,we will try to describe these columns,now since:

<9,6,3,1> $\downarrow S_{18}$ = $\overset{6}{\text{<8,6,3,1>*}}$ + $\overset{3}{\text{<9,5,3,1>*}}$ + $\overset{2}{\text{<9,6,2,1>*}}$ + $\overset{2}{\text{<9,6,3>}}$

<9,5,3,2> $\downarrow S_{18}$ = $\overset{2}{\text{<8,5,3,2>*}}$ + $\overset{1}{\text{<9,4,3,2>*}}$ + $\overset{3}{\text{<9,5,3,1>*}}$

<8,6,3,2> $\downarrow S_{18}$ = $\overset{2}{\text{<7,6,3,2>*}}$ + $\overset{2}{\text{<8,5,3,2>*}}$ + $\overset{6}{\text{<8,6,3,1>*}}$

Then we can take the columns Y_1, Y_2 (see table $3(R^{(3)}_{19,7})$)

- Since (<8,6,3,1>* – <8,5,3,2>*) is m.s. for S_{18} (see the Appendix)

$\therefore$(<8,6,3,1>* – <8,5,3,2>*) $\uparrow^{(2,6)} S_{19}$ is m.s. for S_{19}

$\therefore x \geq y$ (1)

And since (<8,7,3> + <8,7,3>' + <8,5,3,2>* – <8,6,3,1>*)ism.s. for S_{18}

$\therefore$ (<8,7,3> + <8,7,3>' + <8,5,3,2>* – <8,6,3,1>*) $\uparrow^{(2,6)} S_{19}$ is m.s.for S_{19} (see table $3(R^{(3)}_{19,7})$)

$\therefore y \geq x$ (2)

$\Longrightarrow x = y$ (from (1)&(2))

- Since (<9,6,3> + <9,6,3>' – <7,6,3,2>*) ism.s. for S_{18} (see the Appendix)

$\therefore$ (<9,6,3> + <9,6,3>' – <7,6,3,2>*) $\uparrow^{(0,1)} S_{19}$ is m.s. for S_{19}

$\therefore x \geq z$ (3)

And since (<8,6,3,1>* + <10,4,3,1>* – <9,5,3,1>*) ism.s. for S_{18}

$\therefore$ (<8,6,3,1>* + <10,4,3,1>* – <9,5,3,1>*)

$\uparrow^{(2,6)} S_{19}$ is m.s. for S_{19}

$\therefore z \geq y$

And since $x = y \Rightarrow z \geq x$ (4)

$\Rightarrow z = x$ (from (3)&(4))

$\Rightarrow x = y = z \neq 0$ (Since $x \neq 0$ from assumption)

$\Rightarrow K_6$ is splits

Therefore C_{17}, C_{18} correspond with i.m.s. φ<9,5,3,2>,φ<9,5,3,2>' respectively.

Then we get

$$C_1,\dots,C_{12},K_5,C_{15}, C_{16}, C_{17}, C_{18}$$

(17 columns)

But <8,6,3,2>≠<8,6,3,2>' on (7, α)-regular classes

Then we must find another two columns, But the number of columns become19 ,which lead to contradiction

$\therefore K_5$ must split and get C_{13}, C_{14}

Therefore C_{13}, C_{14} correspond withi.m.s. φ<9,5,3,2>,φ<9,5,3,2>' respectively.

- Since <8,6,3,2>≠<8,6,3,2>' on (7, α)-regular classes.

Then either K_6 splits or there are another two columns[6]

If there are another two columns then we get 19columns,which lead to contradiction.

$\therefore K_6$ must split to get C_{17}, C_{18}.

Therefore C_{17}, C_{18} correspond withi.m.s. φ<8,6,3,2>,φ<8,6,3,2>' respectively.

$3(R^{(3)}_{19,7})$																		
<17,2>	1																	
<17,2>'		1																
<16,3>	1		1															
<16,3>'		1		1														
<14,3,2>*			1	1	1	1												
<13,3,2,1>					1		1											
<13,3,2,1>'						1		1										
<10,9>			1						1									
<10,9>'				1						1								
<10,7,2>*	1	1	1	1	1	1			1	1	1	1						
<10,6,2,1>					1		1				1		1					
<10,6,2,1>'						1		1				1	1					
<10,4,3,2>							1						1					
<10,4,3,2>'								1					1					
<9,7,3>*	1	1							1	1	1	1		1	1			
<9,6,3,1>									1	1	1		1	1		1	x	
<9,6,3,1>'									1	1		1	1		1	1		x
<9,5,3,2>													1			1	y	
<9,5,3,2>'													1			1		y
<8,6,3,2>									1	1				1	1	1	z	
<8,6,3,2>'									1	1				1	1	1		z
<7,6,3,2,1>*														1	1			

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	K_5	C_{15}	C_{16}	K_6	Y_1	Y_2

Then we get the columns $C_1, C_2, \dots, C_{18}$ which are linearly independent and $C_i - C_j$ is not p.s. for S_{19} $\forall 1 \le i < j \le 18$,then the decomposition matrixfor the block B_3 is :

$$D^{(3)}_{19,7}$$

Degree	Spin character	Decomposition matrix for the spin block B_3																	
34560	**<17,2>**	1																	
34560	**<17,2>'**		1																
169728	**<16,3>**	1		1															
169728	**<16,3>'**		1		1														
2889216	**<14,3,2>***			1	1	1	1												
2728704	**<13,3,2,1>**					1		1											
2728704	**<13,3,2,1>'**						1		1										
1244672	**<10,9>**			1						1									
1244672	**<10,9>'**				1						1								
55644160	**<10,7,2>***	1	1	1	1	1	1			1	1	1	1						
48372480	**<10,6,2,1>**					1		1				1		1					
48372480	**<10,6,2,1>'**						1		1				1		1				
21829632	**<10,4,3,2>**							1						1					
21829632	**<10,4,3,2>'**								1						1				
70946304	**<9,7,3>***	1	1							1	1	1	1			1	1		
94595072	**<9,6,3,1>**									1	1	1		1		1		1	
94595072	**<9,6,3,1>'**									1	1		1		1		1		1
58046976	**<9,5,3,2>**													1				1	
58046976	**<9,5,3,2>'**														1				1
58046976	**<8,6,3,2>**									1	1					1	1	1	
58046976	**<8,6,3,2>'**									1	1					1	1		1
18191360	**<7,6,3,2,1>***															1	1		
		d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}

6. SpinBlock (of defect 2) B_2

The decomposition matrix forthe spin block B_2 must have 18 columns since we found 41 i.m.s. characters in the decomposition matrices for the other blocks. All the irreducible modular spin characters in this matrix are non self-associate (non double)[11],and $<\lambda> \neq <\lambda>'$ on (7, α)-regularclasses.

<u>Theorem (6.1):</u>

The decomposition matrixforthe block B_2 is $D^{(2)}_{19,7}$.

<u>proof</u> :Using (0,1)-inducing of p.i.s. for S_{18} (see the Appendix) to S_{19} gives

1. $D_1 \uparrow^{(0,1)} S_{19} = C_1$
2. $D_2 \uparrow^{(0,1)} S_{19} = C_2$
3. $D_3 \uparrow^{(0,1)} S_{19} = K_1$
4. $D_5 \uparrow^{(0,1)} S_{19} = C_5$
5. $D_6 \uparrow^{(0,1)} S_{19} = C_6$
6. $D_7 \uparrow^{(0,1)} S_{19} = C_7$
7. $D_8 \uparrow^{(0,1)} S_{19} = C_8$
8. $D_9 \uparrow^{(0,1)} S_{19} = K_2$

9. $D_{11}\uparrow^{(0,1)} S_{19} = C_{11}$
10. $D_{12}\uparrow^{(0,1)} S_{19} = C_{12}$
11. $D_{13}\uparrow^{(0,1)} S_{19} = C_{13}$
12. $D_{14}\uparrow^{(0,1)} S_{19} = C_{14}$
13. $D_{15}\uparrow^{(0,1)} S_{19} = C_{15}$
14. $D_{16}\uparrow^{(0,1)} S_{19} = C_{16}$
15. $D_{17}\uparrow^{(0,1)} S_{19} = K_3$

- Since <15,4>≠<15,4>' on $(7,\alpha)$-regular classes.

Then either K_1 splits or there are another two columns [6].

Suppose that there are another two columns,we will try to describe these columns.

Now since:

$$\langle 15,4\rangle \downarrow S_{18} = \overset{4}{\langle 14,4\rangle^*} + \overset{2}{\langle 15,3\rangle^*}$$

$$\langle 14,4,1\rangle^* \downarrow S_{18} = \overset{4}{\langle 14,4\rangle^*} + \overset{2}{\langle 13,4,1\rangle} + \overset{2}{\langle 13,4,1\rangle'} + \overset{2}{\langle 14,3,1\rangle} + \overset{2}{\langle 14,3,1\rangle'}$$

$$\langle 11,8\rangle \downarrow S_{18} = \overset{2}{\langle 10,8\rangle^*} + \overset{6}{\langle 11,7\rangle^*}$$

$$\langle 11,7,1\rangle^* \downarrow S_{18} = \overset{6}{\langle 11,7\rangle^*} + \overset{5}{\langle 10,7,1\rangle} + \overset{5}{\langle 10,7,1\rangle'} + \overset{6}{\langle 11,6,1\rangle} + \overset{6}{\langle 11,6,1\rangle'}$$

Then we can take the columns Y_1, Y_2

(see table $1(R^{(2)}_{19,7})$)

- Since (<10,8>* + <12,3,2,1>* – <14,3,1>) is m.s. for S_{18} (see the Appendix)

$\therefore$ (<10,8>* + <12,3,2,1>* – <14,3,1>) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore z \geq y$ (1)

And since (<14,3,1> + <8,7,3> – <10,8>*) is m.s. for S_{18}

$\therefore$ (<14,3,1> + <8,7,3> – <10,8>*) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore y \geq z$ (2)

$\Rightarrow y = z$ (from (1)&(2)) (see table $1(R^{(2)}_{19,7})$)

- Since (<10,7,1> – <10,8>*) is m.s. for S_{18} (see the Appendix)

$\therefore$ (<10,7,1> – <10,8>*) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore w \geq z$ (3)

And since (<10,8>* + <17,1>* + <10,5,2,1>* – <10,7,1>) is m.s. for S_{18}

$\therefore$ (<10,8>* + <17,1>* + <10,5,2,1>* – <10,7,1>) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore z \geq w$ (4)

$\Rightarrow z = w$ (from (3)&(4))

$\Rightarrow z = w = y$

- Since (<15,3>* + <12,3,2,1>* – <14,3,1>) is m.s. for S_{18} (see the Appendix)

$\therefore$ (<15,3>* + <12,3,2,1>* – <14,3,1>) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore x \geq y$ (5)

And since (<14,3,1> + <17,1>* – <15,3>*) is m.s. for S_{18}

$\therefore$ (<14,3,1> + <17,1>* – <15,3>*) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore y \geq x$ (6)

$\Rightarrow x = y$ (from (5)&(6))

$\Rightarrow x = y = z = w \neq 0$ (since $x \neq 0$ from assumption)

Then the existence of the two columns lead to split K_1.

$\therefore K_1$ must splits to get C_3, C_4

Therefore C_3, C_4 correspond with i.m.s. φ<15,4>,φ<15,4>' respectively.

$1(R^{(2)}_{19,7})$																	
<18,1>	1																
<18,1>'		1															
<15,4>	1	1	1													x	
<15,4>'	1	1	1														x
<14,4,1>*	1	1	2	1	1											y	y
<12,4,2,1>				1		1											
<12,4,2,1>'					1		1										
<11,8>	1	1	1					1								z	
<11,8>'	1	1	1					1									z
<11,7,1>*	3	3	2	1	1			2	1	1						w	w
<11,5,2,1>				1		1			1		1						
<11,5,2,1>'					1		1			1		1					
<11,4,3,1>						1					1						
<11,4,3,1>'							1					1					
<9,5,4,1>									1		1		1				
<9,5,4,1>'										1		1		1			
<8,7,4>*	2	2						2	1	1			1	1	2		
<8,6,4,1>	1	1						2	1				2	1	2		
<8,6,4,1>'	1	1						2		1			1	2	2		
<8,5,4,2>													1	1	1		
<8,5,4,2>'													1	1	1		
<7,5,4,2,1>*													1	1	2		
	C_1	C_2	K_1	C_5	C_6	C_7	C_8	K_2	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	K_3	Y_1	Y_2

Since <11,8>$\neq$<11,8>' on (7, α)-regular classes.

Then either K_2 split or there are another two columns[6] .Suppose that there are another two columns,we will try to describe these columns.

Now since:

$$\langle 8,7,4\rangle^* \downarrow S_{18} = \overset{8}{\langle 8,6,4\rangle} + \overset{8}{\langle 8,6,4\rangle'} + \overset{4}{\langle 8,7,3\rangle} + \overset{4}{\langle 8,7,3\rangle'}$$

$$\langle 8,6,4,1\rangle \downarrow S_{18} = \overset{6}{\langle 7,6,4,1\rangle^*} + \overset{1}{\langle 8,5,4,1\rangle^*} + \overset{6}{\langle 8,6,3,1\rangle^*} + \overset{8}{\langle 8,6,4\rangle}$$

Then we can take the columns Y_1, Y_2 (see table $2(R^{(2)}_{19,7})$)

Such that $z = w$ from table $1(R^{(2)}_{19,7})$.

○ Since(<8,6,3,1>* + 2<17,1>* + <9,5,3,1>* – <8,7,3> – <8,7,3>')is m.s. for S_{18} (see the Appendix)

$\therefore$(<8,6,3,1>* + 2<17,1>* + <9,5,3,1>* – <8,7,3> – <8,7,3>')$\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore 2y \geq 2h$

$\Rightarrow y \geq h$ (7)

And since (<8,7,3> + <8,7,3>' + <9,5,3,1>* – <8,6,3,1>*) is m.s. for S_{18}

$\therefore$ (<8,7,3> + <8,7,3>' + <9,5,3,1>* – <8,6,3,1>*)$\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore 2h \geq 2y$

$\Rightarrow y \geq h$ (8)

$\Rightarrow y = h$ (from (7)&(8))

- Since (<8,7,3> + <14,3,1> – <10,7,1>) is m.s. for S_{18} (see the Appendix)

$\therefore$ (<8,7,3> + <14,3,1> – <10,7,1>) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore h \geq w$ (9)

And since (<10,7,1> + <7,5,3,2,1> – <8,7,3>) is m.s. for S_{18}

$\therefore$ (<10,7,1> + <7,5,3,2,1> – <8,7,3>) $\uparrow^{(4,4)} S_{19}$ is m.s. for S_{19}

$\therefore w \geq h$ (10)

$\Rightarrow w = h$ (from (9)&(10))

$\Rightarrow z = w = h = y$

Then the existence of the two columns lead to split K_2.

$\therefore K_2$ must split and get C_9, C_{10}

Therefore C_9, C_{10} correspond withi.m.s. φ<11,8>,φ<11,8>' respectively .

- Since <8,5,4,2>≠<8,5,4,2>' on(7, α)-regular classes.

Then either K_3 splits or there are another two columns.

If there are another two columns X_1, X_2 then we get the columns $C_1,\ldots,C_{16},K_3,X_1,X_2$ (19 columns).which lead tocontradiction

$\therefore K_3$ must split and get C_{17}, C_{18}

Therefore C_{17}, C_{18} correspond withi.m.s. φ<8,5,4,2>,φ<8,5,4,2>' respectively.

$2(R^{(2)}_{19,7})$																		
<18,1>	1																	
<18,1>'		1																
<15,4>	1	1	1															
<15,4>'	1	1		1														
<14,4,1>*	1	1	1	1	1	1												
<12,4,2,1>					1		1											
<12,4,2,1>'						1		1										
<11,8>	1	1	1						1								z	
<11,8>'	1	1		1					1									z
<11,7,1>*	3	3	1	1	1	1			2	1	1						w	w
<11,5,2,1>					1		1			1		1						
<11,5,2,1>'						1		1			1		1					

<11,4,3,1>							1					1						
<11,4,3,1>'								1					1					
<9,5,4,1>										1		1		1				
<9,5,4,1>'											1		1		1			
<8,7,4>*	2	2							2	1	1			1	1	2	h	h
<8,6,4,1>	1	1							2	1				2	1	2	y	y
<8,6,4,1>'	1	1							2		1			1	2	2	y	y
<8,5,4,2>														1	1	1		
<8,5,4,2>'														1	1	1		
<7,5,4,2,1>*														1	1	2		
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	K_2	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	K_3	Y_1	Y_2

Now we will discuss the cases:

Case (1) :

Suppose C_3 is not subtracted from C_1 .

⇒(<15,4>–<18,1> – <18,1>') is m.s. for S_{19}

But (<15,4>–<18,1> – <18,1>') $\downarrow S_{18}$ is not m.s. for S_{18} (see the Appendix)

which lead tocontradiction

$\therefore$ $C'_1 = C_1 - C_3$ then when $C'_2 = C_2 - C_3$ (since C_1, C_2 are associate columns [11]) .

⇒(<15,4>' –<18,1> – <18,1>') is m.s. for S_{19}

But (<15,4>'–<18,1> – <18,1>') $\downarrow S_{18}$ is not m.s. for S_{18} (see the Appendix)

$\therefore C'_2 = C_2 - C_4$

Now suppose that C_{10} is not subtracted from C'_1 , then C_9 is not subtracted from C'_2 (since C'_1, C'_2 are associate columns (C. Bessenrodt)) .

⇒(<8,6,4,1> + <8,6,4,1>' – <8,7,4>*) is m.s. for S_{19}

But (<8,6,4,1> + <8,6,4,1>' – <8,7,4>*) $\downarrow_{(4,4)} S_{18}$ is not m.s. for S_{18} (see the Appendix)

which lead to contradiction

$\therefore$

$$C''_1 = C'_1 - C_{10} \quad , C''_2 = C'_2 - C_9$$

Case(2) :

Suppose that C_{18} is not subtractedfrom C_{16}

⇒(<8,7,4>* – <8,5,4,2>) is m.s. for S_{19}

But (<8,7,4>* – <8,5,4,2>) $\downarrow_{(4,4)} S_{18}$ is not m.s. for S_{18} (see the Appendix)

which lead tocontradiction

$\therefore$ $C'_{16} = C_{16} - C_{18}$ then when $C'_{15} = C_{15} - C_{18}$ (since C_{15}, C_{16} are associate columns [11]) .⇒ (<8,7,4>* – <8,5,4,2>') is m.s. for S_{19}

But (<8,7,4>* – <8,5,4,2>') $\downarrow_{(4,4)} S_{18}$ is not m.s. for S_{18} (see the Appendix)

$\therefore C'_{15} = C_{15} - C_{17}$

Then we get the columns

$C''_1, C''_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C'_{15}, C'_{16}, C_{17}, C_{18}$ which are linearly independent and the subtraction of them is not p.s. for S_{19}.

Then the decomposition matrixfor the spin block B_2 is:

$$D_{19,7}^{(2)}$$

Degree	*Spin character*	*Decomposition matrix for the spin block* B_2																	
4352	<18,1>	1																	
4352	<18,1>'		1																
574464	<15,4>		1	1															
574464	<15,4>'	1			1														
4299776	<14,4,1>*			1	1	1	1												
13643520	<12,4,2,1>					1		1											
13643520	<12,4,2,1>'						1		1										
3055104	<11,8>			1						1									
3055104	<11,8>'				1						1								
21498880	<11,7,1>*	1	1	1	1	1	1			1	1	1	1						
33488640	<11,5,2,1>					1		1				1		1					
33488640	<11,5,2,1>'						1		1				1		1				
25798656	<11,4,3,1>							1						1					
25798656	<11,4,3,1>'								1						1				
58212352	<9,5,4,1>											1		1		1			
58212352	<9,5,4,1>'												1		1		1		
38697984	<8,7,4>*	1	1							1	1	1	1					1	1
70946304	<8,6,4,1>									1	1	1				1		1	1
70946304	<8,6,4,1>'									1	1		1				1	1	1
49116672	<8,5,4,2>																**1**	1	
49116672	<8,5,4,2>'															**1**			1
1131520	<7,5,4,2,1>*																	1	1
		d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}

Appendix

Decomposition matrices afforded by blocks of S_{18} modulo 7

The blocks of defect 0 of S_{18} are B_7, B_8, B_9, B_{10}, B_{11}, B_{12} which contain the characters <12,5,1> , <12,5,1>' , <11,4,2,1>* , <10,5,3> , <10,5,3>' , <8,5,4,1>* respectively .

Spin character	*Decomposition matrix for the principal spin block* B_1																	
<18>	1																	
<18>'		1																
<14,4>*	1	1	1	1														
<13,4,1>			1		1													
<13,4,1>'				1		1												
<12,4,2>					1		1											
<12,4,2>'						1		1										
<11,7>*	1	1	1	1					1	1								
<11,6,1>	1	1	1		1				1		1							
<11,6,1>'	1	1		1		1				1		1						

<11,5,2>					1		1				1		1					
<11,5,2>'						1		1				1		1				
<11,4,3>							1						1					
<11,4,3>'								1						1				
<9,5,4>											1		1		1			
<9,5,4>'												1		1		1		
<8,6,4>	1	1							1	1	1				1		1	1
<8,6,4>'	1	1							1	1		1				1	1	1
<7,6,4,1>*									1	1					1	1	1	1
<7,5,4,2>*															1	1	1	1
<6,5,4,2,1>																	1	
<6,5,4,2,1>'																		1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}

Spin character	*Decomposition matrix for the spin block* B_2								
<17,1>*	1								
<15,3>*	1	1							
<14,3,1>		1	1						
<14,3,1>'		1	1						
<12,3,2,1>*			1	1					
<10,8>*		1			1				
<10,7,1>	1	1	1		1	1			
<10,7,1>'	1	1	1		1	1			
<10,5,2,1>*			1	1		1	1		
<10,4,3,1>*				1			1		
<9,5,3,1>*						1	1	1	
<8,7,3>	1				1	1			1
<8,7,3>'	1				1	1			1
<8,6,3,1>*					2	1		1	2
<8,5,3,2>*								1	1
<7,5,3,2,1>									1
<7,5,3,2,1>'									1
	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}	D_{26}	D_{27}

	Decomposition matrix for the spin block B_3		
<16,2>*	1		
<9,7,2>	1	1	
<9,7,2>'	1	1	
< 9,6,2,1>*		1	1
<9,4,3,2>*			1
	D_{28}	D_{29}	D_{30}

Spin character	*Decomposition matrix for the spin block* B_4					
<15,2,1>	1					
<15,2,1>'		1				
<9,8,1>	1		1			
<9,8,1>'		1		1		
<8,7,2,1>*			1	1	1	1
<8,4,3,2,1>					1	
<8,4,3,2,1>'						1
	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}

Spin character	*Decomposition matrix for the spin block* B_5

Spin character	*Decomposition matrix for the spin block* B_6

<13,5>*	1		
<12,6>*	1	1	
<7,6,5>		1	1
<7,6,5>'		1	1
<6,5,4,3>*			1
	D_{37}	D_{38}	D_{39}

<13,3,2>	1					
<13,3,2>'		1				
<10,6,2>	1		1			
<10,6,2>'		1		1		
<9,6,3>			1		1	
<9,6,3>'				1		1
<7,6,3,2>*					1	1
	D_{40}	D_{41}	D_{42}	D_{43}	D_{44}	D_{45}

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مصفوفة التجزئة للمشخصات الاسقاطية لـ S19 قياس 7

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الخلاصة

الغرض الرئيسي لهذا البحث هو حساب مصفوفات التجزئة للمشخصات الاسقاطية للزمرة التناظرية S_{19} *قياس 7 وذلك باستعمال الطريقة* $(r,\bar{r})$- *للتوليد.*