On Certain Types of Lower Minimal structure Actions

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Abstract:

The main aim of this work is to create a new types of Lower minimal structure action namely S_{Ind}^{ms} -action, W_{Ind}^{ms} -action and Vk_{Ind}^{ms} -action. Also, we gave restriction and the relation among the certain types of Lower minimal structure action.

Key words: lower minimal structure, minimal action.

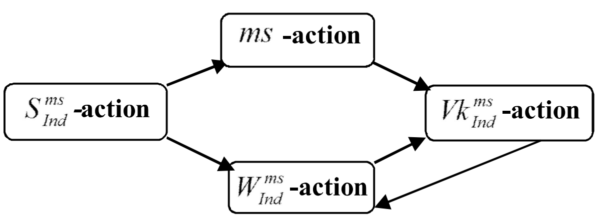
Mathematical Classification QA 440-699

Introduction:

In 1950 Maki H., Umehara J. and Noiri T. introduced the notions of minimal structure and minimal space. They achieved many important results compatible by the general topology case. We recall the basic definitions and facts concerning minimal structures and minimal spaces.

In 2012 Sattar H. and Fieras J. create a new types of minimal continuous, minimal closed and minimal proper functions.

In this work we give the definitions of certain types of Lower minimal structure action. And prove that the restriction of *m*-action (Vk_{Ind}^{ms}-action) from *ums*-space into *ums*-space on an *ms*-closed subset is *ms*-action (Vk_{Ind}^{ms}-action) respectively (2.2 and 2.5). Also the restriction of S_{Ind}^{ms}-action (W_{Ind}^{ms}-action) on a closed subset is S_{Ind}^{ms}-action (W_{Ind}^{ms}-action) respectively (2.3 and 2.4). we give the relation among these types and from (3.1, 3.2, 3.3, 3.4 and 3.5) we have the following diagram.



1. Basic Definitions and Notations:

1.1 Definition [1], [3]:

Let X be a non-empty set and $P(X)$ the power set of X. A subfamily M_X of $P(X)$ is called a minimal structure (briefly *ms*-structure) on X if $\phi, X \in M_X$. In this case (X, M_X) is said to be minimal structure space (briefly *ms*-space). A set $A \in P(X)$ is said to be an *ms*-open set if $A \in M_X$. $B \in P(X)$ is an *ms*-closed set if $B^c \in M_X$.

1.2 Example:

Let $X = \{a,b,c,d\}$ and $M_X = \{\phi, X, \{a\}, \{c\}, \{c,d\}\}$. Then M_X is an *m*-structure on X, and (X, M_X) is an *ms*-space.

1.3 Remark:

Every topological space is *ms*-space but the converse is not necessarily true as the following example shows. Let $X = \{a,b,c\}$ then $M_X = \{\phi, X, \{a\}, \{c\}\}$ is *ms*-space but not topological space. Since $\{a\} \cup \{c\} = \{a,c\} \notin M_X$.

1.4 Remark [6]:

If (X, M_X) is *ms*-space then there is always a subfamilies T_{M_X} of M_X satisfies the conditions of topological spaces (at least the family $\{\phi, X\}$) and the intersection of these families represent the indiscrete topology on X. T_{M_X} called induced topology from minimal structure .

Note: in this work:

i. every word (minimal) is mean (minimal structure).
ii. if A is open set in X is mean $A \in T_{M_X}$. Also if B is closed set in X mean that $B^c \in T_{M_X}$.

1.5 Definition [7]:

Let X be a non-empty set and M_X an *m*-structure on X. For a subset A of X, the minimal closure of A (briefly $\overline{A}^m$) and the minimal interior of A (briefly $A^{\circ m}$), are defined as follows:

$$\overline{A}^m = \bigcap\{F : A \subseteq F, F^c \in M_X\}$$

$$A^{\circ m} = \bigcup\{V : V \subseteq A, V \in M_X\}$$

1.6 Proposition [1],[3],[4]:

Let X be a non-empty set and M_X an m-structure on X. For $A,B \subseteq X$ the following properties hold:

i. $A \subseteq \overline{A}^m$ and $A^{\circ m} \subseteq A$;

ii. if $A^c \in M_X$, then $\overline{A}^m = A$ and if $A \in M_X$, then $A^{\circ m} = A$;

iii. $\overline{\phi}^m = \phi$, $\overline{X}^m = X$, $\phi^{\circ m} = \phi$ and $X^{\circ m} = X$;

iv. $\overline{(\overline{A}^m)}^m = \overline{A}^m$ and $(A^{\circ m})^{\circ m} = A^{\circ m}$.

v. $(A^c)^{\circ m} = (\overline{A}^m)^c$ and $\overline{(A^c)}^m = (A^{\circ m})^c$;

vi. if $A \subseteq B$, then $\overline{A}^m \subseteq \overline{B}^m$ and $A^{\circ m} \subseteq B^{\circ m}$;

vii. $(A \cap B)^{\circ m} = A^{\circ m} \cap B^{\circ m}$ and $A^{\circ m} \cup B^{\circ m} \subseteq (A \cup B)^{\circ m}$;

viii. $\overline{(A \cup B)}^m = \overline{A}^m \cup \overline{B}^m$ and $\overline{(A \cap B)}^m \subseteq \overline{A}^m \cap \overline{B}^m$.

1.7 Remark:

Let (X, M_X) be an ms-space, if A,B are ms-open sets then $A \cap B, A \cup B$ not necessarily ms-open set as the following example shows . Let $X = \{a,b,c,d\}$, $M_X = \{\phi, X, \{a\}, \{b\}, \{a,b,c\}, \{a,b,d\}\}$ be an m-structure on X then $\{a\}, \{b\}, \{a,b,c\}, \{a,b,d\} \in M_X$ but $\{a\} \cup \{b\} = \{a,b\} \notin M_X$ and $\{a,b,c\} \cap \{a,b,d\} = \{a,b\} \notin M_X$. So, we introduce the following definition .

1.8 Definition [6]:

An ms-space (X, M_X) is called an

(i) ums-space if the arbitrary union of ms-open sets is an ms-open set.

(ii) ims-space if the any finite intersection of ms-open sets is an ms-open set.

1.9 Proposition [6]:

Let (X, M_X) be a ums-space, and A be a subset of X then:

i. $A \in M_X$ if and only if $A^{\circ m} = A$;

ii. A is an ms-closed if and only if $\overline{A}^m = A$.

iii. $A^{\circ m} \in M_X$ and $(\overline{A}^m)^c \in M_X$.

1.10 Remark [6]:

If X be a ums-space and A,B be ms-closed set in X then $A \cap B$ is ms-closed set in X.

Note that, if (X, M_X) is an ms-space and $A \subseteq X$ then $M_A = \{W \cap A : W \in M_X\}$ is a minimal structure on A .[2]

1.11 Definition [2]:

Let (X, M_X) be an ms-space and $A \subseteq X$ then the pair (A, M_A) is called the minimal subspace (briefly ms-subspace) of (X, M_X).

1.12 Proposition [6]:

Let A be an ms-subspace of a ums-space X such that A be an ms-closed set in X, and let $B \subseteq A$, then B is ms-closed set in A if and only if B is ms-closed set in X.

1.13 Theorem [8]:

Let (X, M_X) and (Y, M_Y) be two ms-spaces, then $M_{X \times Y} = \{U \times V : U \in M_X \text{ and} V \in M_Y\}$ is an m-structure on $X \times Y$.

Now, we can introduce the following definition.

1.14 Definition [6]:

Let (X,M_X) and (Y,M_Y) be two *ms*-space then the pair $(X\times Y,M_{X\times Y})$ is called minimal product space (briefly *ms*-product space).

1.15 Proposition [6]:

Let (X,M_X) and (Y,M_Y) be two *ims*-spaces, then the *ms*-product space $(X\times Y,M_{X\times Y})$ is an *ims*-space.

1.16 Definition [7]:

Let $f:(X,M_X)\to(Y,M_Y)$ be a function from *ms*-space X into *ms*-space Y then f is called a minimal continuous (briefly *ms*-continuous) if $f^{-1}(B)\in M_X$, for every $B\in M_Y$.

1.17 Remark [6]:

In general if $f:(X,M_X)\to(Y,M_Y)$ be a function from *ms*-space X into *ms*-space Y, $B\in T_{M_Y}$ then it is not necessarily $f^{-1}(B)\in M_X$ for all non-indiscrete topology T_{M_Y} induced from M_Y. As the following example shows.

1.18 Example:

Let $X=\{a,b,c\}$ and $Y=\{1,2,3\}$ such that $M_X=\{\phi,X,\{a\},\{b\}\}$, $M_Y=\{\phi,Y,\{1\},\{1,3\}\}$ are *m*-structure on X and Y respectively and let $f:(X,M_X)\to(Y,M_Y)$ be a function defined as $f(a)=2,f(b)=3,f(c)=1$, then non-indiscrete topologies T_{M_Y} is $T_{1M_Y}=\{\phi,Y,\{1\}\}$. Then $\{1\}\in T_{1M_Y}$, $f^{-1}(\{1\})=\{c\}\notin M_X$.

So we introduce the following definition.

1.19 Definition [6]:

Let (X,M_X) and (Y,M_Y) be two *ms*-spaces and $f:(X,M_X)\to(Y,M_Y)$ be a function, then f is called:

i. ms_*-continuous if there is non-indiscrete topology T_{M_Y} such that $f^{-1}(B)\in M_X$, $\forall B\in T_{M_Y}$.

ii. $_*ms$-continuous if there is non-indiscrete topology T_{M_X} such that $f^{-1}(B)\in T_{M_X}, \forall B\in M_Y$.

iii. $_*ms_*$-continuous if there are non-indiscrete topologies T_{M_X} and T_{M_Y} such that $f^{-1}(B)\in T_{M_X}, \forall B\in T_{M_Y}$.

1.20 Example:

i. Let $X=\{1,2,3\}$ and $Y=\{a,b,c\}$ such that $M_X=\{\phi,X,\{2\},\{3\}\}$, $M_Y=\{\phi,Y,\{a\},\{b\},\{b,c\}\}$ are *m*-structure on X and Y respectively and let:

A. $f:(X,M_X)\to(Y,M_Y)$ be a function defined as $f(a)=2,f(b)=3,f(c)=1$. Then f is:

a. ms_*-continuous since there is non-indiscrete topology $T_{M_Y}=\{\phi,Y,\{a\}\}$ which satisfies the conditions of definition (1.19-i) .

b. $_*ms_*$-continuous since there are non-indiscrete topologies $T_{M_X}=\{\phi,X,\{3\}\}$ and $T_{M_Y}=\{\phi,Y,\{1\}\}$ which satisfies the conditions of definition (1.19-iii).

c. not $_*ms$-continuous since all non-indiscrete topologies T_{M_X} are $T_{1M_X}=\{\phi,X,\{2\}\}$ and $T_{2M_X}=\{\phi,X,\{3\}\}$ which are not satisfies the conditions of definition (1.19-ii).

B. $g:(X,M_X)\to(Y,M_Y)$ be a constant function defined as $f(a)=f(b)=f(c)=1$. Then g is ${}_*ms$-continuous since there is non-indiscrete topology $T_{M_X}=\{\phi,X,\{3\}\}$ which satisfies the conditions of definition (1.19-ii).

ii. Let $X=\{1,2,3\}$ and $Y=\{a,b,c\}$ such that $M_x=\{\phi,X,\{1\},\{2\},\{3\}\}$, $M_Y=\{\phi,Y,\{b\}\}$ are m-structure on X and Y respectively and let $h:(X,M_X)\to(Y,M_Y)$ be a function defined as $h(1)=h(2)=2, h(3)=c$, then h is neither ms_*-continuous nor ${}_*ms_*$-continuous since only non-indiscrete topologies T_{M_Y} is $T_{M_Y}=\{\phi,Y,\{b\}\}$ which is not satisfies the conditions of definition (1.19-i) and definition (1.19-iii).

1.21 Remark:

If the function f be:

i. ms_*-continuous then f is not necessarily ms-continuous nor ${}_*ms$-continuous.

ii. ms-continuous then f is not necessarily ${}_*ms$-continuous.

iii. ${}_*ms_*$-continuous then f is not necessarily ${}_*ms$-continuous.

As the following examples shows.

1.22 Example:

i. Let $X=\{a,b,c\}$ be a set such that $M_X=\{\phi,X,\{a\},\{a,b\}\}$ be m-structure on X and let $Y=\{1,2,3\}$ be a set such that $M_Y=\{\phi,Y,\{2\},\{3\}\}$ be m-structure on Y. If $f:X\to Y$ be a function defined $f(a)=2, f(b)=3, f(c)=1$. Then f is:

a. ms_*-continuous function but not ms-continuous function, because $\{3\}$ is ms-open set in Y and $f^{-1}(\{3\})=\{b\}$ which is not ms-open set in X.

b. ms_*-continuous function but not ${}_*ms$-continuous function, because $\{3\}$ is ms-open set in Y but there is not topological space induced by M_X such that $f^{-1}(\{3\})=\{b\}$ be open set in X.

ii. Let $X=\{a,b,c\}$ be a set such that $M_X=\{\phi,X,\{a\},\{a,b\}\}$ be m-structure on X and let $Y=\{1,2,3\}$ be a set such that $M_Y=\{\phi,Y,\{1\},\{2\},\{3\}\}$ be m-structure on Y. If $f:X\to Y$ be a function defined $f(a)=3, f(b)=2, f(c)=1$. Then f is:

a. ms-continuous function but not ${}_*ms$-continuous function, because $\{1\}$ is ms-open set in Y and $f^{-1}(\{1\})=\{c\}$ which is not ms-open set in X.

b. ${}_*ms_*$-continuous function but not ${}_*ms$-continuous function, because $\{1\}$ is ms-open set in Y but there is not topological space induced by M_X such that $f^{-1}(\{1\})=\{c\}$ be open set in X.

1.23 Definition [5]:

A minimal group is a set G with two structures:

i. (G,μ) is a group.

ii. (G,μ_G) is a minimal space.

Such that the two structures are compatible, i.e; the multiplication function $\mu : G \times G \to G$ which is defined by $\mu(g_1, g_2) = g_1 g_2$, for every $g_1, g_2 \in G$ and the inversion function $\upsilon : G \to G$ which is defined by $\upsilon(g) = g^{-1}$ for all $g \in G$, are both *ms*-continuous functions.

1.24 Theorem [5]:

The product of minimal groups is a minimal groups.

1.25 Definition [5]:

The minimal group $G = \prod G_i$ in theorem (1.27) called minimal product group.

1.26 Definition [5]:

Let G be a minimal group and X be a minimal space. A left minimal action of G on X is an *ms*-continuous map $\varphi : G \times X \to X$ such that:

i. $\varphi(e, x) = x$, for all $x \in X$ where e is the identity element in G.

ii. $\varphi(g_1, \varphi(g_2, x) = \varphi(\mu(g_1, g_2), x)$, for all $x \in X$ and $g_1, g_2 \in G$.

The *ms*-space X together with minimal action φ is called minimal group space and denoted by msG-space, more precisely (left msG-space). In similar way one can define a right msG-space.

1.27 Definition:

Let G be a minimal group and X be a minimal space.

1. A left S_{Ind}^{ms}-action (minimal strong induced by minimal structure) of G on X is an $_*ms$-continuous map $\varphi : G \times X \to X$.
2. A left W_{Ind}^{ms}-action (minimal weak induced by minimal structure) of G on X is an $_*ms_*$-continuous map $\varphi : G \times X \to X$.
3. A left Vk_{Ind}^{ms}-action (minimal very weak induced by minimal structure) of G on X is an ms_*-continuous map $\varphi : G \times X \to X$.

such that:

i. $\varphi(e, x) = x$, for all $x \in X$ where e is the identity element in G.

ii. $\varphi(g_1, \varphi(g_2, x) = \varphi(\mu(g_1, g_2), x)$, for all $x \in X$ and $g_1, g_2 \in G$.

The *ms*-space X together with S_{Ind}^{ms}-action (W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) φ is called minimal strong group space (minimal weak group space, minimal very weak group space) respectively and denoted by $msSG$-space ($msWG$-space, $msVG$-space) respectively more precisely left $msSG$-space (left $msWG$-space, left $msVG$-space) respectively. In similar way one can define a right $msSG$-space (right $msWG$-space, right $msVG$-space) respectively.

1.28 Remark:

The difference between the left and right minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) is not a trivial one, however there is a one to one correspondence between them as follow: if φ is a left minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) respectively of G on X, then $\varphi' : X \times G \to X$ defined by $\varphi'(x, g) = \varphi(g^{-1}, x)$ is a right minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) respectively of G on X, and similarly for right minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) respectively.

Thus for every left minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) is a conjugate right minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) respectively and vise versa, so every proposition that is true of left minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) respectively has a conjugate proposition for right minimal action (S_{Ind}^{ms}-action, W_{Ind}^{ms}-action, Vk_{Ind}^{ms}-action) respectively. Because of this, we will usually use a left minimal action.

1.29 Example:

Let G be a minimal group, then G is msG-space ($msSG$-space, $msWG$-space, $msVG$-space) by multiplication $\varphi = \mu : G \times G \to G$, $(g_1, g_2) \to g_1 g_2$, φ is an ms-continuous ($_*ms$-continuous, $_*ms_*$-continuous, ms_*-continuous) because G is minimal group.

1.30 Example:

Let G be a minimal group, then G is msG-space ($msSG$-space, $msWG$-space, $msVG$-space) respectively by conjugation.

$\varphi : G \times G \to G$, $(g_1, g_2) \to g_1 g_2 g_1^{-1}$, φ is an ms-continuous ($_*ms$-continuous, $_*ms_*$-continuous, ms_*-continuous) respectively since $\varphi = R_{b_1}^{-1} \circ \mu$ and:

i. $\varphi(e, g) = ege^{-1} = g$ for all $g \in G$.

ii. $\varphi(g_1, \varphi(g_2, g_3))$

$= \varphi(g_1, g_2 g_3 g_2^{-1})$

$= g_1(g_2 g_3 g_2^{-1}) g_1^{-1}$

$= (g_1 g_2) g_3 (g_1 g_2)^{-1} = \varphi(g_1 g_2, g_3)$

for all $g_1, g_2, g_3 \in G$.

2. Restriction of Certain Type of *ms*-action:

2.1 Remark:

Let $\varphi : G \times X \to X$ be *ms*-action (S_{Ind}^{mS}-action, W_{Ind}^{mS}-action, Vk_{Ind}^{mS}-action) and let $A \subseteq X$ the restriction action $\varphi|_{G \times A}$ is not necessarily *ms*-action (S_{Ind}^{mS}-action, W_{Ind}^{mS}-action, Vk_{Ind}^{mS}-action).

2.2 Proposition

Let $\varphi : G \times X \to X$ be an *ms*-action from *ums*-space $G \times X$ into *ms*-space X and let A be an *ms*-closed subset of X then $\varphi|_{G \times A} : G \times A \to A$ is *ms*-action.

Proof: Let $\psi = \varphi|_{G \times A}$

To prove $\psi : G \times A \to A$ is an *ms*-continuous. Let B be an *ms*-closed set in A then B be an *ms*-closed set in X then $\varphi^{-1}(B)$ is an *ms*-closed set in $G \times X$ (φ is an *ms*-continuous function) and then by definition (1.11) $(G \times A) \cap \varphi^{-1}(B)$ is an *ms*-closed set in $G \times X$, hence $(G \times A) \cap \varphi^{-1}(B)$ is an *ms*-closed set in $G \times A$ by using Proposition (1.12). Therefore $\psi^{-1}(B) = (G \times A) \cap \varphi^{-1}(B)$ is an *ms*-closed set in $G \times A$.

2.3 Proposition:

Let $\varphi : G \times X \to X$ be an S_{Ind}^{mS}-action from *ms*-space $G \times X$ into *ms*-space X and let A be a closed subset of X then $\varphi|_{G \times A} : G \times A \to A$ is S_{Ind}^{mS}-action.

Proof: Let $\psi = f|_{G \times A}$

To prove $\psi : G \times A \to A$ is an $_*ms$-continuous. Let B be an *ms*-closed set in A then B be an *ms*-closed set in X then $\varphi^{-1}(B)$ is a closed set in $G \times X$ (φ is a $_*ms$-continuous function) and then $(G \times A) \cap \varphi^{-1}(B)$ is a closed set in $G \times A$. Hence $\psi^{-1}(B) = (G \times A) \cap \varphi^{-1}(B)$ is a closed set in $G \times A$.

2.4 Proposition:

Let $\varphi : G \times X \to X$ be a W_{Ind}^{mS}-action from *ms*-space $G \times X$ into *ms*-space X and let A be a closed subset of X then $\varphi|_{G \times A} : G \times A \to A$ is W_{Ind}^{mS}-action.

Proof: Let $g = f|_{G \times A}$

To prove $\psi : G \times A \to A$ is a $_*ms_*$-continuous. Let B be a closed set in A then B be an closed set in X then $\varphi^{-1}(B)$ is a closed set in

$G\times X$ (φ is a ${}_*ms_*$-continuous function) and then $(G\times A)\cap\varphi^{-1}(B)$ is a closed set in $G\times A$. Hence $\psi^{-1}(B)=(G\times A)\cap\varphi^{-1}(B)$ is a closed set in $G\times A$.

2.5 Proposition:

Let $\varphi: G\times X\to X$ be an Vk_{Ind}^{mS}-action from *ums*-space $G\times X$ into *ms*-space X and let A be an *ms*-closed subset of X then $\varphi|_{G\times A}: G\times A\to A$ is Vk_{Ind}^{mS}-action.

Proof: Let $\psi=f|_{G\times A}$

To prove $\psi: G\times A\to A$ is an ms_*-continuous.

Let B be a closed set in A then B be an closed set in X then $\varphi^{-1}(B)$ is an *ms*-closed set in $G\times X$ (φ is an ms_*-continuous function) and then by Remark (1.10) $(G\times A)\cap\varphi^{-1}(B)$ is an *ms*-closed set in $G\times X$, hence by Proposition (1.12) we have $(G\times A)\cap\varphi^{-1}(B)$ is an *ms*-closed set in $G\times A$. Therefore $\psi^{-1}(B)=(G\times A)\cap\varphi^{-1}(B)$ is an *ms*-closed set in $G\times A$.

3. Relation Among Types of *ms*-action:

3.1 Proposition:

Every *ms*-action is Vk_{Ind}^{ms}-action.

Proof: Let $\varphi: G\times X\to X$ be an *ms*-action then φ is *ms*-continuous.

To prove φ is ms_*-continuous. Let $B\in \mathrm{T}_{Mx}$ then $B\in M_X$ thus $\varphi^{-1}(B)\in M_{G\times X}$ (φ is *ms*-continuous).

Thus φ is ms_*-continuous.

3.2 Proposition:

Every S_{Ind}^{ms}-action is *ms*-action.

Proof:

Let $\varphi: G\times X\to X$ be an S_{Ind}^{ms}-action then φ is ${}_*ms$-continuous.

To prove φ is *ms*-continuous function. Let $B\in M_X$ then $\varphi^{-1}(B)\in \mathrm{T}_{M_{G\times X}}$ (φ is ${}_*ms$-continuous function) and then $\varphi^{-1}(B)\in M_X$.

3.3 Proposition:

Every S_{Ind}^{ms}-action is Vk_{Ind}^{ms}-action.

Proof: Let $\varphi: G\times X\to X$ be an S_{Ind}^{ms}-action then φ is ${}_*ms$-continuous.

To prove φ is ms_*-continuous.

Let $B\in \mathrm{T}_{M_X}$ then $B\in M_X$ therefore $\varphi^{-1}(B)\in \mathrm{T}_{M_{G\times X}}$ (φ is ${}_*ms$-continuous function) and then $\varphi^{-1}(B)\in M_{G\times X}$.

3.4 Proposition:

Every S_{Ind}^{ms}-action is W_{Ind}^{ms}-action.

Proof: Let $\varphi: G\times X\to X$ be an S_{Ind}^{ms}-action then φ is ${}_*ms$-continuous.

To prove φ is ${}_*ms_*$-continuous. Let $B\in \mathrm{T}_{M_X}$ then $B\in M_X$ therefore $\varphi^{-1}(B)\in \mathrm{T}_{M_{G\times X}}$ (φ is ${}_*ms$-continuous).

3.5 Proposition:

Every W_{Ind}^{ms}-action is Vk_{Ind}^{ms}-action.

Proof: Let $\varphi: G\times X\to X$ be an W_{Ind}^{ms}-action then φ is ${}_*ms_*$-continuous. To prove φ is ms_*-continuous. Let $B\in \mathrm{T}_{M_X}$ then $\varphi^{-1}(B)\in \mathrm{T}_{M_{G\times X}}$ (φ is ${}_*ms_*$-continuous function). Thus $\varphi^{-1}(B)\in M_{G\times X}$.

3.6 Proposition:

Every Vk_{Ind}^{ms}-action is W_{Ind}^{ms}-action.

Proof: Let $\varphi: G\times X\to X$ be an Vk_{Ind}^{ms}-action then φ is ms_*-continuous.

To prove φ is ${}_*ms_*$-continuous.

Since φ is ms_*-continuous then there is a topological space T'_{M_X} induced by M_X such that $\forall B\in T'_{M_X}$ then $\varphi^{-1}(B)\in M_{G\times X}$.

Take $B_1 \in T'_{M_X}$ then $\varphi^{-1}(B_1) \in M_{G\times X}$ therefore $T''_{M_X} = \{\varphi, X, B_1\}$ is the topological space induced by M_X which exist φ is an m_*-continuous because $\varphi^{-1}(B_1) \in M_{G\times X}$, $\varphi^{-1}(\phi) = \phi \in M_{G\times X}$ and $\varphi^{-1}(X) = G\times X \in M_{G\times X}$. But $T'_{M_{G\times X}} = \{\phi, X, \varphi^{-1}(B_1)\}$ is a topological space induced by $M_{G\times X}$. Then there are non-indiscrete topologies $T'_{M_{G\times X}}$ and T''_{M_X} such that $\varphi^{-1}(B) \in T'_{M_{G\times X}}, \forall B \in T''_{M_Y}$ thus by definition (1.19-iii) we have φ is ${}_*ms_*$-continuous.

3.7 Remark:

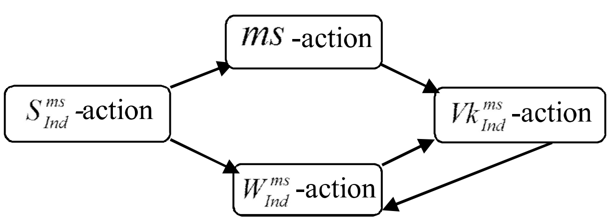
From the remark (1.21) if φ be:

i. Vk^{ms}_{Ind}-action then φ is not necessarily ms-action nor S^{ms}_{Ind}-action.

ii. ms-action then φ is not necessarily S^{ms}_{Ind}-action.

iii. ${}_*ms_*$-continuous then φ is not necessarily S^{ms}_{Ind}-action.

The following diagram shows the relation among types of minimal structure actions.



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حـــول أنماط معينة من افعال البنية الأصغرية

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الخلاصة:

الهدف الرئيسي من هذا العمل هو تقديم نوع جديد من الافعال المتولدة بواسطة البنية الاصغرية وبالتحديد الافعال الأصغرية S^{ms}_{Ind}-action و W^{ms}_{Ind}-action ، Vk^{ms}_{Ind}-action . أيضاً درسنا القصر لهذه الدوال وكذلك أوضحنا العلاقة فيما بين أنواع الافعال الأصغرية.

الكلمات المفتاحية: lower minimal structure. ، minimal action .

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