Some Properties of Contra θ gs-Closed Functions

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Abstract:

The aim of this paper is to introduce and study some properties of a new generalization of contra closed set called contra θgs-closed function in topological spaces, as well as , some theorems and examples of these functions .

Key words— Contra θgs-continuous, θgs-closed set. Contra θgs-closed functions

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1- Introduction:

In 1970, Levine [1] first considered the concept of generalized closed (briefly, g-closed) sets. Arya and Nour [2] defined generalized semi-open (briefly, gs-open) sets using semi openness and obtained some characterization of s-normal space . In 1996, Dontchev introduced the notion of contra continuity and strong S-closedness in topological spaces. In 1999, Dontechev and H. Maki [3], introduced the notion of θ-generalized closed sets ,θ - generalized semi closed(briefly, θ gs-closed) set. In 2014 , Md. Hanif Page [9] introduced and studied of a new generalization of Contra θgs-generalized closed sets. The aim of this paper is to introduce and study of a new θ -generalized semi closed generalization of contra closed called contra -θgs-closed functions utilizing -θgs-closed set.

2- Basic Definitions

Through this paper(X, τ) and (Y, σ) two topological space denote the spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.

Definition 2.1[1] : A subset A of a space (X, τ) is called a generalized closed set (briefly g-closed) if Cl(A) ⊆ U whenever A ⊆ U and U is open.

Definition 2.2[5] : A subset A of a topological space X is called

1- a semi-open set if A ⊂ Cl(Int(A)) ,
2- a semi-closed set if A ⊂ Int(Cl(A)) .

Definition 2.3[5] : The semi-closure of a subset of X is the intersection of all semi-closed sets that contain A and is denoted by sCl(A).

Definition 2.4[7] : The θ-closure of a set A is denoted by Clθ(A) and is defined by Clθ(A) ={x ∈ X: Cl(U) ∩ A = φ, U ∈ τ, x ∈ U} and a set A is θ-closed if and only if A = Clθ(A).

Definition 2.5[6] : A subset A of a topological space X is called θ-generalized semi closed (briefly, θgs-closed) if sClθ(A) ⊂ U whenever A ⊂ U and U is open. The complement of θgs-closed set is θ-generalized-semi open (briefly, θgs-open). We denote the θgs-closed sets of X by θGSC(X, τ) and θgs open sets by θGSO(X, τ) .

***Definition 2.6[8]*:** A topological space X is called

1- T_b-space if every gs-closed set of X is closed set.
2- $T_{\theta gs}$ space if every θgs -closed set in it is closed set.

3- Some Properties Of Contra – θgs Closed Mapping

In this section, the notion of a new class of function called contra θgs- closed functions is introduce and obtain some of their characterizations and properties. Also, the relationships with some other related.

1) A mapping f:(x, σ) →(y , τ) is said to be contra – θ**gs** closed if A ⊂ X is closed then f(A) is θ**gs** – open in Y .
2) A mapping f:(x, σ) →(y , τ) is said to be contra – (θ**gs) *** - closed if A ⊂X is θ**gs -** closed then f(A) is open in Y .
3) A mapping f: x →y is said to be contra – (θ**gs) *** - closed if A ⊂ X is θ**gs -** closed then f(A) is θ**gs** – open .

From the following example, it is clear that both contra *θ* gs-closed and *θ* gs-closed both are independent notions of each other.

Definition 3.1: A function f: X→Y is contra θgs- closed if $f^{1(}V)$ is θgs-closed set in X for each open set V of Y .
Definition 3.2 [9]**:** A function f: X→ Y is called θ -generalized semi-continuous (in briefly, θ**gs** -continuous), if f-1 (F) is θ**gs** -closed in X for every closed set F of Y.

Example 3.3: Let X=Y= {a, b, c, d} and σ1 = {X, ϕ, {b}, {d}, {a, b, d}, {b, c, d}},
σ2 = {Y, ϕ, {b, d}, {a}}
be topologies on X and Y respectively. We have θgs-closed sets in X are {X, ϕ, {a}, {b}, {c}, {d}, {a, b},{a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, c, d}}.

1- Define a function g: X→Y by g(a) = c, g(b) = b, g(c) = a and g(d) = d. Then g is θgs-continuous function.

but not contra θgs-continuous, because for open set {b, d} in Y, g-1({b, d}) = {b, d} is not θgs-closed set in X.

2- Define a function g: X→Y by g(a) = b, g(b) = c, g(c) = d and g(d) = a. Then g is contra θgs-continuous function but not θgs-continuous, because for closed set {a, c} in Y, g-1({a, c}) = {b, d} is not θgs-closed set in X. We denote the family of θgs-closed sets of X by θGSC(X, τ) and θgs-open.

Remark 3.4 :

1- Any intersection of θgs-closed sets is θgs-closed set. Hence, by complement, any union of θgs-open sets is θgs-open.
2- Union of θgs-closed sets may fail to be θgs-closed set.

Definition 3.5[4] :The First Separation Axiom
A topological space X satisfies the first separation axiom T_1 :
$\forall$ x, y $\in$ X, x $\neq$ y if each one of any two points of X has a neighborhood that does not contain the other point. In other words, there exist open sets U_x and V_y such that

$x \in U_x$, $y \notin u_x$ and $y \in U_y$, $x \notin u_y$

Definition 3.6[4] : The Second Axiom
We start with the second axiom, which is most important. Besides the notation T_2, it has a name:(Hausdorff axiom). A topological space X satisfying T_2 is a Hausdorff space. This axiom is stated as follows: any two distinct points possess disjoint neighborhoods. We can state it more formally: $\forall$ x, y $\in$ X, x $\neq$ y $\exists U_x, V_y$: $U_x \cap V_y = \varnothing$.

Definition 3.7 [8]: A topological space (X, τ) is called

1- θgs−T_0 if for any pair of distinct points x and y of X, there exists a θgs-open set containing x but not y or a θgs-open set containing y but not x.

2- θ gs-T_1 if for every pair of distinct points x and y of X, there exists a θgs-open set containing x but not y and a θgs-open set containing y but not x.

3- θ gs-T_2 if for each pair of distinct points x and y of X, there exist disjoint θgs-open sets, one containing x and the other containing y .

4- θgs-Open And θgs-Closed Functions

Definition 4.1 : A function f: (X, τ) → (Y, σ) is said to be θgs-open (resp., θgs-closed) if f (V) is θgs-open (resp., θgs-closed) in Y for every open set (resp., closed) V in X.

Theorem 4.2 : A function f : X → Y is θgs-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a θgs-open set V of Y such that S $\subseteq$ V and $f^{1}(V)$ $\subseteq$ U .

Proof : Assume that f is θgs-closed. Let S be a subset of Y and U be an open set of X such that S $\subseteq$ f (U), that is, $f^{-1}(S) \subseteq U$. Now, U^c is closed set in Y. Then f (U^c) is θgs-closed in X, since f is θgs-closed. So, Y /f (U^c) is θgs-open in. Thus V = Y \f (U^c) is a θgs-open set containing S such that $f^{1}(V) \subseteq U$.
Conversely, suppose that F is a closed set in X. Then $f^{-1}(Y \setminus f(F)) \subseteq X \setminus F$ and X \ F is open. By hypothesis , there is a θgs-open set V of Y such that Y \ f (F) $\subseteq$ V and f−1(V) $\subseteq$ X \ F and so F $\subseteq$ X \ $f^{1}(V)$.
Hence Y \ V $\subseteq$ f (F) $\subseteq$ f (X \ $f^{-1}(V)$) $\subseteq$ Y \ V which implies f (F) = Y \ V .
Since Y \ V is θgs-closed, f (F) is θgs-closed and thus f is θgs-closed.

Theorem 4.3 : Let f : X → Y and g : Y → Z are θgs-closed functions and Y be Tθgs − space. Then their composition gof is θgs-closed.
Proof : Let A be a closed set of X. Then by hypothesis f (A) is a θgs-closed set in Y. Since Y is Tθgs − space, f (A) is closed in Y. Since g is θgs-closed , g(f (A)) is θgs-closed in Z. But g(f (A)) = (gof)(A). Hence g of is θgs-closed[9].

Theorem 4.4 : Let h : X → Y and k : Y → Z be two functions such that
their composition koh : X → Z is θgs-closed function. Then statement holds;
If h is continuous and surjective (onto) , then k is θgs-closed.
θ-generalized semi-open and θ-generalized semi-closed functions .

Proof : Let A be a closed set in Y. Then $h^{-1}(A)$ is closed in X as h is continuous. Since hok is θgs-closed and if h is surjective , (koh)($h^{-1}(A)$) = k(h (h −1(A))) = k(A) is θgs-closed in Z. Therefore k is a θgs-closed function in X.

Theorem 4.5: For any bijection (one to one) f : X → Y , the following statements are equivalent

1- Inverse of f is θgs-continuous.
2- f is a θgs-open function.
3- f is a θgs-closed function.

Proof : 1→ 2 Let U be an open set of X. By assumption θgs-continuous, $(f^{-1})^{-1}(U)$ = f (U) is θgs-open in Y and so f is θgs-open function.
2 → 3: Let F be a closed set of X. Then F C is open in X. By assumption f (F^{C}) is θgs-open in Y , that is , f (F^{C}) = $(f (F))^{C}$ is θgs-open in Y and therefore f (F) is θgs-closed in Y . Hence f is θgs-closed.
3 → 1: Let F be a closed set in X. By assumption f (F) is θgs-closed in Y. But f (F) = $(f^{-1})^{-1}(F)$ = f (F) is θgs-closed and therefore inverse image of f is θgs-continuous[9].

Example 4. 6 : Let X = Y = {a, b, c} , τ1 = {X, φ, {a} , {b} , {a, b}} and
τ2 = {Y, φ, {a} , {b, c}}.We have θgs-closed sets in X are {X, φ, {a} , {b, c}} and θgs-closed sets in Y are {Y, φ, {a} , {b, c}}.

Theorem 4.7: Let f: X→Y be a contra θgs-continuous onto. Then the property hold: If X is θgs-closed compact, then Y is compact.

Proof : Let {Vα: α ∈I} be an open cover of Y . Since f is contra θgs-continuous, then {f^{-1}(Vα) : α∈I} is θgs-closed cover of X. Since X is θgs-closed compact, there exists a finite subset Io of I such that X = ∪ {f^{-1} (Vα): α∈ Io } {f^{-1}(Vα): α∈I_o}. Since f is surjective, Y = ∪ {f^{-1}(Vα): α∈ I_o}, which is finite subcover of Y. Therefore Y is compact.

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بعض الخواص للدوال المغلقة من النوع (Contra θgs)

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المستخلص:

في هذا البحث سنقدم بعض الخواص لأنواع جديدة من الدوال المغلقة ، تسمى عكس الدوال المغلقة من النوع **θgs** في الفضاءات التبولوجية بالاعتماد على أنواع من المجموعات المغلقة تدعى الدوال المغلقة من **(Contra θgs)** مع بعض المبرهنات والامثلة لهذه الدوال.

الكلمات المفتاحية : الدوال المغلقة ، الدوال المغلقة من النوع **θgs** ، الدوال المغلقة من **(Contra θgs)**

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