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Hydraulically Fractured Formations: Parameters Controlling Performance and Maximum Number of Fractures

Abstract- *Horizontal wells essentially increase the area of contact between wellbores and reservoir fluids to some extent. Hydraulic fractures increase this area significantly and develop the vertical permeability. Because of these two techniques, well deliverability or productivity index can be increased to the limit required by the worldwide needs. Several models have been derived for the productivity index of fractured formations and the maximum number of fractures for both finite and infinite reservoirs. The models were developed based on the idea that the total pressure drop in the wellbore can be estimated as the sum of different pressure drops caused by different flow regimes. This pressure drop is necessary for the fluid to flow from the reservoir toward the wellbore. It is well known that the developed flow regimes in the area around the horizontal wells or the hydraulic fractures are not the same as the flow regimes at far distance from wellbores, which is close to the outer boundaries. Therefore, four flow regimes were expected to develop in infinite acting reservoir: pseudo radial flow at the outer boundaries, elliptical flow in the area between wellbores and the regions close to the outer boundaries, formation linear flow in the area between fractures toward wellbores and fractures, and finally fractures linear flow, while pseudo-steady state flow was the expected flow regime for the case of limited reservoirs. Each one of these flow regimes contributes to the total pressure drop necessary for producing certain flow rate in addition to the pressure drop caused by the damage zones resulted from horizontal well drilling and completion, hydraulic fracturing process, and fluid flow chocking effect. In this study, the effects of the anisotropy, fracture dimensions, radius of drainage area, number of fractures and fracture conductivity on productivity index had been investigated. A novel approach for the maximum number of fractures necessary for a specific productivity index was introduced in this paper. The model had been examined for two field cases taken from literatures. The calculated flow rates by this model showed good agreement with the measured flow rates.*

Keywords- *Hydraulic Fractures; Fractures number; Controlling Fractures*

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1. Introduction

Hydraulic Fracturing is one of the most important techniques of stimulation, which involves the creation of fractures in the carbonate reservoirs. This technique has been commonly used in low permeability of oil and tight gas reservoirs to improve the productivity of the horizontal wells. The multi-stage hydraulic fractures have been widely applied to enhance the naturally fractured connectivity in the carbonate formations as well as increase the contact area between wellbore and the rock matrix.

In the production process and management of oil and gas reservoirs, the productivity index is important parameter, which refers to the volume of produced hydrocarbons due to pressure drop in the reservoir. This index and geometry of drainage area are depended on permeability and fluid properties in porous media. Recently, many models have been investigated the productivity index in the fractured carbonate reservoirs [1-4]. The effect of the number of fractures and fracture half-length were investigated by developing models to predict the performance of hydraulic fractured reservoirs [5].

The performance of hydraulic fractures in horizontal wells depends on the penetration ratio in the vertical and horizontal direction. Several models were introduced by researchers to investigate the effect of flow regimes and drainage area shape on the productivity wells in heterogeneous and tight gas fractured reservoirs [6-8].

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The flow regimes in transverse fractures have been studied by Daal and Economides in 2006 [9] when they used analytical models to predict production outcome from horizontal completions and introduced approach to determine productivity index under two-phase flow in horizontal wells. This approach also can be applied to detect the fracture stages number that required for the horizontal wells.

2. Model Description

By considering, a horizontal well is extending in an infinite acting reservoir having circular drainage area and the hydraulic fractures are propagating transversely to the well. Figure 1 shows the configuration of the whole system. The following assumptions are necessary to derive the model:

1- The fractures are symmetrical in dimensions.
2- The distance (spacing) between fractures are equal.
3- Reservoir fluids are slightly compressible.
4- Single-phase fluid flow.

3. Fluid Flow Regimes

It is well known that the developed flow regimes in the area around the horizontal wells or the hydraulic fractures are not the same as the flow regimes at far distance from wellbores, which is close to the outer boundaries. Therefore, four flow regimes are expected to develop in infinite acting reservoir. The following flow regimes are expected to develop:

I. Pseudo-radial Flow

Pseudo-radial flow regime is the dominant flow in the reservoir drainage area far from the vicinity of the wellbore when reservoir fluids flow in the XY plane radially toward the fractures area such as shown in Figure 2. This flow is characterized by constant value (0.5) for the dimensionless pressure derivative curves on log-log plot of dimensionless pressure and dimensionless time. The pressure drop due to the pseudo-radial flow can be calculated by assuming that the hydraulic fractures area represents the wellbore radius. Therefore, the governing equation for this flow derived from Darcy law is:

$$\Delta P_{RF} = \frac{q\mu B}{2\pi kh}\ln(2r_e / L) \quad (1)$$

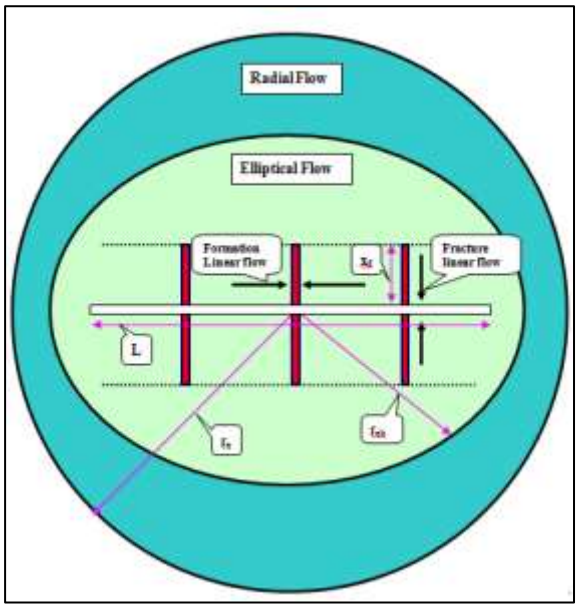

Figure 1: Horizontal well and hydraulic fractures in a circular reservoir

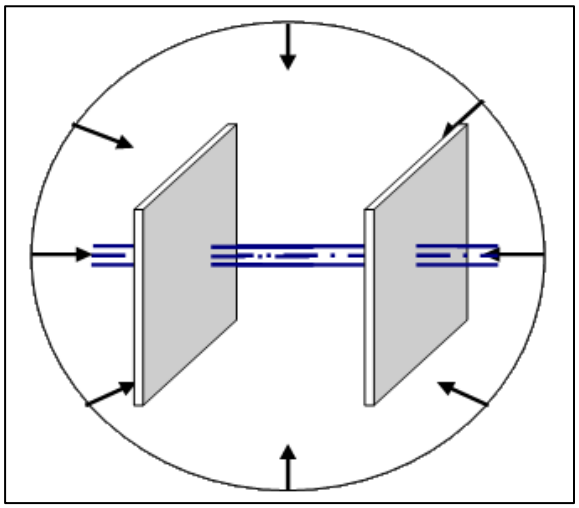

Figure 2: Pseudo-radial flow regime of hydraulic fractures

II. Elliptical flow

Elliptical flow regime indicates elliptical flow toward the fracture such as shown in Figure 3. This flow regime was described initially by Tiab [10]. It often occurs in the case of infinite conductivity fractures. However, it can be seen in a few cases of uniform flux fractures. This type of flow depends on the number of fractures and spacing between them. The governing equation for elliptical flow is:

$$\Delta P_{EF} = \frac{q\mu B}{2\pi kh}\left[\ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right)+\left(\frac{I_{ani}h}{L}\ln\left(\frac{I_{ani}h}{r_{we}(I_{ani}+1)}\right)\right)\right]$$

(2)

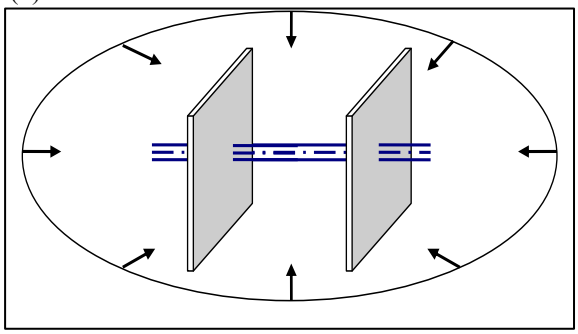

Figure 3: Elliptical flow regime for hydraulic fractures

III. Formation Linear Flow

Reservoir fluids flow from the areas around each fracture toward the fractures vertical face in the XZ plan as shown in Figure 4. This flow takes place in the vicinity of the wellbore. It is one direction linear flow. The pressure drop caused by this flow can be written as:

$$\Delta P_{LF} = \frac{q\mu BLh}{2n^2 x_f h_f k}$$

(3)

IV. Fracture Linear Flow

This flow regime is observed inside the fractures where reservoir fluids flow linearly in one direction toward the wellbore as shown in Figure 5. In general, the pressure drop resulted from fractures linear flow is not significantly important at all times. The governing equation for the pressure drop of this flow regime can be approximated as:

$$\Delta P_{FLF} = \frac{q\mu B x_f}{2nw_f k_f h_f}. \qquad (4)$$

V. Boundary dominated flow

For finite reservoir, boundary dominated flow definitely develops at late time when the production pulse reaches the boundary. The time for this flow to be developed depends on the reservoir properties. The equation that describes the pressure drop due to the boundary dominated flow in fractured formation is:

$$\Delta P_{BDF} = \frac{q\mu B}{2\pi k\left(\frac{1}{2}\ln\left(\frac{4A}{\gamma C_{Af} r_f^2}\right)\right)} \qquad (5)$$

$$r_f^2 = \frac{2Lx_f}{\pi} \qquad (6)$$

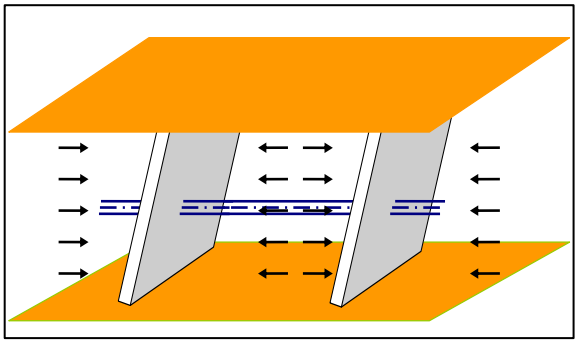

Figure 4: Formation linear flow

VI. Pressure drop due to skin factor

Skin factor has remarkable impact on the pressure drop of hydraulically fractured horizontal wells. Two types of skin are considered in this case. The first type is the mechanical skin factor resulted from drilling, completion, and fracturing process. The second one is the chock flow skin factor or the skin factor due to the moving from wide drainage area toward narrow one as the fluid reaches the fractures area. The following model is proposed by Brown and Economides [11]:

$$s = s_m + s_C = s_m + \frac{kh}{k_f w_f}\left[\ln\left(\frac{h}{2r_w}\right)-\frac{\pi}{2}\right]. \qquad (7)$$

The pressure drop due to skin factor is:

$$\Delta P_s = \frac{q\mu B}{4n\pi kh_f}.$$

(8)

4. Total Pressure Drop

The total pressure drop in the wellbore can be estimated as the sum of different pressure drops caused by different flow regimes in addition to the pressure drop caused by the damage zones resulted from horizontal well drilling and completion, hydraulic fracturing process, and fluid flow chocking effect. Then, total pressure drop can be written as:

$$\Delta P_t = \Delta P_{RF} + \Delta P_{EF} + \Delta P_{LF} + \Delta P_{FLF} + \Delta P_S .$$
(9)

Substitute Eqs. (1), (2), (3), (4), and (6) into Eq. (9), then total pressure drop is given by:

$$\Delta P_t = \frac{q\mu B}{2\pi kh}\left[\begin{array}{l} \ln(2r_e/L)+\ln\left(\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right) \\ +\left(\dfrac{I_{ani}h}{L}\ln\left(\dfrac{I_{ani}h}{r_{we}(I_{ani}+1)}\right)\right) \\ +\dfrac{\pi hL}{n^2x_fh_f}+\dfrac{\pi x_f kh}{nw_fk_fh_f} \\ +\dfrac{1}{2n}\left[s_m+\dfrac{kh}{k_fw_f}\left(\ln\left(\dfrac{h}{2r_w}\right)-\dfrac{\pi}{2}\right)\right] \end{array}\right] \quad (10)$$

Based on Eq. (10), three models for dimensionless pressure, flow rate, and productivity index can be introduced. Assuming constant flow rate and $(Lw = 2r_{eh})$, Eq. (10) can be written for dimensionless pressure as:

$$P_D = \ln(2r_{eD})+\ln(2.058)+\left(I_{ani}h_D\ln\left(\frac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right) + \frac{\pi h_D}{n^2x_{fD}h_{fD}}+\frac{\pi h_D}{nC_{fD}h_{fD}}+ \frac{1}{2n}\left[s_m+\frac{h_D}{x_{fD}C_{fD}}\left\{\ln\left(\frac{h_D}{2r_{wfD}}\right)-\frac{\pi}{2}\right\}\right] \quad (11)$$

Eq. (10) can also be written for the dimensionless flow rate assuming constant pressure drop as:

$$q_D = \frac{1}{\left[\begin{array}{l} \ln(2r_{eD})+\ln(2.058)+\left(I_{ani}h_D\ln\left(\dfrac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right)+ \\ \dfrac{\pi h_D}{n^2x_{fD}h_{fD}}+\dfrac{\pi h_D}{nC_{fD}h_{fD}}+ \\ \dfrac{1}{2n}\left[s_m+\dfrac{h_D}{x_{fD}C_{fD}}\left\{\ln\left(\dfrac{h_D}{2r_{wD}}\right)-\dfrac{\pi}{2}\right\}\right] \end{array}\right]} \quad (12)$$

The steady state productivity index of hydraulically fractured horizontal wells can be written in dimensionless form as:
In Field units, the productivity index is given by:

$$J_D = \frac{1}{\left[\begin{array}{l} \ln(2r_{eD})+\ln(2.058)+\left(I_{ani}h_D\ln\left(\dfrac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right) \\ +\dfrac{\pi h_D}{n^2x_{fD}h_{fD}}+\dfrac{\pi h_D}{nC_{fD}h_{fD}}+ \\ \dfrac{1}{2n}\left[s_m+\dfrac{h_D}{x_{fD}C_{fD}}\left\{\ln\left(\dfrac{h_D}{2r_{wD}}\right)-\dfrac{\pi}{2}\right\}\right] \end{array}\right]} \quad (13)$$

$$J = CJ_D \quad (14)$$

C - $\dfrac{141.2\mu\beta}{kh}$ (15)

$$r_e = \frac{43560A}{\eta} \quad (16)$$

$$a = \frac{L}{2}\sqrt{0.5+\left[0.25+(2r_{eh}/L)^4\right]^{0.5}} \quad (17)$$

$$r_{we} = \sqrt{\frac{x_fh_f}{\pi}} \quad (18)$$

$$q_D = \frac{141.2q\mu B}{kh\Delta P_t} \quad (19)$$

$$P_D = \frac{kh\Delta P_t}{141.2q\mu B} \quad (20)$$

$$r_{eD} = r_e/L \quad (21)$$

$$h_D = h/L \quad (22)$$

$$h_{fD} = h_f/L \quad (23)$$

$$x_{fD} = x_f/L \quad (24)$$

$$r_{weD} = r_{we}/L \quad (25)$$

$$r_D = r_w/L \quad (26)$$

$$w_{fD} = w_f/L \quad (27)$$

$$C_{fD} = \frac{k_fw_f}{kx_f} \quad (28)$$

For reservoirs having rectangular drainage area rather than circular area as shown in Figure 6, the

dimensionless pseudo-steady state productivity index can be written as:

$$J_D = \frac{1}{\left[\begin{array}{l} \ln\sqrt{\left(2\pi x_{eD} y_{eD} / \gamma x_{fD} C_{Af}\right)} \\ +\ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right) \\ +\left(I_{ani} h_D \ln\left(\frac{I_{ani} h_D}{r_{weD}(I_{ani}+1)}\right)\right) \\ +\frac{\pi h_D}{n^2 x_{fD} h_{fD}} + \frac{\pi h_D}{n C_{fD} h_{fD}} \\ +\frac{1}{2n}\left[s_m + \frac{h_D}{x_{fD} C_{fD}}\left\{\ln\left(\frac{h_D}{2r_{wD}}\right) - \frac{\pi}{2}\right\}\right] \end{array}\right]} \quad \text{.....(29)}$$

Where:

$$x_{eD} = 2x_e / L \qquad (30)$$

$$y_{eD} = 2y_e / L \qquad (31)$$

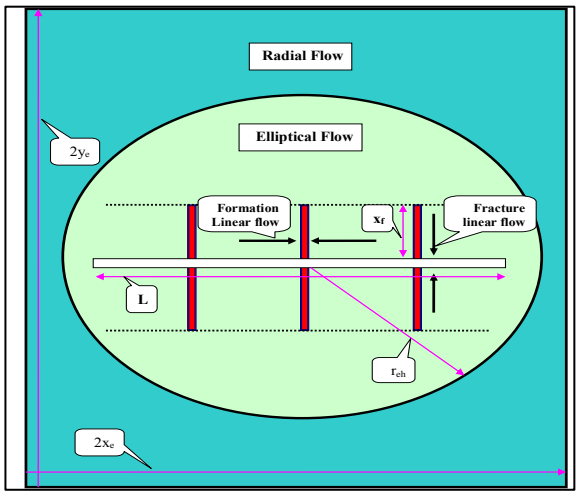

Figure 6: Horizontal well and hydraulic fractures in a rectangular reservoir

5. Maximum Number of Fractures

From the two model of the dimensionless productivity index presented by equations (13) and (29), the maximum number of hydraulic fractures can be estimated knowing the fact that this number represents the fractures that do not change the productivity index:

$$\frac{\partial J_D}{\partial n} = 0 \qquad (32)$$

$$n_{\max} = \left| \frac{-\frac{\pi h_D}{2x_{fD} h_{fD}}}{\left[\frac{\pi h_D}{C_{fD} h_{fD}} + \frac{1}{2}\left[s_m + \frac{h_D}{x_{fD} C_{fD}}\left\{\ln\left(\frac{h_D}{2r_{wD}}\right) - \frac{\pi}{2}\right\}\right]\right]} \right| \qquad (33)$$

Therefore, the maximum number of fractures for the two cases, the steady-state and the pseudo-steady state can be written as:

6. Effect of Proppant Number

The modified proppant number is calculated based on different configurations reservoir. For rectangular shape drainage area, the proppant number is:

$$N_P = nI_x I_y C_{fD} \qquad (34)$$

Using the definition of the above-mentioned proppant number, the productivity index for rectangular reservoirs can be written as:

$$J_D = \frac{1}{\left[\begin{array}{l} \ln\left(\frac{\sqrt{2\pi x_{eD} y_{eD}}}{\gamma x_{fD} C_{Af}}\right) + \ln\left(\frac{a+\sqrt{a^2-\left(\frac{L}{2}\right)^2}}{\frac{L}{2}}\right) + \left(I_{ani} h_D \ln\left(\frac{I_{ani} h_D}{r_{weD}(I_{ani}+1)}\right)\right) + \frac{\pi h_D}{n^2 x_{fD} h_{fD}} + \\ \frac{4\pi h_D x_{fD}^2}{y_{eD} x_{eD} N_P h_{fD}} + \frac{1}{2n}\left[s_m + \frac{4\pi h_D x_{fD}}{x_{eD} y_{eD} N_P}\left\{\ln\left(\frac{h_D}{2r_{wD}}\right) - \frac{\pi}{2}\right\}\right] \end{array}\right]}$$

(35)

The following indications can be inferred from the relationship between the proppant number and the productivity index:

1-There two impacts for the proppant number on productivity index as shown in Figures. 7, 8, 9 and 10: The first impact, for short fracture length, is positive i.e. the productivity index increase as the fracture length increases. The second one, for long fracture length, is negative i.e. the productivity index decreases as the fracture length increases. Therefore, for each proppant number there is certain value for the fracture length where the productivity index reaches its maximum value. The reason for this behavior refers to the fact that the increasing of the fracture length leads to increase the area of the fracture's surface exposures to the reservoir fluid. This definitely means increasing the flow rate and hence the productivity index. At same point, the productivity index is no longer increases with the increasing of fracture length. This point indicates the maximum productivity index for certain proppant number. Beyond this point, the increasing of the fracture length leads to decreasing the productivity index due to the fact the great percentage of reservoir fluid flows only to the outermost fractures only. No more fluid flows to the inner fractures, therefore, flow rate decreases and productivity index decreases also.

2-This impact is observed for different numbers of fractures and different horizontal wellbore penetration ratios.

3- For the designed productivity index and reservoir configurations, the fracture dimensions can be determined based on the maximum productivity index for each proppant number.

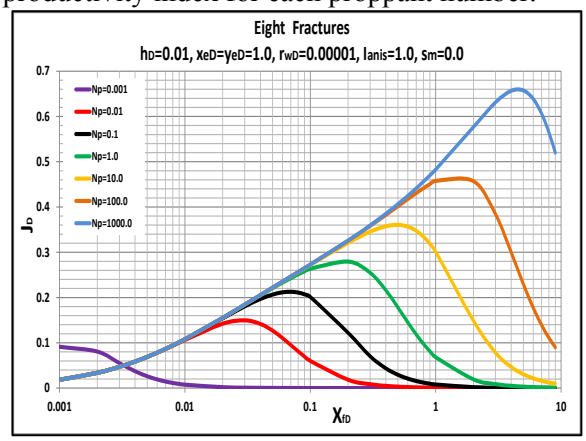

Figure 7: Productivity Index for Eight Fractures

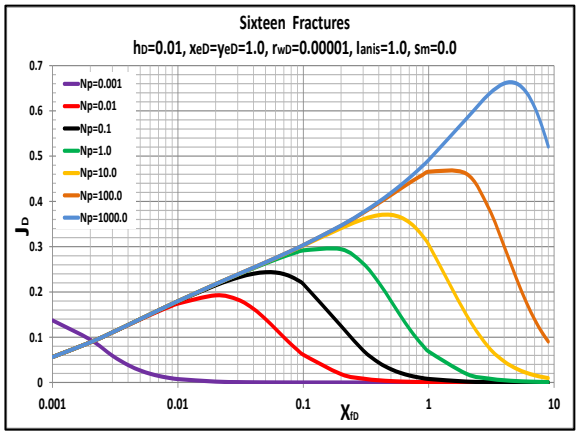

Figure 8: Productivity Index for Sixteen fractures.

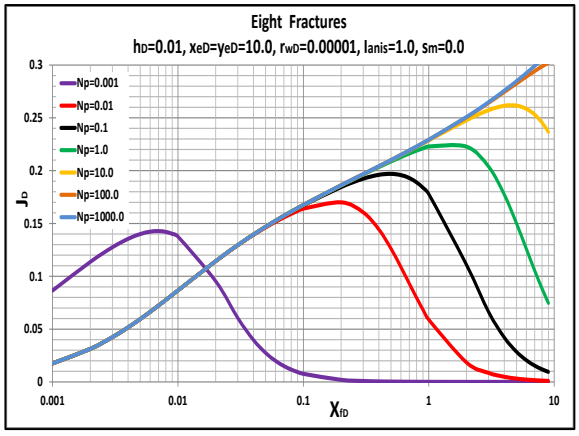

Figure 9: Productivity Index for Eight Fractures.

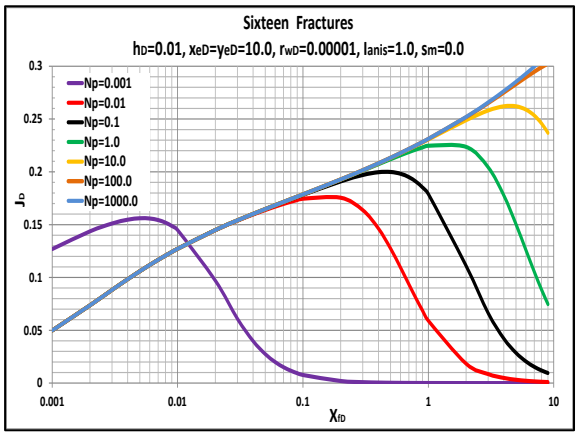

Figure 10: Productivity Index for Sixteen fractures.

7. Effect of Vertical Penetration Ratio

Even though hydraulic fracturing process has been a common application in the petroleum industry during the last two decades, the final output of this process is significantly affected by several factors. The successful process has to produce maximum actual production from the total reserve in the formation. Fracture dimensions (half fracture length, fracture width, and fracture height) are of great importance in the performance as are the orientation of the fractures as well as the rock and fluid properties. Typically, it is preferred that the fracture height be equal to the formation height, where fully-penetrating fractures can be produced. Unfortunately, the fractures can't always penetrate totally the formation where partially penetrating fractures may be produced. Partially penetrating hydraulic fractures are undesirable stimulation process due to the possibility of reducing the expected production rate of the fractured formation. However, fully penetrating fractures in a reservoir with water and oil in contact may lead to an early or immediate water production. Therefore, partially penetrating fractures may be the only way to prevent the production of unwanted water.

The productivity index of fractured formation is greatly affected by the vertical penetration ratio. This ratio is defined as the parentage of fracture height to the formation height. It can be seen in Figures. 11, 12, 13 and 14 that the productivity index has similar behavior for the penetration ratio regardless the number of fractures or the type of reservoir. The following points are observed:

1- For all cases, there is no remarkable difference in the productivity index as the penetration ratio decreases for both short $(x_{fD} < 0.001)$ and long $(x_{fD} > 1.0)$ fracture length. For short-length fractures, the reason for this conclusion might be referred to the non-sensible change in the surface area of fracture that allows for reservoir fluid to through fractures. While for long-length fractures,

the reason is the flowing of reservoir fluid toward the outermost fractures only when the penetration ratio increases gradually to reach the formation height.

2- The significant impact of the penetration ratio on productivity index is seen in the moderate fracture length where the productivity index increase as the penetration ratio increase. The reason for this behavior can be explained due to the fact that belongs to the increasing of the surface area of flow given by the fractures when the penetration ratio increases. As a result, the flow rate increase and the productivity index increases also.

3- The productivity index is constant $(J_D = 0.7)$ for steady-state flow regardless the number of fractures, fracture conductivity as shown in Figures 15 and 16, the horizontal wellbore penetration ratio and the vertical penetration ratio when the fracture length is long. The constant productivity index corresponds to the beginning of the pseudo-radial flow in the infinite acting reservoirs. In addition, the point at which $(J_D = 0.7)$ can be used to determine fracture dimensions and proppant number.

5- The productivity index increases significantly at high values of fracture length as shown in Figures 11 and 12. The relationship between productivity index and fracture length is positive constant slope straight line. The slope is almost equal to (5) regardless the number of fractures, fracture conductivity, vertical penetration ratio and horizontal wellbore penetration ratio. The reason for this behavior might be understood as the proportional decreasing in the flow rate and pressure drop. The beginning of the straight line is an indication for the fracture length at which the boundary dominated flow is reached.

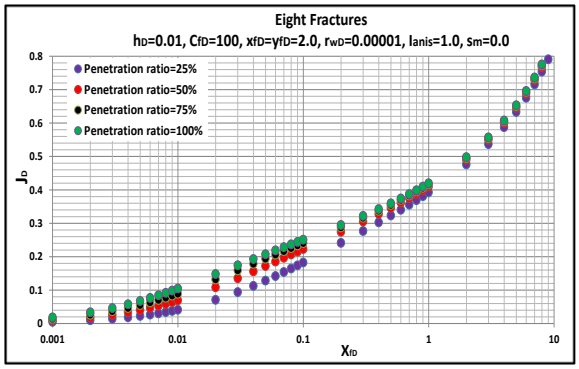

Figure 11: Pseudo-steady state Productivity Index for Eight Fractures

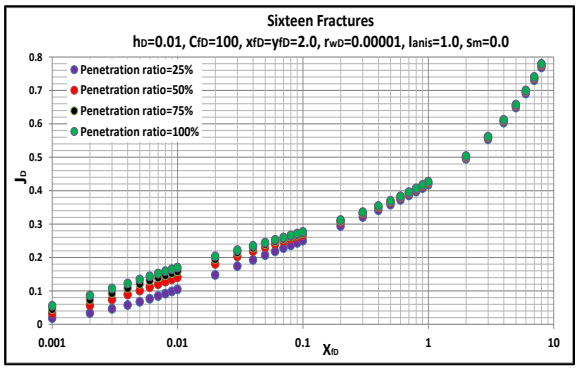

Figure 12: Pseudo-steady state Productivity Index for Sixteen

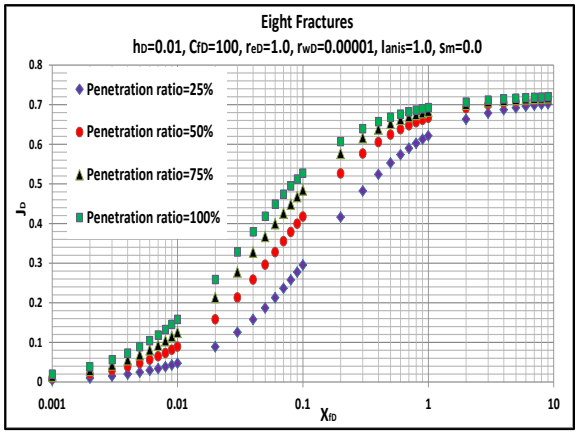

Figure 13: Productivity Index for Eight Fractures

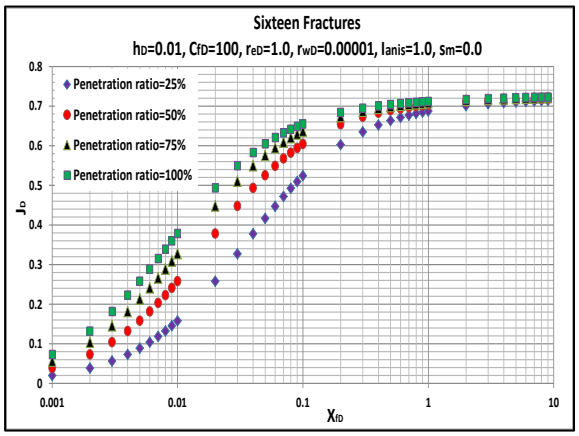

Figure 14: Productivity Index for Sixteen fractures.

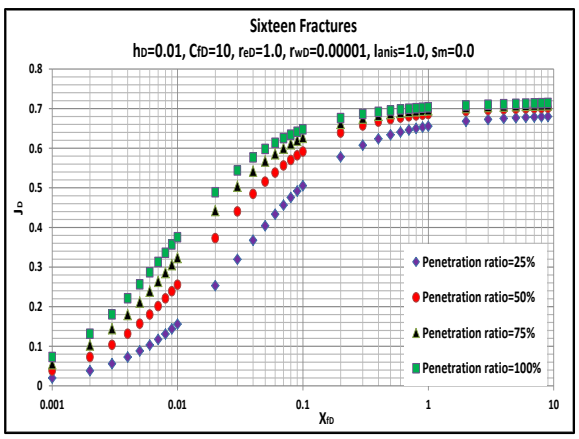

Figure 15: Productivity Index for Eight Fractures, C_{fD}=10

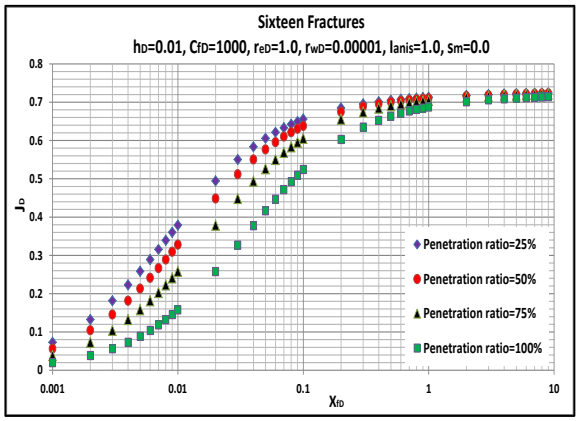

Figure 16: Productivity Index for Sixteen fractures, C_{fD}=1000.

8. Effect of Horizontal Wellbore Penetration Ratio

Horizontal wellbore penetration ratio is defined as the ratio of the horizontal wellbore length to the reservoir boundary length parallel to the wellbore. The best scenario for this ratio is to be $\left(\frac{L}{2y_{eD}} = 1.0\right)$, i.e. the wellbore is fully penetrating the formation. This would increase the drainage area of the reservoir that can be undergone production. As this ratio decreases less than (1.0), the flow rate rate and the productivity index notably decreases as shown in Figures. 17 and 18. For short length fractures, there are no recognizable differences in the productivity index for different horizontal wellbore lengths. However, the differences are easily recognized for long length fractures. This behavior shows identical trend for different numbers of fractures and fracture conductivities.

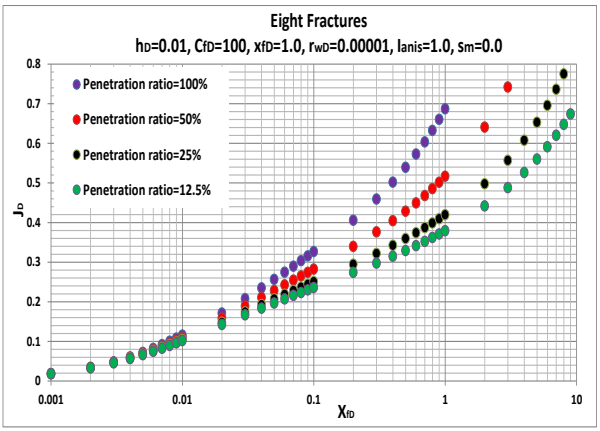

Figure 17: Productivity Index for Eight Fractures

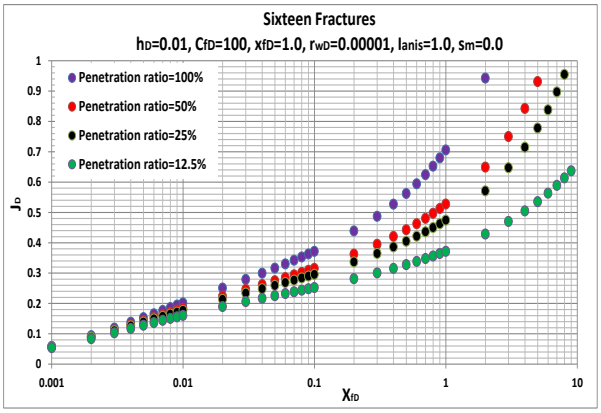

Figure 18: Productivity Index for Sixteen fractures

9. Model Application

The developed model has been examined for two field cases. The two cases have been taken from literatures [5, 7].

Case 1:

Pay zone thickness	46 ft
Reservoir permeability	1.3 md
Average reservoir	2380 psi
Oil viscosity	3.5 cp
Horizontal wellbore	939 ft
Wellbore radius	0.222 ft
Skin factor	4
Fracture length	
Fracture No. 1	836.4 ft
Fracture No. 2	688.8 ft
Fracture No. 3	705.2 ft
Estimated fracture width	0.2 in
Fracture permeability	30000 md
Drainage Area	60 acres
Bottom hole flowing	910 psi
Measured flow rate	41 STB/D
Formation volume	1.13 res-

Due to a symmetricity of fractures lengths, average fracture length has been used in the calculations$\left(x_{favg} = 371.73\ ft\right)$.

$$r_e = \sqrt{\frac{435604}{\pi}} = 9123 ft$$

$$a = \frac{L}{2}\sqrt{0.5 + \left[0.25 + (2r_{eh}/L)^4\right]^{0.5}} = 9746$$

$$r_{we} = \sqrt{\frac{x_f h_f}{\pi}} = 73.8 ft$$

$$r_{eD} = r_e / L = 0.9716$$

$$h_D = h / L = 0.049$$

$$h_{fD} = h_f / L = 0.049$$

$$x_{fD} = x_f / L = 0.396$$

$$r_{weD} = r_{we} / L = 0.0786$$

$$r_{wD} = r_w / L = 0.000236$$

$$w_{fD} = w_f / L = 0.0000177$$

$$C_{fD} = \frac{k_f w_f}{k x_f} = 1.03465$$

Pseudo-radial flow is not expected to be developed. Therefore, elliptical flow, formation linear flow and fracture linear flow are the only flow regimes that expected to be developed. The dimensionless flow rate can be calculated as:

$$q_D = \frac{1}{[1.36 - 0.057 + 0.8813 + 1.0116 + 0.7279]} = 0.255$$

Then, the calculated flow rate is:

$$q_{calculated} = \frac{q_D kh\Delta P_t}{141.2uB} = 40.12\ stb/day$$

The error can be estimated using the measured and calculated flow rate:

$$Error\% = \frac{41 - 40.12}{41} * 100 = 2\%.$$

Case 2

Pay zone thickness	39 ft
Reservoir permeability	7.5 md
Average reservoir pressure	2602 psi
Oil viscosity	4.8 cp
Horizontal wellbore length	1820 ft
Wellbore radius	0.19 ft
Skin factor	0
Fracture length	
Fracture No. 1	246 ft
Fracture No. 2	246 ft
Fracture No. 3	246 ft
Fracture No. 4	246 ft
Estimated fracture width	0.9 in
Fracture permeability	30000 md
Drainage Area	16 acres
Bottom hole flowing	1279 psi
Measured flow rate	128 STB/D
Formation volume factor	1.084res-

$$r_e = \sqrt{\frac{43560A}{\pi}} = 471.1ft$$

$$r_{we} = \sqrt{\frac{x_f h_f}{\pi}} = 39.1\ ft$$

Because $r_e < L/2$, the pseudo-radial flow regime close to the outer boundaries is not expected to occur. The configuration of the reservoir is rectangular (1820*383 ft). Therefore elliptical flow, formation linear flow and fracture linear flow regimes only might be developed in the vicinity of the wellbore.

$$h_D = h/L = 0.0214$$

$$h_{fD} = h_f/L = 0.0214$$

$$x_{fD} = x_f/L = 0.0676$$

$$r_{weD} = r_{we}/L = 0.0215$$

$$r_{wD} = r_w/L = 0.0001$$

$$w_{fD} = w_f/L = 0.0000412$$

$$C_{fD} = \frac{k_f w_f}{k x_f} = 2.44$$

The dimensionless flow rate is:

$$q_D = \frac{1}{[0.7217 - 0.015 + 2.904 + 0.322 + 0.05]} = 0.25$$

Then, the calculated flow rate is:

$$q_{calculated} = \frac{q_D kh\Delta P_t}{141.2uB} = 132 stb/day$$

The error can be estimated using the measured and calculated flow rate:

$$Error\% = \frac{132 - 128}{128} * 100 = 3\%$$

10. Conclusions

1- The modified proppant number and the two penetration ratios, the vertical and horizontal, have significant impacts on the productivity index of hydraulically fractured formations.

2- The impacts are identical regardless the number of fractures, fracture conductivity and reservoir configurations.

3- For certain proppant number, the productivity index has two different behaviors with fracture length. For short fracture length, the index increases with the increasing of fracture length. For long fracture length, the index decreases with the increasing of fracture length.

4- There is specific fracture length for each proppant number that gives maximum productivity index.

5- For infinite acting reservoir, the productivity index increases with the increasing of fracture length. At a certain fracture length, the index is no longer increased with fracture length. The maximum value of productivity index is (0.7). This value indicates the beginning of late radial flow.

6- For finite acting reservoir, the productivity index has constant slope straight line with long fracture length. The slope is almost (5). The beginning point of this line indicates the beginning of boundary dominated flow.

7- There are no significant impacts for the vertical penetration ratio on productivity index for both short and long fracture length. However, the remarkable impacts can be observed for moderate fracture length.

8- There is no significant impact for the horizontal wellbore penetration ratio on productivity index for short fracture length, but

there is distinguished impact for long fracture length.

Nomenclatures

A	Drainage area, acres.
a	Half the major axis of drainage ellipse and defined in Eq. (A-2), ft
B	Formation volume factor res-bbl/STB.
C_{Af}	Shape factor of fractured reservoir.
C_{fD}	Fracture conductivity
h	Reservoir height, ft.
h_f	Fracture height, ft.
I_{ani}	Anisotropy factor
k	Reservoir permeability, md.
k_f	Fracture permeability, md.
L	Horizontal wellbore length, ft
N_P	Proppant number.
n	Number of fractures.
ΔP	Pressure drop, psi
q	Flow rate, STB/D.
r_e	Reservoir radius, ft.
r_{eh}	Hydraulic radius of drainage area, ft.
r_f	Radius defined in Eq. (10), ft
r_w	Wellbore radius, in.
r_{we}	Radius defined in Eq. (A-3), ft
s	Skin factor.
x_f	Fracture half length, ft.
x_e	Reservoir width, ft
w_f	Fracture width, in.
y_e	Reservoir length, ft.
μ	Viscosity, cp.
γ	Constant

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