Jordan Right (ϕ,θ)-derivation of prime Rings

Farhan D. Shiya
Department of Mathematics,
College of Education,
University of AL-Qudisiya

Abstract

Let R be a2-torsion free prime ring. Suppose that ϕ,θ are automorphisms of R. In the present paper it is established that if R admits non zero Jordan right (θ,θ)derivation, then R is commutative .Further ,as an corollary of this result it is show that every Jordan right (θ,θ) derivation on R is a right (θ,θ)-derivation on R .

Finally ,in case of an arbitrary prime ring it is proved that if R admits a right (ϕ,θ) –derivation which acts also as a homorphisms on anon zero ideal of R ,then d=0 on R where d: R→R **is a right** (θ,θ)-derivation on R

1- Introduction

Thought the present paper R will denote an associative ring with center Z (R) .Recall that R is prime if aRb={0} implies that a=0 or b=0.As usually [x,y] will denote the commutator xy-yx .An additive subgroup U of R is called a Lie ideal of R if [u,r]∈U for all u∈U and r∈R .Suppose that ϕ,θ are endomorphisms of R .An additive mapping d:R→R is called a(ϕ,θ)–derivation if $d(xy)=d(x)\phi(y)+\theta(x)d(y)$, and a Jordan ($\phi,\theta$)-derivation if $d(x^2)= d(x)\theta(x) + d(x)\phi(x)$ For all x,y ∈ R .In this present paper we shall show that if a 2-torsion free prime ring R admits an additive mapping satisfying $d(u^2)=2d(u)\theta(u)$ for all u ∈U, then either d(U)={0} or U⊆Z (R) where U is a Lie ideal of R with u^2∈U for all u ∈U and θ is automorphism of R . Further, some more related results are also obtain. Final section of the present paper deals with the study of right (ϕ,θ)-derivation which acts also a homomorphism of the ring.

1-1 Defintion

An additive Mapping d:R→R is called a right (ϕ,θ)–derivation if d(xy)=d(x)ϕ(y) +d(y)θ(x) and a Jordan right(ϕ,θ)-derivation if d(x^2)= d(x)ϕ(x) +d(x) θ(x) for all x,y ∈ R.

2- Preliminaries

(2.1)Lemma: [7, Lemma 2]

If $U \not\subset Z(R)$ is a lie ideal of a 2-torsion free prime ring R and $a,b \in R$ such that $\boldsymbol{aUb=\{0\}}$, then $a=0$ or $b=0$.

(2.2) Lemma:[8 , Lemma 4]

Let G and H be additive groups and let R be a 2-trosion free ring, Let $f: G \times G \to H$ and $g: G \times G \to R$ be biadditive mappings Suppose that for each pair $a,b \in G$ either $f(a,b)=0$ or $g(a,b)^2=0$, in this case either $f=0$ or $g(a,b)^2=0$ for all $a,b \in G$.

(2.3)Lemma:[9]

Let R be a 2-torsion free prime ring and U a Lie ideal of R if admits a derivation d such that $\boldsymbol{d(U)^n=0}$ for all $u \in U$, where $n \geq 1$ is a positive integer then $d(u)=0$ for all $u \in U$.

(2.4)Lemma: [10 , Lemma 13]

Let R be a 2-torsion free semi prime ring if U is a commutative Lie ideal of R, then $\boldsymbol{U \subseteq Z(R)}$.

(2.5)<u>Lemma</u>:

Let R be a 2-torsion free ring and let U be a Lie ideal of R such that $u^2 \in U$ for all $u \in U$, suppose that θ is an endomorphism of R if $d : R \to R$ is an additive mapping satisfying $d(u^2) = 2d(u)\theta(u)$ for all $u, v \in U$,then.

(i) $d(uv + vu) = 2d(u)\theta(v) + 2d(v)\theta(u)$

(ii) $d(uvu) = d(v)\theta(u^2) + 3d(u)\theta(u)\theta(v) - d(u)\theta(u)\theta(v)$

(iii) $d(u)\theta(u)[\theta(u), \theta(v)] = d(u)[\theta(u), \theta(v)]\theta(v)$

(iv) $d[u, v][\theta(u), \theta(v)] = 0$ for all $u, v \in U$

(v) $d(vu^2) = d(v)\theta(u^2) + d(u)[3\theta(v)\theta(u) - \theta(u)\theta(v)] - d[u, v]\theta(u)$ for all $u, v \in U$.

Proof:

(i) $d(u + v)^2 = 2d(u + v)\theta(u + v)$

$$= 2d(u)\theta(u) + 2d(u)\theta(v) + 2d(v)\theta(u) + 2d(v)\theta(v)$$

and on the hand

$$d(u + v)^2 = d(u^2 + v^2 + uv + vu) = d(u^2) + d(v^2) + d(uv + vu)$$

Combining two relations

$$d(uv + vu) = 2d(u)\theta(v) + 2d(v)\theta(u)$$

(ii) $d(u(vu + uv) + (vu + uv)u)$ by (i)

$$= 2d(u)\theta(vu + uv) + 2d(vu + uv) + \theta(u)$$

$$= 2d(u)[\theta(v)\theta(u) + \theta(u)\theta(v)] + 2[d(v)\theta(u) + 2d(u)\theta(u)]\theta(u)$$

$$4d(v)\theta(u^2) + 6d(u)\theta(u) + 2d(u)\theta(u)\theta(v)$$

on the other hand.

$$d(u(vu + uv) + (vu + (uv)u) = d(uvu + u^2v + vu^2 + uvu)$$

$$= d(u^2v + vu^2) + 2d(uvu)$$

combining the above equation we get

$$d(uvu) = 2d(v)\theta(u^2) + 3d(u)\theta(v)\theta(u) - d(u)\theta(u)\theta(v)$$

(iii) by linearzing (i) on u we get

$$d(u + w)v(u + w)$$

$= d(v)\theta(u+w)^2 + 3d(u+w)\theta(v)\theta(u+w) - d(u+w)\theta(u+w)\theta(v)$

$= d(v)\theta(u^2) + d(v)\theta(w^2) + d(v)[\theta(u)\theta(w) + \theta(w)\theta(u)] + 3d(u)\theta(v)\theta(u) + 3d(u)\theta(v)\theta(w)$
$+ 3d(w)\theta(v)\theta(u) + 3d(w)\theta(v)\theta(w) - d(u)\theta(u)\theta(v) - d(w)\theta(u)\theta(v) - d(u)\theta(w)\theta(v) - d(w)\theta(w)\theta(v)$
----------------(2)

on the other hand.

$d(u+w)v(u+w) = d(uvu) + d(wvw) + d(uvw + d(uvw + wvu)$ ----------------(3)

combining (2) and (3) we arrive at

$d(uvw + wvu) = d(v)[\theta(u)\theta(v) + \theta(w)\theta(u)] + 3d(u)\theta(v)\theta(w)$

$+ 3d(w)\theta(v)\theta(u) - d(w)\theta(u)\theta(v) - d(u)\theta(w)\theta(v)$ --
--(4)

since $uv + vu$ and $uv - vu$ both belong to u we find that $2uv \in U$ for all $u, v \in U$.

Hence, by our hypothesis we find that

$d((2uv)^2) = 2d(2uv)\theta(2uv)$

i.e. $4d(uv)^2 = 8d(uv)\theta(uv)$ since char $R \neq 2$

we have $d(uv)^2 = d(uv)\theta(u)\theta(v)$. Replace w by $2uv$ in ---(4) and use the fact that char $R \neq 2$ to get

$d(uv(uv) + (uv)vu) = d(v)[\theta(u)\theta(uv) + \theta(uv)\theta(u)] + 3d(u)\theta(v)\theta(uv) + 3d(uv)\theta(v)\theta(u)\theta(v)$

$= d(v)[\theta(u^2)\theta(v) + \theta(u)\theta(v)\theta(u)] + 3d(u)\theta(v)\theta(u)\theta(v) - d(uv)\theta(v)\theta(u)$

$= d(v)[\theta(u^2)\theta(v) + \theta(u)\theta(v)\theta(u)] + 3d(u)[\theta(v^2)\theta(u)] - d(uv)\theta(v)\theta(u)$ -----------(5)

on the other hand

$d((uv)^2 + uv^2u) = 2d(uv)2\theta(u)\theta(v) + 2d(v)\theta(u^2)\theta(v) + 3d(u)\theta(u)\theta(v^2) - d(u)\theta(v^2)\theta(u)$ ------(6)

combining 5 and 6 we get

$d(uv)[\theta(u), \theta(v)] = d(v)[\theta(u), \theta(v)]\theta(u) + \theta(v)[\theta(u), \theta(v)]\theta(v)$ --------(7)

Replacing v by $u + v$ in (7) we have

$2d(u)\theta(u)[\theta(u), \theta(v)] + d(uv)[\theta(u), \theta(v)] = 2d(u)[\theta(u), \theta(v)]\theta(u) + d(v)[\theta(u), \theta(v)]\theta(u) + d(u)[\theta(u), \theta(v)]\theta(v)$

Now application of (7) yields (iii)

(iv) Lineariz (iii) on u to get

$d(u)\theta(u)[\theta(u), \theta(v)] + d(v)\theta(v)[\theta(u), \theta(v)] + d(v)\theta(u)[\theta(u), \theta(v)] + d(u)\theta(v)[\theta(u), \theta(v)]$

$= d(v)[\theta(w), \theta(v)]\theta(u) + d(v)[\theta(u), \theta(v)]\theta(u) + d(u)[\theta(u), \theta(v)]\theta(v) + d(v)[\theta(u), \theta(v)]\theta(v)$

for all $u,v \in U$

Now application of (7) and (ii) yields that

$d(v)\theta(u)[\theta(u),\theta(v)] + d(u)\theta(v)[\theta(u),\theta(v)]$

$= d(uv)[\theta u,\theta v]$ and hence

$\{d(uv) - d(v)\theta(u) - d(u)\theta(v)\}[\theta(u),\theta(v)] = 0$ for all $u,v \in U$ ---------(8)

combining

$d(uv+vu) = 2d(v)\theta(u) + 2d(u)\theta(v)$ and (8) we find that

$\{d(uv) - d(v)\theta(u) - d(w)\theta(v)\}[\theta(u),\theta(v)] = 0$--------------------(9)

for all $u,v \in U$

Further, combining of (8) and (9) yields the required result.

(iv) Replace v by $2vu$ in $d(uv+vu)$ and use the fact char $R \neq 2$ to get

$d(u2vu + 2vu^2) = 2d(2vu)\theta(u) + 2d(u)\theta(2vu)$

$= 4(d(vu)\theta(u) + d(u)\theta(v)\theta(u))$

$\therefore d(uvu + vu^2) = 2(d(vu)\theta(u) + d(u)\theta(v)\theta(u))$ for all $u,v \in U$------------(10)

Again me placing v by $2uv$ in $d(uv+vu)$

$d(u^2v + uvu) = 2d(uv)\theta(u) + d(u)\theta(u)\theta(v)$ for all $u,v \in U$ -----------(11)

Now combining (10) and (11), we get

$d(u^2v - vu^2) = 2d[u,v]\theta(u) + d(u)[\theta(u),\theta(v)]$ for all $u,v \in U$ -------(12)

Replacing u by u^2 in $d(uv+vu)$, we have

$d(u^2v + vu^2) = 2d(v)\theta(u^2) + 2d(u^2)\theta(v) + 2d(u)\theta(u)\theta(v)$

$= 2\{d(v)\theta(u^2) + 2d(u)\theta(u)\theta(v)\}$--------------------(13)

Hence, subtracting (12) from (13) and using the fact that characteristic of $R \neq 2$ we find that

$d(vu^2) = d(v)\theta(u^2) + d(u)\{3\theta(v)\theta(u) - \theta(u)\theta(v)\} - d[u,v]\theta(u)$ for all $u,v \in U$.

3- Right derivation and commutativity of prime ring .

(3-1)Theorem:

Let R be a 2-torsion free prime ring and let U be a lie ideal of R such that $u^2 \in U$ for all $u \in U$ suppose that θ is an outomorphism of R if $d : R \to R$ is an additive mapping satisfying $d(u^2) = 2d(u)\theta(u)$ for all $u \in U$ then either $\boldsymbol{d(U) = \{0\}}$ or $U \subseteq Z(R)$

proof: suppose that $U \not\subset Z(R)$ by Lemma(2-5) (iii) we have

$d(u)\{\theta(u^2)\theta(v) - 2\theta(u)\theta(v)\theta(u) + \theta(v)\theta(u^2)\} = 0$ for all $u, v \in U$ ---------(3.1)

Replacing $[u, w]$ for u in (3.1) we get

$d[u, w][\theta(u), \theta(w)]^2\theta(v) - 2[\theta(u), \theta(w)]\theta(v)[\theta(u), \theta(w)]\theta(v)\theta[u, w]^2$

$d[u, w]\{[\theta(u), \theta(w)]^2\theta(v) - 2[\theta(u), \theta(w)]\theta(v)[\theta(u), \theta(w)] + \theta(v)[\theta(u), \theta(w)]\}$ for all $u, v, w \in U$

Now, application of Lemma (2-5) (ii) yields that

$\theta^{-1}d([u, w]\theta^{-1}[\theta(u), \theta(w)]^2)U = \{0\}$

hence by Lemma 2.1 we find that R or each pair $u, w \in U$ either $[\theta(v), \theta(w)]^2 = 0$ or $d([u, w]) = 0$. This implies that either $[u, w]^2 = 0$ or $d([u, w]) = 0$. Note that the mappings $(u, w) \to [u, w]$ and $(u, w) \to d([u, w])$
satisfy the requirements of the Lemma (2-2).

Hence either $[u, w]^2 = 0$ for all $u, w \in U$ or $d([u, w]) = 0$ for all $u, w \in U$. If $[u, w]^2 = 0$ for all $u, w \in U$, then for each $u \in U$,
$(I_n(w))^2 = 0$ for all $w \in U$, where I_n is the inner derivation such that $I_n(w) = [u, w]$. Thus by the application of Lemma (2-3). we find that U is a commutative Lie ideal of R, and hence by Lemma (2.4) $U \subseteq Z(R)$, a contradiction. Hence we consider the remaining case that $d([u, w]) = 0$ for all $u, w \in U$
i.e. $d(uw) = d(wu)$ for all $u, w \in U$, since $wu - uw$ and $wu + uw$ both belong to U, we find that $2wu \in U$ for all $u, w \in U$.

This yields that $d((2wu)u) = d(u(2wu))$

Since $d(uv + vu)$ is valid in the present situation, we find that

$4d((wu)u) = d((2wu)u + u(2wu)$

$= 4d(u)\theta(w)\theta(u) + 2d(2wu)\theta(u)$

$= 4d(u)\theta(w)\theta(u) + 2d(wu + uw)\theta(u)$

$= 4[d(u)\theta(w)\theta(u) + d(u)\theta(u)\theta(w) + d(w)\theta(u^2)]$

since R is 2-torsion free, we obtain

$d((wu)u) = d(w)\theta(u^2) + d(u)\theta(w)\theta(u) + d(u)\theta(w)\theta(u)$ for all $u, w \in U$ ------------------(3-2) since $d([u,w])=0$ for all $u,w\in U$, using Lemma (2-5) (iv) and (3-2), we get

$2d(u)[\theta(u),\theta(w)]=0$ this implies that

$d(w)[\theta(u),\theta(w)]=0$ for all $u,w\in U$ ---------(3-3)

Now, replacing w by $2wv$ in (3-3) and using the fact that char $R\neq 2$ we get

$d(u)[\theta(u),\theta(w)]\theta(v)=0$ i.e., $\theta^{-1}d(u)\theta^{-1}[\theta(u),\theta(w)]U=\{0\}$

thus by Lemma (2-1), we find that for each $u\in U$, $\theta^{-1}[\theta(u),\theta(w)]=0$ or $\theta^{-1}d(w)=0$.

This implies that $[u,w]=0$ or $d(u)=0$

Now let $U_1=\{u\in U/[u,w]=0\, for \quad all \quad w\in U\}$

And $U_2=\{u\in U/d(u)=0\}$. Clearly U_1,U_2 are additive subgroups of U whose union is U, but a group can not be written as a union of two of its proper subgroups and hence by brauer's trick either $U=U_1$ or $U=U_2$ if $U=U_1$, then $[u,w]=0$ for all $u,w\in U$ and by using the similar arguments as above we get $U\subseteq Z(R)$, again a contradiction.

Hence we have the remaining possibility that $d(u)=0$ for all $u\in U$. i.e

$d(u)=\{0\}$ this completes the proof of the theorem.

<u>(3-2)Theorem :</u>

Let R be a 2-torsion free prime ring and U a Lie ideal of R such that

$u^2\in U$ for all $u\in U$. Suppose that θ is an automorphism of R. if $d:R\to R$ is an additive mapping satisfying $d(u^2)=2d(u)\theta(u)$ for all $u\in U$, then

$d(uv)=d(v)\theta(u)+d(u)\theta(v)$ for all $u,v\in U$.

<u>Proof:</u> suppose that $d=0$ on U. Since $2uv\in U$, $uv-vu$ both belong to U, we find that $2d(uv)=d(2uv)=0$. This implies that $d(uv)=0$ for all $u,v\in U$.

Hence the result is obvious in the present case. Therefore now assume that $d(U)\neq\{0\}$. Then by above theorem $U\subseteq Z(R)$. This R satisfies the property $d(u^2)=d(u)\theta(u)+\theta(u)d(u)$ for all $u\in U$. By theorem (3-2) of [3] we find that d(uv)=d(u) θ(v)+ θ(u)d(v) for all $u,v\in U$. for all $u,v\in U$.Further since $\theta(u)\subseteq Z(R)$.

We find that $d(uv)=d(v)\theta(u)+d(u)\theta(v)$ holds for all $u,v\in U$.

<u>(3-3)Corollary :</u>

Let R be a 2-torsion free prime ring if $d:R\to R$ if is a Jordan right derivation, then d is right derivation.

<u>(3-4)Theorem</u>:

Let R be a 2-torsion free ring and U a lie ideal of R such that $u^2\in U$ for all $u\in U$. Suppose that θ is an endomrphism of R and R has a commutator which is not a zero divisor. If $d:R\to R$ is an additive mapping satisfying $d(u^2)=2d(u)\theta(u)$ for all $u\in U$, then $d(uv)=d(v)\theta(u)+d(u)\theta(v)$

<u>Proof:</u> for any $u,v\in U$, define a map $f:U\times U\to R$, $f(u,v)=d(uv)-d(v)\theta(u)-d(u)\theta(v)$ since θ and d both are additive, f is additive in both the arguments and is zero if J is a right $\boldsymbol{(\theta,\theta)-derivation}$ **.**

Not that (8) is still valid in the present situation and hence we have

$[\theta(u),\theta(v)]f(u,v)=0$ for all $u,v\in U$ --------(3-4)

let a,b be an elements of U such that

$[\theta(a),\theta(b)]c=0$ limits that $c=0$

application of (3-4) yields that

$f(a,b)=0$ ---------------------------(3-5)

Replacing u by $u+a$ in (3-4) and using (3-4) , we find that

$[\theta(u),\theta(v)]f(a,v)+[\theta(a),\theta(v)]f(u,v)=0$ for all $u,v\in U$ ------------(3-6)

Replacing v by b in (3-6) and using (3-6) we have

$f(u,b)=0$ for all $u\in U$ -------(3-7)

Further, substituting $v+b$ for v in (3-6) and using (3-5) and (3-7) we get

$[\theta(u),\theta(b)]f(a,v)+[\theta(a),\theta(b)]f(u,v)=0$ for all $u,v\in U$ ---------(3-8)

Now replacing u by a in (3-8) and using the fact char $R\neq 2$, we have

$f(a,v)=0$ for all $v\in U$ ---------------(3-9)

combining of (3-8) and (3-9) yields that $[\theta(a),\theta(b)]f(u,v)=0$ this implies that $f(u,v)=0$ for all $u,v\in U$. i.e. d is a right $(\theta,\theta)-derivation$.

<u>(3-5)Theorem</u>: [2.Theorem 3-2]:

Let R be a prime ring and k a non-zero ideal of R, and let θ,ϕ be automorphisms of R.

Suppose that $d:R\to R$ is a $(\theta,\varphi)-\text{derivation}$ of R.

(i) if d acts as a homomorphism on k then $d=0$ on R.

(ii) if d acts as anti-homomorphism on k then $d=0$ on R.

In the present section our objective is to extend the above study to the right derivation of a prime ring R which acts either as a homomorphism or as an anti-homomorphism of R.

<u>(3-6)Theorem:</u>

Let R be a prime ring and k anon-zero ideal of R, and let θ be outomorphisms of R suppose $d:R\to R$ is a right $(\theta,\phi)-derivation$ of R.

(i) if d acts as an anti-homomorphism on K, then $d=0$ on R.

(ii) if d acts as a homomorphism on K, then $d=0$ on R.

proof:

(i) Let d act as an anti-homomorphism on k by our hypothesis we have

$d(yx)=d(x)\theta(y)+d(y)\phi(x)$ ---------(3-10)

in (3-10) replacing *y* by *yx*

$d(x)d(yx)=d(yx(x))=d(x)\theta(yx)+d(yx)\phi(x)$ ---------(3-11)

Now multiplying (3-10) in the left by $d(x)$

$d(x)d(yx) = d(x)d(x)\theta(y) + d(x)d(y)\phi(x)$

$d(x)d(yx) = d(x)d(x)\theta(y) + d(yx)\phi(x)$ --------------(3-12)

combining (3-11) and (3-12) we get

$d(x)d(x)\theta(y) = d(x)\theta(y)\theta(x)$ ---------------------(3-13)

in (3-13) replace y by yr to get

$\mathrm{d(x)d(x)\theta(y)\theta(r) = d(x)\theta(y)\theta(r)\theta(x)}$ ------------(3-14)

for all $x, y \in K$, and $r \in R$.

multiplying (3-13) on right by $\theta(r)$ and combining with (3-14), we obtain

$d(x)\theta(y)[\theta(r), \theta(x)] = 0$ ---------------------(3-15)

in (3-15) replacing y by ys we get

$d(x)\theta(y)\theta(s)[\theta(r), \theta(x)] = 0$ for all $x, y \in K$ and $r, s \in R$.

And hence

$\theta^{-1}d(x)yR[r, x] = \{0\}$ for all $x, y \in K$ and $r \in R$. Thus for each $x \in K$, the primeness of R forces that either $[r, x] = 0$ or $d(x)\theta(y) = 0$

let $K_1 = \{x \in K / d(x)\theta(y) = 0$ for all $y \in K$ and $r \in R$. Thus for each $x \in K$, the prime ness of R forces that either $[r, x] = 0$ or $d(x)\theta(y) = 0$.

Let $K_1 = \{x \in K / d(x)\theta(y) = 0 \quad for \quad all \quad y \in K\}$ and $K_2 = \{x \in K / [r, x] = 0 \quad for \quad all \quad r \in R\}$

Then clearly K_1 and K_2 are additive subgroups of K whose union is K by braur's trick, we have

$d(x)\theta(y) = 0$ for all $x, y \in K$ or $[r, x] = 0$ for all $x \in K$ and $r \in R$,

if $[r, x] = 0$, replace x by sx to get $[r, s]x = 0$ for all $x \in K$ and $r, s \in R$, this implies that $[r, s]Rx = \{0\}$.

The prime ness of R forces that either $x = 0$ or $[r, s] = 0$, but $K \neq \{0\}$.

We have $[r, s] = 0$ for all $r, s \in R$, i.e. R is commutative so

$d(xy) = d(x)\phi(y) + \theta(x)d(y)$ for all $x, y \in K$, i.e. d is a (ϕ, θ)**-*deriveation*** which acts as an anti-homomorphism on K . Hence by theorem (3-5) (ii) we have $d = 0$ on R.

Hencefor, we have remaining possibility that

$d(x)\theta(y) = 0$ for all $x, y \in K$.

Replace y by ry in (3-17), to get
$d(x)\theta(r)\theta(y)=0$ for all $x,y\in K$ and $r\in R$, and hence
$\theta^{-1}d(x)Ry=\{0\}$ this implies that
$\theta^{-1}(d(x))=0$ that is $d(x)=0$ for $x\in K$ (3-18)
Replace x by sx in (3-18) to get
$\theta(x)d(s)=0$ for all $x\in K$, $s\in R$---------------(3-19)
Replacing x by xr in (3-19) we get
$\theta(x)\theta(r)d(s)=0$ for all $x\in K$ and $r,s\in R$, and hence $xR\theta^{-1}d(s)=\{0\}$
Since R is prime, and K a non-zero ideal of R, we find that $d=0$ on R.
(ii) if d acts as a homomorphism on K then we have
$d(y)d(x)=d(yx)=d(x)\theta(y)+d(y)\phi(x)$ for all $x,y\in K$---------(3-20)
Replacing x by yx in (3-20) we get
$d(xy)d(y)=d(y)\theta(x)\theta(y)+d(xy)\theta(y)$ for all $x,y\in K$
$d(y)d(yx)=d(yx)\theta(y)+d(y)\phi(y)\phi(x)$
Now application of (3-20) yields $d(y)\phi(y)\phi(x)=d(y)\phi(y)\phi(x)$ this implies
$d(y)(d(y)-\phi(y))\phi(x)=0$ for all $x,y\in K$---------(3-21)
Replace x by rx in (3-21) to get
$d(y)d(y)-\phi(y)\phi(r)\theta(x)=0$ for all $x,y\in K$ and $r\in R$, and hence
$\theta^{-1}(d(y)(d(y)-\phi(y))Rx=\{0\}$ for all $x,y\in K$.
The prime ness of R forces that either $x=0$ or $\theta^{-1}d(y)(d(y)-\phi(y))=0$
Since K is a non-zero ideal of R, we have $\theta^{-1}d(y)(d(y)-\phi(y))=0$ this yields that
$d(y)(d(y)-\phi(y))=0$ this is $\boldsymbol{d(y^2)=d(y)\theta(y)}$, since d is a right $\boldsymbol{(\theta,\phi)-derivition}$ we find that $\boldsymbol{d(y)\theta(y)=0}$.
Linear zing the latter relation we have
$d(y)\theta(x)+d(x)\theta(y)=0$ for all $x,y\in K$-----------(3-22)
Replace x by xy in (3-22) to get
$d(y)\theta(x)\theta(y)=0$ for all $x,y\in K$ -----------------(3-23)
Substituting xs for $\boldsymbol{x}$ in (3-23) we get
$d(y)\theta(x)\theta(s)\theta(y)=0$ for all $x,y\in K$ and $s\in R$, and hence $\theta^{-1}d(y)xRy=\{0\}$.

This for each $y \in K$, the prime ness of R forces that either $y=0$ also implies that

$\theta^{-1}d(y)x=0$ that is $\theta(x)d(y)=0$------------------(3-24)

Now using similar techniques as used to get (I) from (3-17) we get the required result.

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