Blocks of defect zero for the symmetric group

Najlaa Sadeq Abdullah

Ministry of education in Basrah, Alanwar Intermediary School for girls, Basrah –Khor al Zubair

Abstract

The main result of this paper is to find (if exist) the spin blocks of defect zero to thedecomposition matrix ofthe spin characters of the symmetric group $S_n$on a field with characteristic $p = 3$, also to find the general formula of the spin blocks of defect zero (if exist) to the decomposition matrices of the spin characters of the symmetric group S_n on fields with characteristic $p = 5,7$.

1. *Introduction*

The group $\bar{G}$ is called a representation group (covering group) to the group$G$ if and only if there is a homomorphism from $\bar{G}$ to G such that the kernel of this homomorphism is contained in the center of $\bar{G}$[1].

Schur showed that the symmetric group S_n has a representation group $\overline{S_n}$ and showed that $\overline{S_n}$ has a central subgroup $Z = \{-1,1\}$such that $\overline{S_n}/Z \cong S_n$[2].

Where the representations of $\overline{S_n}$ fall into two classes:

1. The representations which have Z in their kernel; theseare called the representationsof S_n .
2. The representations which do not have Z in their kernel; these are called the spin representations of S_n .

The spin characters of the spin representations of S_n are labeled by the distinct parts of partitions of$n$. The spin characters of S_n are distributed into blocks[3],this blocks are determined by the $\bar{p}$–core of characters (partitions) [3]. If the $\bar{p}$–core of a character equal to its corresponding partition then the character has defect zero[4]The decomposition matrix of the spin characters of S_n is a relationshipsbetween the irreducible spin characters and the irreducible modular spin characters of S_n .In this paper we concentrate on blocks of defect zero, such that Theorem(3.1) is finding (if exist) the spin blocks of defect zero to the decomposition matrix of the spin characters of$S_n$when$p = 3$. Theorems (3.2), (3.3),are finding the general formula of the spin blocks of defect zero (if exist) to the decomposition matrix of the spin characters of S_n when $p = 5,7$.

2. *Preliminaries*

Definition(2.1)[5]*:*A sequence $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$of positive integers numbers is called distinct partition to n and denoted by λ›–n if $\lambda_1 > \lambda_2 > \cdots > \lambda_k > 0$ and $\sum \lambda_i = n$.

Theorem(2.2)[6]*:*Let $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k) \succ n$,then there is:

i. A self-associate (double) spin character for the symmetric group S_n denoted by $\langle\lambda\rangle^*$ if $n - k$is even.
ii. A pair of associate spin characters for the symmetric group $S_n$denoted by $\langle\lambda\rangle, \langle\lambda\rangle'$ if $n - k$ is odd.

Definition(2.3)[7]*:*, p.37) There are two kinds of bars to $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k) \succ n$ with distinct parts:

i. $(i,1)_h$ −bar where $i < h \leq k$, consists of $\lambda_i + \lambda_h$nodes in the $i_{th}$and h_{th} rows.
ii. $(i,j)_{(i)}$ − bar consists of (i,j) − node together with $\lambda_i - j$ nodes to the right of it.

Definition(2.4):[6] The $\bar{p}$–core of the partition λ is the remaining diagram after removing all $p -$bars from the diagram of λ and denoted by $\lambda(\bar{p})$.

Examples:

1. $\langle\lambda\rangle = \langle 10,7,2,1\rangle^*$, $p = 3$, $\lambda(\bar{3}) = \langle 4,1\rangle$
2. $\langle\lambda\rangle = \langle 15,9,1\rangle^*$, $p = 5$, $\lambda(\bar{5}) = \langle\ \rangle$ empty
3. $\langle\lambda\rangle = \langle 9,4,2,1\rangle^*$, $p = 7$, $\lambda(\bar{7}) = \langle 9,4,2,1\rangle$
4. $\langle\lambda\rangle = \langle 13,8,3,5\rangle$, $p = 11$, $\lambda(\overline{11}) = \langle 5,2\rangle$

Definition(2.5)[5] If the $\bar{p}$–core of a character equal to its corresponding partition then the character has defect zero.

Theorem(2.6)[5]*:*The spin characters $\langle\lambda_1\rangle, \langle\lambda_2\rangle$ are in the same $p -$ block of S_n if and only if $\lambda_1(\bar{p}) = \lambda_2(\bar{p})$.

i. The associate spin characters $\langle\lambda\rangle, \langle\lambda\rangle'$ belong to the same $p -$ block of S_n if $\langle\lambda\rangle \neq \lambda(\bar{p})$, $\langle\lambda\rangle$ and $\langle\lambda\rangle'$ each form a $p -$ block of defect zero on their on if $\langle\lambda\rangle = \lambda(\bar{p})$.

3. *<u>Number of spin blocks of defect zero</u>*

This paper was on the spin blocks of defect zero to the decomposition matrices of the spin characters of S_n on the fields with characteristics $(p) = 3, 5, 7,$; equivalently, the spin characters which is corresponding partitionshave no bars[7],this characters are irreducible modular spin and principle indecomposable modular spin characters.The main result which was presented in this paper is findingthe spin blocks of defect zero (if exist) to the decomposition matrix of the spin characters of the symmetric group$S_n$on a field with characteristic $p = 3$, also the general formula of the spin blocks of defect zero (if exist) to the decomposition matrices of the spin characters of S_n on the fields withcharacteristics$(p) = 5,7$.

And the results will be as below:

Theorem(3.1):

Let $t \neq 0, t \in \mathbb{N}$, $r \in \{1,2\}, m = 3t + r$, then there is (if exist) one spin block of defect zero$\langle m, m-3, m-6, \dots, r\rangle^*$ to the decomposition matrix of the spin charactersof the symmetric group S_n, where $n = \sum_{i=0}^{t}(m-3i)$,for aprime number $p = 3$, if $n-(t+1)$ even number or there aretwo spin blocks $\langle m, m-3, m-6, \dots, r\rangle, \langle m, m-3, m-6, \dots, r\rangle'$if $n-(t+1)$odd number.

Proof:

To prove the spin character $\langle m, m-3, m-6, \dots, r\rangle$ form 3- block of defect zero to the decomposition matrix(D_n) of the spin characters of the symmetric group S_n, where $n = \sum_{i=0}^{t}(m-3i)$(by using definitions (2.3) , (2. 4) &(2.5)):

1- $\because 3 \nmid m$ (from assumption)

$\Rightarrow 3 \nmid (m-3i)$,$\forall i \in \{0,1,\dots,t\}$

2- $\because 3 \nmid 2m$

$\Rightarrow$ $2m-3(i+j) \not\equiv 0$ mod $3 \forall i,j \in \{0,1,\dots,t\}$

$\Rightarrow (m-3i)+(m-3j) \not\equiv 0$ mod $3 \forall i,j \in \{0,1,\dots,t\}$

3- $\because$if $(m-3i)-3 > 0, i \in \{0,1,\dots,t-1\} \Rightarrow \exists$ j $\in \{0,1,\dots,t\}$

$\ni (m-3i)-3 = m-3j$

Then from the above and by using theorem (2.6), $\langle m, m-3, m-6, \dots, r\rangle$form 3- block of defect zero.

Now suppose there is another spin character, $\langle l_1, \dots, l_v\rangle$, form 3-block of defect zero to D_n of S_n.

if$l_i - 3 > 0, i \in \{1,\dots,v\} \Rightarrow \exists j \in \{1,\dots,v\} \ni l_i - 3 = l_j$ (since $\langle l_1, \dots, l_v\rangle$, 3-block of defect zero)

If $l_j - 3 > 0 \Rightarrow \exists k \in \{1,\dots,v\} \ni l_j - 3 = l_k$

⋮

$l_h < 3, h \in \{1,\dots,v\}$

Also if $l_f - 3 > 0$, $f \in \{1,\dots,v\} \Rightarrow \exists e \in \{1,\dots,v\} \ni l_f - 3 = l_e$ (since $\langle l_1, \dots, l_v\rangle$, 3-block of defect zero)

If $l_e - 3 > 0 \Rightarrow \exists z \in \{1,\dots,v\} \ni l_e - 3 = l_z$

⋮

$l_u < 3, u \in \{1,\dots,v\}$

Then $\langle l_1, \dots, l_v\rangle$can be written as in the form $\langle l_i, l_f, l_j, l_e, \dots, l_h, l_u\rangle \ni l_h, l_u \in \{1,2\}$

but $l_i + l_j \not\cong 0$ mod 3 $\forall i,j \in \{1,\dots,v\}$

$\Rightarrow$contradiction (since $\langle l_1, \dots, l_v\rangle$, form 3-block of defect zero to D_n of S_n)

$\therefore \langle l_1, \dots, l_v\rangle$must be written as $\langle l_1, l_1-3, l_1-6, \dots, l_1-3(v-1)\rangle$, $\ni$ $l_1 - 3(v-1) < 3$

$\because m = 3t + r$, $l_1 = 3n_1 + r$, $r \in \{1,2\}$, $t, n_1 \neq 0, t, n_1 \in \mathbb{N}$ (since $\langle l_1, \dots, l_v\rangle, \langle m, m-3, m-6, \dots, r\rangle$, are spin blocks of defect zeroto S_n)

Now if $l_1 > m$

$\Rightarrow$ $l_1 + (l_1 - 3) + \dots + \big(l_1 - 3(v-1)\big) > m + (m-3) + \dots + r$

$\Rightarrow$ $l_1 + (l_1 - 3) + \dots + \big(l_1 - 3(v-1)\big) > n$

$\Rightarrow l_1 + l_2 + \dots + l_v > n$

⇒contradiction (since $\langle l_1, \dots, l_v\rangle$ correspond a distinct partition to n)

If $l_1 < m$

$$\Rightarrow l_1 + (l_1 - 3) + \cdots + \left(l_1 - 3(v-1)\right) < m + (m-3) + \cdots + r$$

$\Rightarrow \quad l_1 + (l_1 - 3) + \cdots + \left(l_1 - 3(v-1)\right) < n$

$\Rightarrow l_1 + l_2 + \cdots + l_v < n$

⇒contradiction
(since $\langle l_1, \dots, l_v\rangle$correspond a distinct partition to n)

$\therefore l_1 = m$

$$\Rightarrow l_1 + (l_1 - 3) + \cdots + \left(l_1 - 3(v-1)\right) = m + (m-3) + \cdots + r$$

$\Rightarrow v - 1 = t$

$\Rightarrow l_i = m - 3(i-1), \forall i \in \{2, \dots, v\}$

$\therefore \langle l_1, \dots, l_v\rangle = \langle m, m-3, \dots, r\rangle$

Then $\langle m, m-3, m-6, \dots, r\rangle^*$isthe 3-block of defect zeroto D_n if $n-(t+1)$ even number or $\langle m, m-3, m-6, \dots, r\rangle, \langle m, m-3, m-6, \dots, r\rangle'$ are the3-blocks of defect zero toD_n if $n-(t+1)$ odd number, where $n = \sum_{i=0}^{t}(m-3i)$, from Theorem(2.2).And when $n \neq \sum_{i=0}^{t}(m-3i)$, then there are no spin blocks of defect zero toD_n of S_n .

Examples:

1. For $m = 16 \Rightarrow m = 3\times 5 + 1 \Rightarrow$ $\langle 16,13,10,7,4,1\rangle$, $\langle 16,13,10,7,4,1\rangle'$, are the spin blocks of defect zero toD_{51} of S_{51} for $p = 3$.
2. For $m = 11 \Rightarrow m = 3\times 3 + 2 \Rightarrow \langle 11,8,5,2,\rangle^*$ is the spin block of defect zero to D_{26} of S_{26} for $p = 3$.

Theorem(3.2):

Let $m_i, t_i \geq 0, r_i \in \{0,1,2,3,4\}, m_i = 5t_i + r_i$, $5 \nmid m_i, i = \{1,2\}$, $r_1 + r_2 \neq 5$ then there is (if exist) at least one spin block of defect zero to the decomposition matrix of the spin characters of the symmetric group S_n ,where $n = \sum_{j=0}^{t_1}(m_1 - 5j) + \sum_{k=0}^{t_2}(m_2 - 5k)$, for a prime number $p = 5$ and these blocks areinthe form $\langle m_1, m_2, m_1 - 5, m_2 - 5, m_1 - 10, m_2 - 10, \dots, r_1, r_2\rangle$.

Proof:

To prove the spin character $\langle m_1, m_2, m_1 - 5, m_2 - 5, m_1 - 10, m_2 - 10, \dots, r_1, r_2\rangle$ form 5-block of defect zero to the decomposition matrix (D_n) of the spin characters of the symmetric group S_n, where $n = \sum_{j=0}^{t_1}(m_1 - 5j) + \sum_{k=0}^{t_2}(m_2 - 5k)$ (by using definitions (2.3) , (2. 4) &(2.5)):

1- $\because \; 5 \nmid m_i$, $i = \{1,2\}$ (from assumption)

$\Rightarrow \; 5 \nmid (m_i - 5v_i)$,$\forall v_i \in \{0,1, \dots, t_i\}$, i={1,2}

2- $\because r_1 + r_2 \neq 5$ (from assumption)

$\Rightarrow m_1 + m_2 \not\equiv 0 \mod 5$

$\Rightarrow m_1 + m_2 - 5\,(v_1 + v_2) \not\equiv 0 \mod 5 \forall v_1 \in \{0,1,\dots,t_1\}$, $v_2 \in \{0,1,\dots,t_2\}$

$\Rightarrow (m_1 - 5v_1) + (m_2 - 5v_2) \not\equiv 0 \mod 5 \forall\ v_1 \in \{0,1,\dots,t_1\}$, $v_2 \in \{0,1,\dots,t_2\}$

3- if$(m_i - 5v_i) - 5 > 0$, $v_i \in \{0,1,\dots,t_i - 1\}$, $i \in \{1,2\}$

$\Rightarrow \exists g_i \in \{0,1,\dots,t_i\} \ni (m_i - 5v_i) - 5 = m_i - 5g_i$

Then from the above and by using theorem (2.6), $\langle m_1, m_2, m_1 - 5, m_2 - 5, m_1 - 10, m_2 - 10, \dots, r_1, r_2\rangle$, form spin block of defect zero.

Now suppose there is another spin character $\langle h_1, h_2, \dots, h_l\rangle$ form 5- block of defect zero to D_n of S_n not in the form $\langle m_1, m_2, m_1 - 5, m_2 - 5, m_1 - 10, m_2 - 10, \dots, r_1, r_2\rangle$.

if $h_i - 5 > 0, i \in \{1,\dots,l\} \Rightarrow \exists j \in \{1,\dots,l\} \ni h_i - 5 = h_j$ (since $\langle h_1, h_2, \dots, h_l\rangle$, 5- block of defect zero)

If $h_j - 5 > 0 \Rightarrow \exists k \in \{1,\dots,l\} \ni h_j - 5 = h_k$

⋮

$h_y < 5, y \in \{1,\dots,l\}$

And if $h_f - 5 > 0$, $f \in \{1,\dots,l\} \Rightarrow \exists e \in \{1,\dots,l\} \ni h_f - 5 = h_e$ (since $\langle h_1, h_2, \dots, h_l\rangle$, 5- block of defect zero)

If $h_e - 5 > 0 \Rightarrow \exists z \in \{1,\dots,l\} \ni h_e - 5 = h_z$

⋮

$h_v < 5, v \in \{1,\dots,l\}$

alsoif $h_w - 5 > 0, w \in \{1,\dots,l\} \Rightarrow \exists d \in \{1,\dots,l\} \ni h_w - 5 = h_d$ (since $\langle h_1, h_2, \dots, h_l\rangle$, 5- block of defect zero)

If $h_d - 5 > 0 \Rightarrow \exists o \in \{1,\dots,l\} \ni h_d - 5 = h_o$

⋮

$h_u < 5, u \in \{1,\dots,l\}$

Then $\langle h_1, h_2, \dots, h_l\rangle$ can be written as in the form $\langle h_i, h_f, h_w, h_j, h_e, h_d, \dots, h_y, h_v, h_u\rangle \ni h_y, h_v, h_u \in \{1,2,3,4\}$

But $h_i + h_j \not\equiv 0 \mod 5 \quad \forall i, j \in \{1,\dots,l\}$

$\Rightarrow$ Contradiction (since $\langle h_1, h_2, \dots, h_l\rangle$, form5- block of defect zero to D_n of S_n)

$\therefore \langle h_1, h_2, \dots, h_l\rangle$ must be in the form $\langle m_1, m_2, m_1 - 5, m_2 - 5, m_1 - 10, m_2 - 10, \dots, r_1, r_2\rangle$

Then there is at least one spin block of defect zero to D_n of S_n, when $n = \sum_{j=0}^{t_1}(m_1 - 5j) + \sum_{k=0}^{t_2}(m_2 - 5k)$, this block is in the form $\langle m_1, m_2, m_1 - 5, m_2 - 5, m_1 - 10, m_2 - 10, \dots, r_1, r_2\rangle$,and there are no spin blocks of defect zero to D_n of S_n, when $n \neq \sum_{j=0}^{t_1}(m_1 - 5j) + \sum_{k=0}^{t_2}(m_2 - 5k)$.

Examples:

1. For $m_1 = 11$, $m_2 = 8 \Rightarrow m_1 = 2\times 5 + 1, m_2 = 1\times 5 + 3 \Rightarrow \langle 11,8,6,3,1\rangle^*$ is a spin block of defect zero to D_{29} of S_{29} for $p = 5$.

2. For $m_1 = 14$, $m_2 = 2 \Rightarrow m_1 = 2\times5+4, m_2 = 0\times5+2 \Rightarrow \langle 14,9,4,2\rangle$, $\langle 14,9,4,2\rangle'$ are a spin blocks of defect zero to D_{29} of S_{29} for $p = 5$.

Theorem(3.3):

Let $m_i, t_i \geq 0$, $r_i \in \{0,1,2,3,4,5,6\}$, $m_i = 7t_i + r_i$, $7 \nmid m_i$, $i = \{1,2,3\}$, $r_i + r_j \neq 7$, $\forall i,j \in \{1,2,3\}$, then there is (if exist) at least one spin block of defect zero to the decomposition matrix of the spin characters of the symmetric group S_n , where $n = \sum_{k=0}^{t_1}(m_1 - 7k) + \sum_{h=0}^{t_2}(m_2 - 7h) + \sum_{l=0}^{t_3}(m_3 - 7l)$, for a prime number $p = 7$,and these blocks are in the form $\langle m_1, m_2, m_3, m_1 - 7, m_2 - 7, m_3 - 7, \dots, r_1, r_2, r_3\rangle$.

Proof: To prove $\langle m_1, m_2, m_3, m_1 - 7, m_2 - 7, m_3 - 7, \dots, r_1, r_2, r_3\rangle$ form 7- block of defect zero to the decomposition matrix of the spin characters (D_n) of S_n , where $n = \sum_{k=0}^{t_1}(m_1 - 7k) + \sum_{h=0}^{t_2}(m_2 - 7h) + \sum_{l=0}^{t_3}(m_3 - 7l)$ (by using definitions (2.3), (2.4) & (2.5))

1- $\because 7 \nmid m_i$, $i = \{1,2,3\}$ (from assumption)

$\Rightarrow 7 \nmid (m_i - 7v_i)$, $\forall v_i \in \{0,1,\dots,t_i\}$, $i = \{1,2,3\}$

2- $\because r_i + r_j \neq 7 \forall i,j \in \{1,2,3\}$ (from assumption)

$\Rightarrow m_i + m_j \not\equiv 0 \mod 7 \forall i,j \in \{1,2,3\}$

$\Rightarrow m_i + m_j - 7\,(v_i + v_j) \not\equiv 0 \mod 7 \forall\ i,j \in \{1,2,3\}$, $v_i \in \{0,1,\dots,t_i\}$, $v_j \in \{0,1,\dots,t_j\}$

$\Rightarrow (m_i - 7v_i) + (m_j - 7v_j) \not\equiv 0 \mod 7$ $\forall i,j \in \{1,2,3\}$

3- if $(m_i - 7v_i) - 7 > 0, i \in \{1,2,3\}$, $v_i \in \{0,1,\dots,t_i - 1\}$

$\Rightarrow \exists g_i \in \{0,1,\dots,t_i\} \ni (m_i - 7v_i) - 7 = m_i - 7g_i$

Then from the above and by using theorem(2.6), $\langle m_1, m_2, m_3, m_1 - 7, m_2 - 7, m_3 - 7, \dots, r_1, r_2, r_3\rangle$ form spin block of defect zero to D_n of S_n.

Now suppose there is another spin character $\langle c_1, c_2, \dots, c_h\rangle$form 7- block of defect zero to D_n of S_n not in the form $\langle m_1, m_2, m_3, m_1 - 7, m_2 - 7, m_3 - 7, \dots, r_1, r_2, r_3\rangle$

if $c_i - 7 > 0, i \in \{1,\dots,h\} \Rightarrow \exists j \in \{1,\dots,h\} \ni c_i - 7 = c_j$ (since $\langle c_1, c_2, \dots, c_h\rangle$, 7- block of zero)

If $c_j - 7 > 0 \Rightarrow \exists k \in \{1,\dots,h\} \ni c_j - 7 = c_k$

$\vdots$

$c_y < 7$, $y \in \{1,\dots,h\}$

Also if $c_f - 7 > 0$, $f \in \{1,\dots,h\} \Rightarrow \exists e \in \{1,\dots,h\} \ni c_f - 7 = c_e$ (since $\langle c_1, c_2, \dots, c_h\rangle$, 7- block of zero)

If $c_e - 7 > 0 \Rightarrow \exists z \in \{1,\dots,h\} \ni c_e - 7 = c_z$

$\vdots$

$c_v < 7$, $v \in \{1,\dots,h\}$

And if $c_w - 7 > 0, w \in \{1, \dots, h\} \Rightarrow \exists d \in \{1, \dots, h\} \ni c_w - 7 = c_d$ (since $\langle c_1, c_2, \dots, c_h \rangle$, 7- block of zero)

If $c_d - 7 > 0 \Rightarrow \exists l \in \{1, \dots, h\} \ni c_d - 7 = c_l$

$\vdots$

$c_u < 7$, $u \in \{1, \dots, h\}$

also if $c_b - 7 > 0, b \in \{1, \dots, h\} \Rightarrow \exists x \in \{1, \dots, h\} \ni c_b - 7 = c_x$ (since $\langle c_1, c_2, \dots, c_h \rangle$, 7- block of zero)

If $c_x - 7 > 0 \Rightarrow \exists a \in \{1, \dots, h\} \ni c_x - 7 = c_a$

$\vdots$

$c_o < 7$, $o \in \{1, \dots, h\}$

Then $\langle c_1, c_2, \dots, c_h \rangle$ can be written as in the form $\langle c_i, c_f, c_w, c_b, c_j, c_e, c_d, c_x, \dots, c_y, c_v, c_u, c_o \rangle$ $, \ni c_y, c_v, c_u, c_o \in \{1,2,3,4,5,6\}$

But $c_i + c_j \not\equiv 0 \mod 7 \forall i, j \in \{1, \dots, h\}$

$\Rightarrow$ Contradiction (since $\langle c_1, c_2, \dots, c_h \rangle$, 7- block of zero to D_n of S_n)

Then $\langle c_1, c_2, \dots, c_h \rangle$ must be in the form $\langle m_1, m_2, m_3, m_1 - 7, m_2 - 7, m_3 - 7, \dots, r_1, r_2, r_3 \rangle$.

Then there is at least one spin block of defect zero to D_n of S_n, when $n = \sum_{k=0}^{t_1}(m_1 - 7k) + \sum_{h=0}^{t_2}(m_2 - 7h) + \sum_{l=0}^{t_3}(m_3 - 7l)$, this block is in the form, $\langle m_1, m_2, m_3, m_1 - 7, m_2 - 7, m_3 - 7, \dots, r_1, r_2, r_3 \rangle$,and there is no spin block of defect zero to D_n of S_n, when $n \neq \sum_{k=0}^{t_1}(m_1 - 7k) + \sum_{h=0}^{t_2}(m_2 - 7h) + \sum_{l=0}^{t_3}(m_3 - 7l)$.

Examples:

1. For $m_1 = 11, m_2 = 9, m_3 = 8 \Rightarrow m_1 = 1\times7 + 4, m_2 = 1\times7 + 2, m_3 = 1\times7 + 1 \Rightarrow \langle 11,9,8,4,2,1 \rangle, \langle 11,9,8,4,2,1 \rangle'$ are spin blocks of defect zero to D_{35} of S_{35} for $p = 7$.
2. For $m_1 = 15$, $m_2 = 9 \Rightarrow m_1 = 2\times7 + 1, m_2 = 1\times7 + 2 \Rightarrow \langle 15,9,8,2,1 \rangle^*$ is a spin block of defect zero to D_{35} of S_{35} for $p = 7$.

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بلوكات ذات تأثير صفر للزمر التناضرية

نجلاء صادق عبد الله

وزارة التربية، متوسطة الأنوار للبنات، خور الزبير البصرة

الخلاصة

النتيجةَ الرئيسيةَ لَهذا البحثِ هيْ ايَجادَ (إن وجِدتُ) البلوكات الاسقاطية ذات المؤثر صفر لمصفوفة التجزئة للبلوكات الاسقاطية للزمرة التناظرية على الحقل ذو مميز 3، كذلك ايجاد الصيغة العامة للكتل الاسقاطية ذات المؤثر صفر(إن وجِدتُ) الى مصفوفات التجزئة للكتل الاسقاطية للزمرة التناظرية على الحقول ذات المميز 5,7.

	Spin blocks of defect zero to D_n of S_n for a prime number p		
n	P=3	P=5	P=7
5	⟨4,1⟩,⟨4,1⟩'	—	—
6	—	⟨7,2⟩*	—
7	⟨5,2⟩,⟨5,2⟩'	⟨6,1⟩,⟨6,1⟩',⟨4,3⟩,⟨4,3⟩'	⟨4,2,1⟩*
8	—	—	⟨6,2⟩*,⟨5,3⟩*
9	—	⟨7,2⟩,⟨7,2⟩',⟨6,21⟩*	⟨6,3⟩,⟨6,3⟩',⟨5,4⟩,⟨5,4⟩',⟨5,3,1⟩*, ⟨8,1⟩,⟨8,1⟩'
10	—	⟨7,2,1⟩,⟨7,2,1⟩',⟨6,3,1⟩, ⟨6,31⟩'	⟨6,4⟩*,⟨5,4,1⟩⟨5,4,1⟩'
11	—	⟨8,3⟩,⟨8,3⟩'	⟨9,2⟩,⟨9,2⟩',⟨6,5⟩,⟨6,5⟩',⟨8,2,1⟩*, ⟨6,3,2⟩*
12	⟨7,4,1⟩,⟨7,4,1⟩'	⟨8,3,1⟩,⟨8,3,1⟩'	⟨9,2,1⟩,⟨9,2,1⟩',⟨6,4,2⟩,⟨6,4,2⟩', ⟨8,3,1⟩,⟨8,3,1⟩'
13	—	⟨9,4⟩,⟨9,4⟩',⟨7,4,2⟩*	⟨10,3⟩,⟨10,3⟩',⟨8,4,1⟩*
14	—	—	⟨10,3,1⟩,⟨10,3,1⟩',⟨8,5,1⟩,⟨8,5,1⟩'
15	⟨8,5,2⟩*	⟨9,4,2⟩*,⟨8,4,3⟩*	⟨10,3,2⟩*,⟨9,4,2⟩*,⟨11,4⟩,⟨11,4⟩'
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