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Bernstein Polynomials Method for Solving Volterra and Fredhom Integro- Differential Equation

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Abstract:

In this paper, Bernstein polynomials method is used to find an approximate solution for Volterra and Fredholm integro-Differential equation of second kind. These polynomials are incredibly useful mathematical tools, because they are simply defined, can be calculated quickly on computer systems and represent a tremendous variety of functions. They can be differentiated and integrated easily.

1. Introduction:

New methods are always needed to solve integral equations because no single method works well for all such equations.

There has been considerable interest in solving differential and integral equation Using techniques which involve Bernstein polynomials method.

In this paper we introduce method to solve the following linear Volterra and Fredholm integro-Differential equations in the, form[6][2].

$$\begin{cases} y'(x) = f(x) + \lambda \int_a^x k(x,t)\,y(t)\,dt \\ y(a) = y_a \end{cases} \qquad a \le x \le b \qquad \ldots(1)$$

$$\begin{cases} y'(x) = f(x) + \lambda \int_a^b k(x,t)\,y(t)\,dt \\ y(a) = y_a \end{cases} \qquad a \le x \le b \qquad \ldots(2)$$

Where the functions $f(x)$ and the kernel $k(x,t)$ are known and $y(x)$ is the solution to be determined.

2. Bernstein Polynomials Method

Polynomials are incredibly useful mathematical tools as they are simply defined, can be calculated quickly on computer systems and represent a tremendous variety of functions.

The Bernstein polynomials of degree n are defined by [4], [2].

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} \qquad \text{for } i = 0,1,2,\ldots,n \qquad \ldots(3)$$

where

$\binom{n}{i} = \dfrac{n!}{i!(n-i)!}$, (n) is the degree of polynomials, (i) is the index of polynomials and (t) is the variable. In [0,1].

The exponents on the (t) term increase by one as (i) increases, and the exponents on the (1-t) term decrease by one as (i) increases.

The Bernstein polynomial of degree (n) can be defined by blending together two Bernstein polynomials of degree (n-1). That is, the nth degree Bernstein polynomial can be written as, [2].

$$B_k^n(t) = (1-t)B_k^{n-1}(t) + tB_{k-1}^{n-1}(t) \qquad \ldots(4)$$

Bernstein polynomials of degree (n) can be written in terms of the power basis. This can be directly calculated using the equation (3) and the binomial theorem as follows, [2].

$$B_k^n(t) = \binom{n}{k} t^k (1-t)^{n-k} = \sum_{i=k}^{n} (-1)^{i-k} \binom{n}{i}\binom{i}{k} t^i$$

Where the binomial theorem is used to Expand $(1-t)^{n-k}$.

The derivatives of the nth degree Bernstein polynomials are polynomials of degree (n-1)

$$\frac{d}{dt} B_k^n(t) = \frac{d}{dt}\binom{n}{k} t^k (1-t)^{n-k} = n\left(B_{k-1}^{n-1}(t) - B_k^{n-1}\right) \quad 0 \le \mathrm{k} \le n \qquad \ldots(5)$$

3. A Matrix Representation for Bernstein Polynomials

In many applications, a matrix formulation for the Bernstein polynomials is useful. These are straight forward to develop if only looking at a linear combination in terms of dot products. Given a polynomial written as a linear combination of the Bernstein basis functions [4][2].

$$B(t) = c_0 B_0^n(t) + c_1 B_1^n(t) + c_2 B_2^n(t) + \ldots + c_n B_n^n(t) \qquad \ldots(6)$$

It is easy to write this as a dot product of two vectors

$$B(t) = \left[B_0^n(t) \quad B_1^n(t) \quad B_2^n(t) \ldots B_n^n(t)\right] \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} \qquad \ldots(7)$$

which can be converted to the following form:

$$B(t)=\begin{bmatrix}1 & \mathrm{t} & \mathrm{t}^2 & \cdots & \mathrm{t}^{\mathrm{n}}\end{bmatrix}\begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & \mathrm{b}_{\mathrm{n}2} & \cdots & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix} \qquad \ldots (8)$$

where b_{nn} are the coefficients of the power basis that are used to determine the respective Bernstein polynomials, we note that the matrix in this case lower

triangular.

The matrix of derivatives of Brenstein polynomials

$$B'(t)=\begin{bmatrix}0 & 1 & 2\mathrm{t} & \cdots & \mathrm{nt}^{\mathrm{n-1}}\end{bmatrix}\begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & \mathrm{b}_{\mathrm{n}2} & \cdots & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix} \qquad \ldots (9)$$

4. Analysis of the Method

In this section, we give some applications of the proposed for obtaining the approximate solution of the linear integro-differential equations.

4.1 Solution of Volterro Integro-Differential Equation with Bernstein Polynomials

In this subsection Bernstein polynomials to find the approximate solution for Volterra integro-differential equation.

Let us reconsider the Volterra integro-differential equation of the second kind in equation (1).[5][1][3]

$$y'(x)= f(x)+\int_a^x k(x,t)y(t)dt \qquad \mathrm{x}\in[\mathrm{a},\mathrm{x}] \qquad \ldots (10)$$

And

Let $y(t)=B(t)=\begin{bmatrix}B_0^n(t) & B_1^n(t) & B_2^n(t)\ldots B_n^n(t)\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix}$

by using equation (7)

$y'(t) = n\left(B_{k-1}^{n-1}(t) - B_k^{n-1}\right)$ by using equation (5)

A applying the Bernstein polynomials method for equation (10), we get the following formula.

$$\begin{bmatrix}0 & 1 & B_{1-1}^{n-1}(t) \ldots B_n^{n-1}(t)\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix} = f(x) + \int_a^x k(x,t)[B_0^n(t)\ \ B_1^n(t) \ldots B_n^n(t)]\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix}dt$$

…(11)

by using equations(8) and (9), which can be converted to the following form:

$$\begin{bmatrix}0 1 & 2\mathrm{t}\cdots \mathrm{nt}^{\mathrm{n-1}}\end{bmatrix}\begin{bmatrix}b_{00} & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & \cdots & 0\\ \mathrm{b}_{20} & \mathrm{b}_{21} & \cdots & 0\\ \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix} = f(x) + \int_a^x k(x,t)\begin{bmatrix}1\,\mathrm{t}\cdots \mathrm{t}^{\mathrm{n}}\end{bmatrix}\begin{bmatrix}b_{00} & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & \cdots & 0\\ \mathrm{b}_{20} & \mathrm{b}_{21} & \cdots & 0\\ \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix}dt$$

… (12)

4.2 Solution of Fredholm Integro-differential Equation with Bernstein Polynomials

In this subsection Bernstein polynomials using to find the approximate solution for Fredholm integro-differential equation.

Let us reconsider the Fredholm integro-differential equation of the second kind in equation (2),[5][1][3].

$$y'(x) = f(x) + \int_a^b k(x,t)y(t)dt \qquad \mathrm{x} \in [\mathrm{a,b}] \qquad \ldots (13)$$

And

Let $y(t) = B(t) = \begin{bmatrix}B_0^n(t) & B_1^n(t) & B_2^n(t) \ldots B_n^n(t)\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix}$ by using equation (7)

$y'(t) = n\left(B_{k-1}^{n-1}(t) - B_k^{n-1}\right)$ by using equation (5)

A applying the Bernstein polynomials method for equation (10), we get the following formula.

$$\begin{bmatrix}0 & 1 & B_{1-1}^{n-1}(t)\dots B_n^{n-1}(t)\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix} = f(x)+\int_a^b k(x,t)[B_0^n(t)\ \ B_1^n(t)\dots B_n^n(t)]\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix}dt$$

…(14)

by using equations (8) and (9), which can be converted to the following form:

$$\begin{bmatrix}0 1 & 2\mathrm{t}\cdots \mathrm{nt}^{\mathrm{n\text{-}1}}\end{bmatrix}\begin{bmatrix}b_{00} & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & \cdots & 0\\ \mathrm{b}_{20} & \mathrm{b}_{21} & \cdots & 0\\ \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix} = f(x)+\int_a^b k(x,t)\begin{bmatrix}1\,\mathrm{t}\cdots \mathrm{t}^{\mathrm{n}}\end{bmatrix}\begin{bmatrix}b_{00} & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & \cdots & 0\\ \mathrm{b}_{20} & \mathrm{b}_{21} & \cdots & 0\\ \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix}dt$$

… (15)

now to find all integration in equations(12) and (15).

Then in order to determine $c_0, c_1, \dots, c_n$, we need n equations;

Now Choice $x_i, i = 1,2,3, \dots n$ in the interval [a, b], which gives (n) equations.

Solve the (n) equations by Gauss elimination to find the values $c_0, c_1, \dots, c_n$.

The following algorithm summarizes the steps for finding the approximate solution for the second kind of linear Volterra and Fredholm integro-differential equations.

5. Algorithm (BPM)

Input: $(f(t), k(t,s), y(s), a, b, x)$,

Output: polynomials of degree n

Step1:

Choice n the degree of Bernstein polynomials

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} \qquad \text{for } i = 0,1,2,\dots,n$$

Step2:

Put the Bernstein polynomials in linear Volterra Fredholm integro-differential equation of second kind.

$$\mathrm{y}'(\mathrm{t}) = \begin{bmatrix}0 & 1 2\mathrm{t} & \dots \mathrm{nt}^{\mathrm{n\text{-}1}}\end{bmatrix}\begin{bmatrix}b_{00} & 0 & 0 & \cdots & 0\\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0\\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ \mathrm{b}_{\mathrm{n}0} & \mathrm{b}_{\mathrm{n}1} & \mathrm{b}_{\mathrm{n}2} & & \mathrm{b}_{\mathrm{nn}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\\ \vdots\\ c_n\end{bmatrix} = f(x)+\int_a^x k(x,t)[B_0^n(t)\ \ B_1^n(t)\dots B_n^n(t)]\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_n\end{bmatrix}dt$$

$$\mathrm{y}'(\mathrm{t}) = \left[0 \;\; 1\,2\mathrm{t} \;\; \ldots \mathrm{nt}^{\mathrm{n-1}}\right] \begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0 \\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{b}_{\mathrm{n0}} & \mathrm{b}_{\mathrm{n1}} & \mathrm{b}_{\mathrm{n2}} & & \mathrm{b}_{\mathrm{nn}} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = f(x) + \int_a^b k(x,t)[B_0^n(t) \;\; B_1^n(t) \ldots B_n^n(t)] \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} dt$$

Step3:

Compute

$$\int_a^x k(x,t)\left[1\,\mathrm{t} \cdots \mathrm{t}^{\mathrm{n}}\right] \begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0 \\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{b}_{\mathrm{n0}} & \mathrm{b}_{\mathrm{n1}} & \mathrm{b}_{\mathrm{n2}} & & \mathrm{b}_{\mathrm{nn}} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} dt$$

$$\int_a^b k(x,t)\left[1\,\mathrm{t} \cdots \mathrm{t}^{\mathrm{n}}\right] \begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0 \\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{b}_{\mathrm{n0}} & \mathrm{b}_{\mathrm{n1}} & \mathrm{b}_{\mathrm{n2}} & & \mathrm{b}_{\mathrm{nn}} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} dt$$

Compute $\left[0\,1\,2\mathrm{t} \cdots \mathrm{nt}^{\mathrm{n}}\right] \begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0 \\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{b}_{\mathrm{n0}} & \mathrm{b}_{\mathrm{n1}} & \mathrm{b}_{\mathrm{n2}} & & \mathrm{b}_{\mathrm{nn}} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

and $\left[0\,1\,2\mathrm{t} \cdots \mathrm{nt}^{\mathrm{n}}\right] \begin{bmatrix} b_{00} & 0 & 0 & \cdots & 0 \\ \mathrm{b}_{10} & \mathrm{b}_{11} & 0 & \cdots & 0 \\ \mathrm{b}_{20} & \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{b}_{\mathrm{n0}} & \mathrm{b}_{\mathrm{n1}} & \mathrm{b}_{\mathrm{n2}} & & \mathrm{b}_{\mathrm{nn}} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

Step4:

Compute $c_0, c_1, \cdots, c_n$, where $x_i, i = 1,2,3,\cdots,n$, $x_i \in [a,b]$

End:

Example1:

In this example we will consider the following linear Volterra integro-differential equation of the second kind with initial value type:

$$y'(x) = 1 + \sin(x) + \int_0^x y(t)dt \qquad y(0) = -1$$

and with the exact solution $y(x) = \frac{1}{4}e^x - \frac{3}{4} - \frac{1}{2}\cos(x)$

Now to derive the solution by using the Bernstein polynomials method, we can use the following scheme:

When Bernstein polynomials algorithm is applied both sides in example. And choose the degree of Bernstein polynomials n=2, we get:

$$-2c_0(1-x) + 2c_1(1-x) - 2c_1 x + 2c_2 x = 1 + \sin(x) + \int_0^x \left[c_0(1-t)^2 + 2c_1 t(1-t) + c_2 t^2\right]dt$$

Next

$$-2c_0(1-x) + 2c_1(1-x) - 2c_1 x) + 2c_2 x = 1 + \sin(x) + \left(2c_0\int_0^x (1-t)^2 dt + 2c_1\int_0^x t(1-t)dt + c_2\int_0^x t^2 dt\right)$$

And after performing the integration.

$$-2c_0(1-x) + 2c_1 - 4c_1 x + 2c_2 x = 1 + \sin(x) - \frac{1}{3}c_0(1-x)^3 + \frac{1}{3}c_0 + c_1 x^2 - \frac{2}{3}c_1 x^3 + \frac{1}{3}c_2 x^3$$

$$(-2(1-x) + \frac{1}{3}c_0(1-x)^3 - \frac{1}{3})c_0 + (2(1-x) - 2x - x^2 + \frac{2}{3}x^3)c_1 + (2x - \frac{1}{3}x^3)c_2 = 1 + \sin(x)$$

Then in order to determine c_0, c_1 *and* c_2, we need three equation;

Now Choose $x_i, i = 1,2$ in the interval [0, 1],with substation in the initial condition in the equation $y'(x) = -2a_0(1-x) + 2a_1 - 4c_1 x + 2c_2 x$ which gives three equations.

$$c_0 = -1$$

$$\frac{-31}{24}c_0 - \frac{1}{6}c_1 + \frac{23}{24}c_2 = 1.4794$$

$$\frac{-1}{3}c_0 - \frac{7}{3}c_1 + \frac{5}{3}c_2 = 1.8415$$

Solve the three equation by Gauss elimination to find the values c_0, c_1 *and* c_2.

To get

$c_0 = -1$
$c_1 = -0.5783$
$c_2 = 0.0953$

Then the solution of linear Volterra integro-differential equation of the second kind is:

$$y(x) = (c_0 - 2c_1 + c_2)x^2 - 2((c_0 - c_1)x + c_0$$

$$y(x) = 0.5219x^2 + 0.8434x - 1$$

Approximated solution for some values of (x) by using Bernstein polynomials method and exact values $y(x) = \frac{1}{4}e^x - \frac{3}{4} - \frac{1}{2}\cos(x)$ of Example1, depending on least square error (L.S.E), $Error = \sum_{k=1}^{m} (y_{Exact}(x) - y_{Approximation}(x))^2$ are presented in Table(1) and figure(1).

Example2:

Consider the following linear Fredholm integro-differential equation of the second kind with initial condition:

$$y'(x) = xe^x + e^x - x + \int_0^1 xy(t)dt \quad , y(0) = 0$$

and with the exact solution $y(x) = xe^x$

Now to derive the solution by using the Bernstein polynomials method, we can use the following scheme:

When Bernstein polynomials algorithm is applied both sides in example. And choice the degree of Bernstein polynomials n=2, we get:

$$-2a_0(1-x) + 2a_1(1-x) - 2c_1x + 2c_2x = xe^x + e^x - x + \int_0^1 x\left[c_0(1-t)^2 + 2c_1t(1-t) + c_2t^2\right]dt$$ Next

$$-2c_0(1-x) + 2c_1(1-x) - 2c_1x) + 2c_2x = xe^x + e^x - x + \left(2c_0\int_0^1 x(1-t)^2dt + 2c_1\int_0^1 xt(1-t)dt + c_2\int_0^1 xt^2dt\right)$$

And after performing the integration.

$$-2c_0(1-x) + 2c_1 - 4c_1x + 2c_2x = xe^x + e^x - x + \left[\frac{c_0}{3} + c_1 + \frac{c_2}{3}\right]x - \frac{2}{3}c_1$$

$$(-2 + \frac{2}{3}x)c_0 + (\frac{8}{3} - 5x)c_1 + (\frac{5}{3}x)c_2 = xe^x + e^x - x$$

Then in order to determine c_0, c_1 *and* c_2, we need three equation;

Now Choose $x_i, i = 1,2$ in the interval [0, 1], with substation in the initial condition in the equation $y'(x) = -2a_0(1-x) + 2a_1 - 4c_1 x + 2c_2 x$ which gives three equations.

$$c_0 = 0$$

$$\frac{-7}{6}c_0 + \frac{1}{6}c_1 + \frac{5}{6}c_2 = \frac{3}{2}e^{0.5} - \frac{1}{2}$$

$$\frac{-1}{3}c_0 - \frac{7}{3}c_1 + \frac{5}{3}c_2 = 2e - 1$$

Solve the three equation by Gauss elimination to find the values c_0, c_1 *and* c_2.
To get

$$c_0 = 0$$

$$c_1 = -0.1839$$

$$c_2 = 2.4045$$

Then the solution of linear Fredholm integro-differential equation of the second kind is:

$$y(x) = (c_0 - 2c_1 + c_2)x^2 - 2((c_0 - c_1)x + c_0$$

$$y(x) = 2.7723x^2 - 0.3678x$$

Approximated solution for some values of (x) by using Bernstein polynomials method and exact values $y(x) = xe^x$ of Example2, depending on least square error (L.S.E), $Error = \sum_{k=1}^{m}(y_{Exact}(x) - y_{Approximaiton}(x))^2$ are presented in Table(2) and figure(2).

6. Conclusion

In This paper presents the use of Bernstein polynomials method, for solving linear Volterra and Fredholm integro-differential equation of the second kind. From solving numerical examples the following points have been identified:

1. This method can be used to solve the all kinds of linear Volterra and Fredholm integro-differential equation.
2. It is clear that using the Bernstein polynomial basis function to approximate when the n^th degree of Bernstein polynomial is increases the error is decreases.
3. We can see also from Figure(1) and Figure(2) that the approximation is good. And when comparisons approximation solution with exact solution that the Bernstein polynomial method is very effective and convenient.

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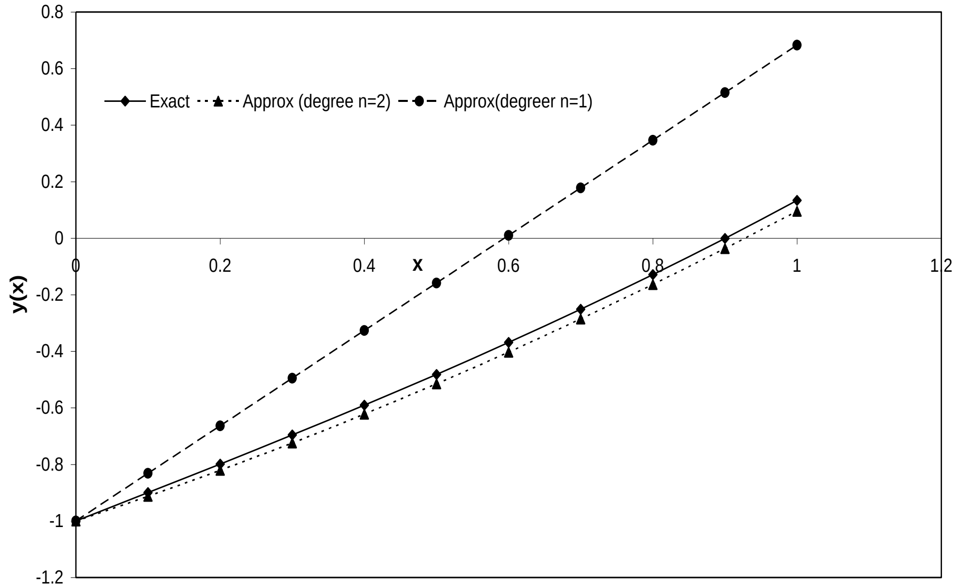
Table (1) The results of Example1 using (BPM) algorithm

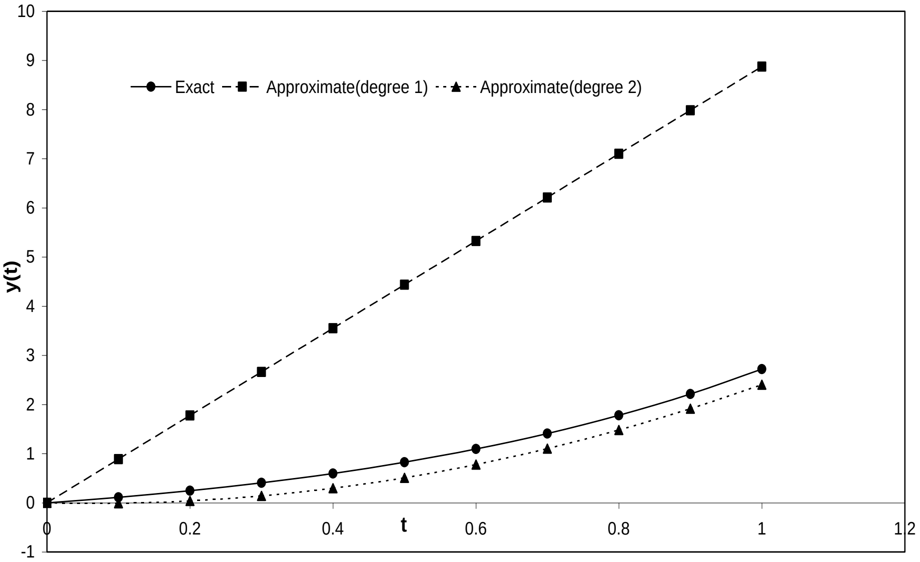
x	*Exact* $y(x)$	Approximation $y(x)$ of degree(n=1)	Approximation $y(x)$ of degree (n=2)
0	-1	-1	-1
0.1	-0.8998	-0.8317	-0.9131
0.2	-0.7987	-0.6634	-0.8212
0.3	-0.6958	-0.4951	-0.7243
0.4	-0.5903	-0.3268	-0.6223
0.5	-0.4815	-0.1586	-0.5153
0.6	-0.3687	0.0097	-0.4033
0.7	-0.2514	0.1780	-0.2862
0.8	-0.1290	0.3463	-0.1641
0.9	-0.00083145	0.5146	-0.0369
1	0.1335	0.6829	0.0953
L.S.E $Error = \sum_{k=1}^{10} (y_{Exact}(x) - y_{Approximation}(x))^2$		1.357912	0.010062



Approxim

Table (2) The results of Example2 using (BPM) algorithm

T	*Exact* $y(t)$	Approximation $y(t)$ of degree(n=1)	Approximation $y(t)$ of degree(n=2)
0	0	0	0
0.1	0.1105	0.8873	-0.0091
0.2	0.2443	1.7746	0.0373
0.3	0.4050	2.6620	0.1392
0.4	0.5967	3.5493	0.2964
0.5	0.8244	4.4366	0.5092
0.6	1.0933	5.3239	0.7773
0.7	1.4096	6.2112	1.1010
0.8	1.7804	7.0986	1.4800
0.9	2.2136	7.9859	1.9145
1	2.7183	8.8732	2.4045
L.S.E $Error = \sum_{k=1}^{10}(y_{Exact}(t) - y_{Approximation}(t))^2$		170.244	0.790595



Figure(2)
Approximation and Exact solution of linear Fredholm integro-differential equation of example2

طريقة متعددة حدود برنشتن لحل معادلة فولتيره وفريدهولوم التفاضلية

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الخلاصة:

في هذا البحث استعملت طريقة متعددة حدود برنشتن لإيجاد الحل التقريبي لمعادلة فولتيرة وفريدهولوم التفاضلية الخطية من النوع الثاني. وأن متعددات الحدود تستعمل بشكل كبير في الطرق الرياضية وذلك لبساطة تعريفها، والحل بهذه الطريقة يتقارب بسرعة وبخطوات قليلة. كما يمكن تفاضلها وتكاملها بسهولة