Analysis of Pretensioned Partially Prestressed Concrete Beams

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Abstract

Deflection of partially prestressed concrete beams is investigated using the finite element method taking in to account the plasticity of steel, nonlinearity of concrete in compression and tension softening of concrete. Embedded bar approach is used to represent the steel reinforcement and prestressing tendon in concrete layer. Elastic perfectly-plastic approach has been employed to model the compressive behaviour of the concrete.

The yield condition is formulated in terms of the first two-stress invariants. The movement of the subsequent loading surfaces is controlled by the hardening rule, which is extrapolated from the uniaxial stress-strain relationship defined by a parabolic function. Concrete crushing is a strain controlled phenomenon, and can be monitored by a fracture surface similar to the yield surface. A smeared fixed crack approach is used to model the behaviour of the cracked concrete, with a tensile strength criterion to predict crack initiation. The steel is considered as an elastic perfectly plastic material with linear strain hardening, steel reinforcement is assumed to have similar tensile and compressive stress-strain relationship. The calculated and the observed effects have shown a satisfactory agreement compared with experimental results.

تحليل ا لعتبات الخرسانية مسبقة الاجهاد جزئيا

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الخلاصة

استخدمت طريقة العناصر المحددة لتحليل العتبات الخرسانية مسبقة الاجهاد جزئيا تم اخذ سلوك الحديد اللدن بنظر الاعتبار. استخدم اسلوب القضيب المطمور في الخرسانة لتمثيل حديد التسليح والتسليح مسبق الشد، فضلا عن السلوك اللاخطي للخرسانة في حالة الانضغاط وانفعالات الشد المرنة للخرسانة مثل سلوك الخرسانة بعد الخضوع في حالة الانضغاط كمادة مرنه تامة اللدونه أو كمادة مرنه مع انفعالات لدنه متصلده. عبر عن شروط الخضوع بدلالة أول متغيرين للإجهاد . حكمت حركة سطوح التحميل بقاعدة التصلب التي تتحدد من علاقة الإجهاد–الانفعال الأحادي المحور والمعبر عنها بدلالة القطع المكافئ. ان سحق الخرسانة هي ظاهرة محكومة بالانفعال ومنظمة بسطح سحق يشبه سطح الخضوع .لقد استخدم أسلوب الشق الثابت لتمثيل سلوك الخرسانة المتشققة مع شرط الشد للتنبؤ بحدوث الشق . كما اخذ بنظر الاعتبار تأثير صلابة الشد في الخرسانة المتشققة .ان حديد التسليح قد اعتبر كمادة مرنة تامة اللدونه مع تصلب انفعال خطي .واعتبر سلوك حديد التسليح متشابها تحت تأثير قوى الشد والانضغاط . تم تحليل عدة أمثله ذات نتائج عمليه منشوره و أظهرت توافقا جيدا مع النتائج العدديةالمستحصلة في الدراسة الحالية .

1. Introduction

Prestressing technique uses tendons placed completely outside the concrete section; the tendons are used to prestress the structural member longitudinally and are generally becoming desirable tools for rehabilitation and strengthening existing structures having insufficient strength and/or excessive deflection and cracking. When the prestressed and non-prestressed reinforced concrete is present in a concrete member (partially prestressed concrete) the flexural strength is essentially supplied by the tendons with the non-prestressed steel playing a minor presented role for certain types of construction, a full combination of prestressed and reinforced concrete may be the best design making use of the advantages of the both.

Nelson [1] presented a method to calculate elastic flexural stress in partially prestressed concrete beams in which cracking can be expected at service load.

Based on a series of laboratory tests on precast prestressed T and I beam, Nawy and Huang [2] proposed cracking width and deflection formula for evaluation of the serviceability of such member.

Kang and Scordelis [3] presented method of accounting for both nonlinearity of reinforced and prestressed concrete frames. The method includes the time dependent effects due to load history, temperature history, creep, shrinkage, aging of concrete and prestress relaxation.

Naaman *et al.* [4] studied flexural ductility in partially prestress concrete member under static loading and identified similarities between reinforced prestress and partially prestress concrete using nonlinearity analyses models.

Cambell and Cheuined [5] studied the influence of the quantity of unbounded non-prestress steel on the stress in the prestress steel at ultimate in an unbounded partially prestressed concrete beams.

Robentrost [6] studied the ductility and nonlinearity over load behaviour of partially prestressed concrete flexural member. These works were limited to post tension bounded partially prestressed concrete beams subject predominantly to flexural under static, short term loading.

Eray *et al.* [7] tested tension for four different type of cast-in-place inserts, wire-formed and bolt-types inserts, and investigated their behaviour under conditions present in prestressed concrete bridge girders. They have studied the effects of parameters such as the level of axial compression, reinforcement details, and interaction of double inserts on the behaviour of the inserts and they have calculated numerically.

Shih-Ho and Naaman [8] proposed a simplified expression for computing the short term elastic deflection of a prestressed two-way slab, which is based on results from extensive finite element analyses and are applicable to the computation of live load deflection.

2. Steel Representation

The strain in the prestressing steel is

$$\varepsilon = -Kz + \lambda + f_{pe} / E_p \tag{1}$$

where Kz is the beam curvature, λ is the strain at the chosen references axis of the beam f_{pe} is the effective prestressing and E_p is Young modulus of prestressing reinforcing steel.

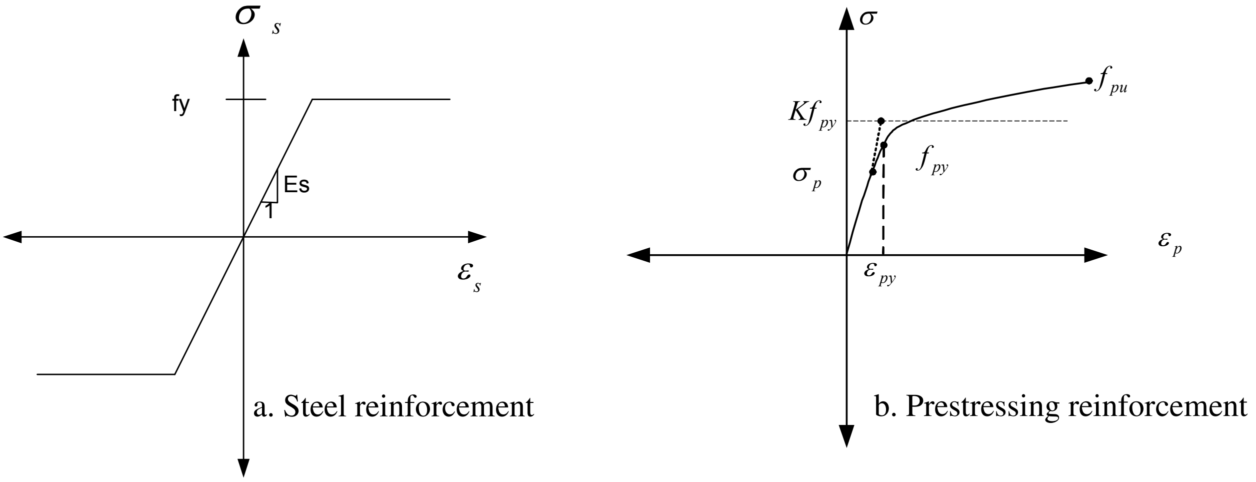
The non-prestress steel is assumed to be elastic-perfectly plastic, characterized by Young's elastic modulus E_s and axial yield stress f_y, see Fig. 1. a.

For the prestressing tendon, the stress strain formula proposed by Menegotto and Pinto which was used by Naaman [4], is adopted in the present study, see Fig. 1. b.

$$\sigma_p = E_P \varepsilon_P \left[Q + (1-Q) \Bigg/ \left[1 + \left(\frac{E_P \varepsilon_P}{K f_{py}} \right)^N \right]^{1/N} \right] \tag{2}$$

where
$$Q = \frac{f_{pu} - K f_{py}}{E_p \varepsilon_{pu} - K f_{py}} \tag{3}$$

and f_{py} is the yield stress, f_{pu} and ε_{pu} are the ultimate stress and strain respectively; and N, K and Q are empirical parameters whose values are recommended by reference [4], as 6.06, 1.0325 , and 0.00625, respectively .



a. Steel reinforcement

b. Prestressing reinforcement

Fig. 1. Stress strain relations for steel and prestressing reinforcement.

3. Finite Element Method

In the present study nonlinear finite model is presented to predict the behaviour of reinforced concrete beams under sustained loading. Eight nodded thick plate (Serendipity) element was used for the idealization of concrete member.

4. Embedded Bar

In the present study, an approach similar to that adopted by Ranjbaran [9] is used. The reinforced bar or pretension tendon which represent two nodded or three nodded axial element is embedded anywhere within an element in the mid-surface. Perfect bond is assumed between the reinforcement and the surrounding concrete.

5. Material Modelling

Based on the flow theory of plasticity, the nonlinear compressive behavior of concrete is modelled. Adopting Kupfer's results [10], the yield condition for the beams can be written in terms of the stress components as:

$$f(\sigma)=\{1.355[(\sigma_x^2+\sigma^2_y-\sigma_x\sigma_y)+3(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2)]+0.355\sigma_0(\sigma_x+\sigma_y)\}^{1/2}=\sigma_0 \qquad (4)$$

where (σ_0) is the equivalent effective strength taken as compressive strength (f_c') obtained from uniaxial test. Both perfect-plastic and strain hardening plasticity approaches are employed and the one dimensional illustration is shown in Fig. 2.

The crushing type of concrete is a strain-controlled phenomenon. A simple way is used by converting the yield criterion in stresses into the yield criterion directly in terms of the strain, thus crushing condition can be expressed in terms of the total strain components as given by Hinton, and Owen [11]

$$1.355\ [(\varepsilon_x^2+\varepsilon^2_y-\varepsilon_x\varepsilon_y)+0.75(\gamma_{xy}^2+\gamma_{xz}^2+\gamma_{yz}^2)]+0.355\varepsilon_u(\varepsilon_x+\varepsilon_y)=\varepsilon_u^2 \qquad (5)$$

When Equation (5) is satisfied, the strain (ε_u) reaches the crushing surface, and the concrete is assumed to lose all its characteristics of strength and stiffness.

The response of concrete in tension is assumed to be linearly elastic until the fracture surface is reached. Cracks are assumed to form in planes perpendicular to the direction of maximum principal tensile stress if the maximum stress reaches the specified concrete tensile strength. After cracking has occurred, a gradual release of the concrete stress component normal to cracked plane is adopted according to a tension-stiffening diagram illustrated in Fig. 3. The process of loading and unloading of cracked concrete is also shown in Fig. 3. A reduced shear modulus taken as a function of the current tensile strain is used to simulate aggregate interlock and dowel action.

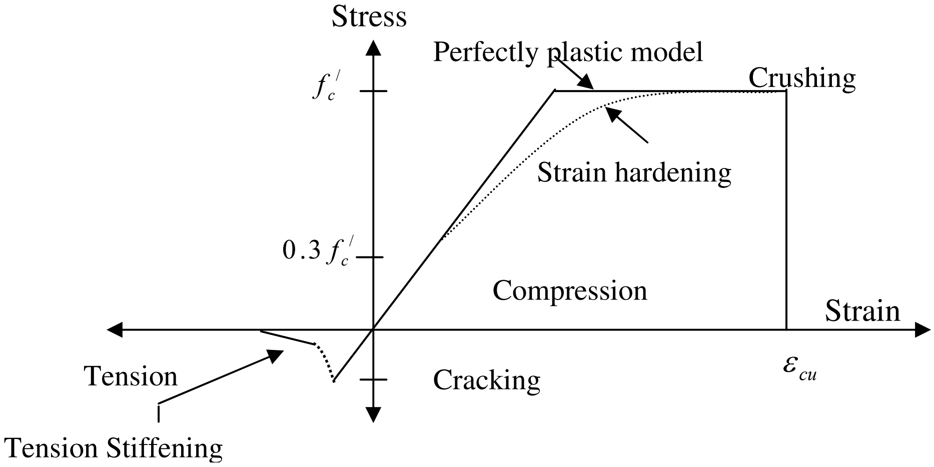


Fig. 2. Uniaxial representation of the concrete constitutive model [11].

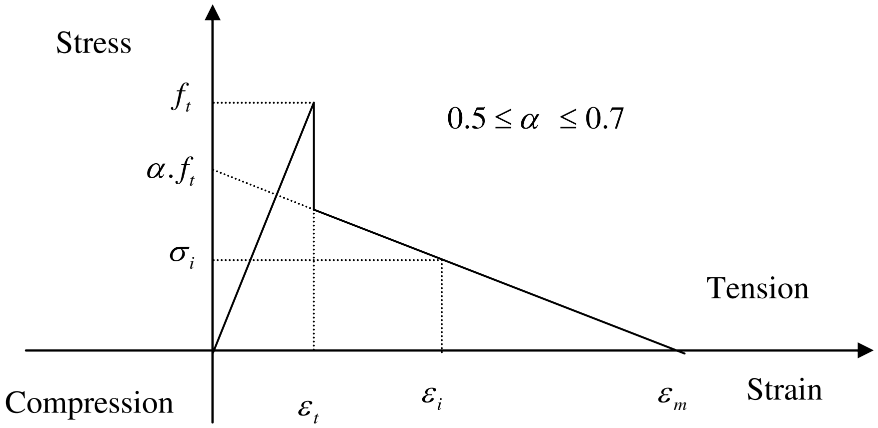


Fig. 3. Tension stiffening in concrete after cracking [11].

The tensile cracks produce damage to concrete with the transverse strain having a degrading effect not only on the compressive strength but also in the compressive stiffness, so that the concrete in this case become softer and weaker than that in a standard cylinder test. In the present study the relationship suggested by Belarbi and Hsu [12] is adopted,

$$f'_{c\max} = \frac{0.9 f'_c}{\sqrt{1+400\ \varepsilon_1}} \tag{6}$$

where $f_c^{/}$ is the concrete cylinder compressive strength and (ε_1) is the average principal tensile strain of concrete in direction (1).

Steel reinforcement is modelled by considering the steel bars as layers of equivalent thickness. Each steel layer exhibits uniaxial response, having strength and stiffness characteristics in the bar direction. A bilinear idealization is adopted in order to model the elasto-plastic stress-strain relationships.

An incremental iterative Newton-Raphson method is employed in order to trace the response of the structure through the loading history

6. Numerical Examples

To verify the numerical method used in the present study a comparison with three beams simply supported, partially prestressed at 0.7 P_u, under concentrated load, given by reference [13] is shown in Fig. 4.

The material properties are given in Table 1. Taking advantage of symmetry only one half of the beam is considered and idealized by 18 elements. Results of load deflection curves are shown in Figs. 5, 6, and 7. Good agreement is obtained by the proposed method compared with the experimental results.

Table 1. Material properties for beams of example (1).

Beam No	E_c MPa	f_c' MPa	ν_c	f_t' MPa	α	ε_m	F_y MPa	F_y MPa	E_{yp} MPa	E_s MPa	A_{st} mm^2	A_{sp} mm^2
1	30459	42.26	0.15	3.4	0.6	0.003	375	1490	210000	214000	78.54	78.54
2	30459	42.26	0.15	3.4	0.6	0.003	375	1490	210000	214000	78.54	235.62
3	30459	42.26	0.15	3.4	0.6	0.003	375	1490	210000	214000	0	314

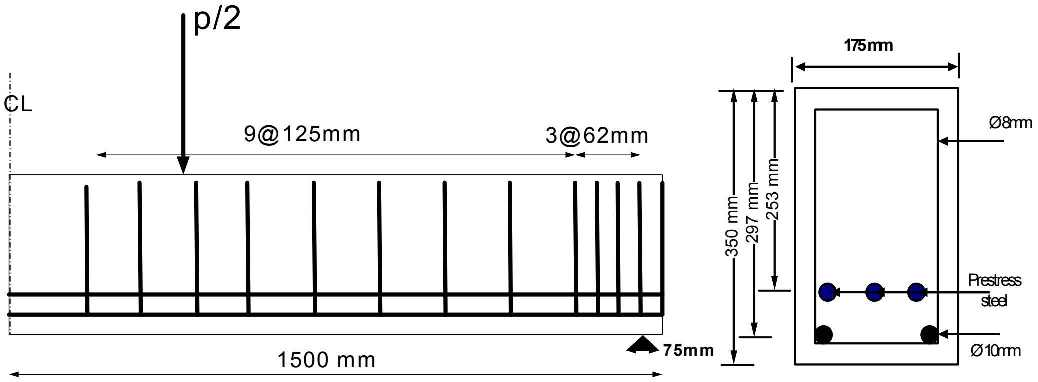


Fig. 4. Detail for Beam reinforcement.

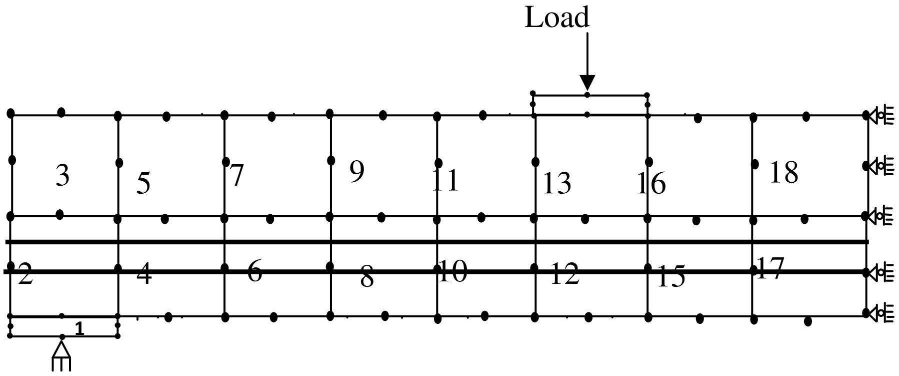


Fig. 5. Finite element mesh used in the present study.

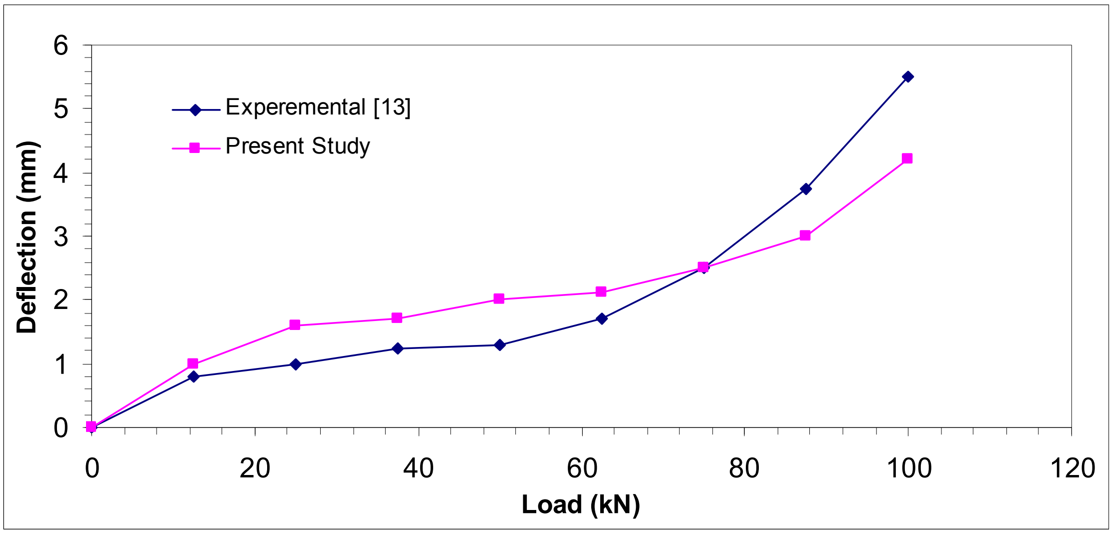


Fig. 6. Load deflection curve for beam No. 1.

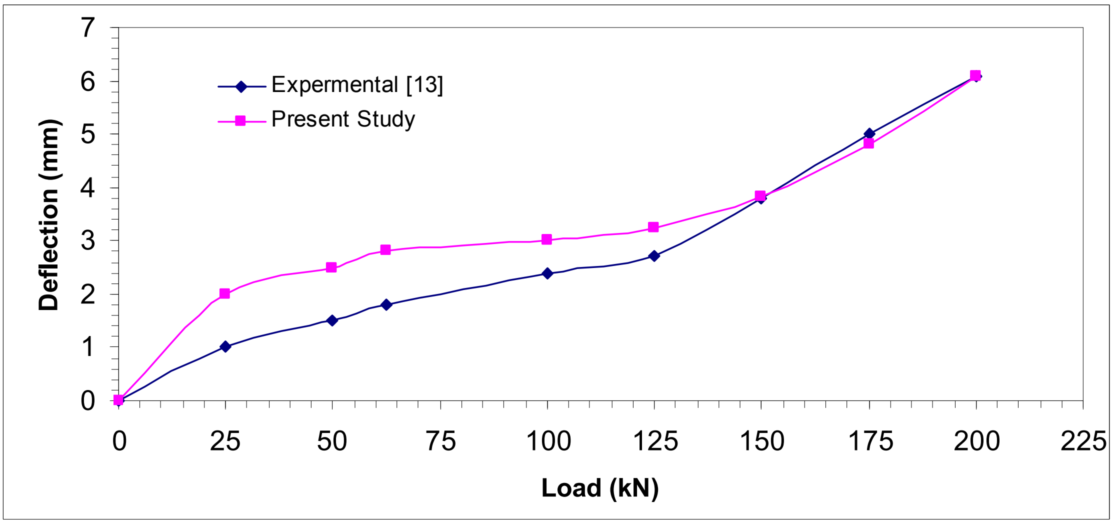


Fig. 7. Load deflection curve for beam No. 2.

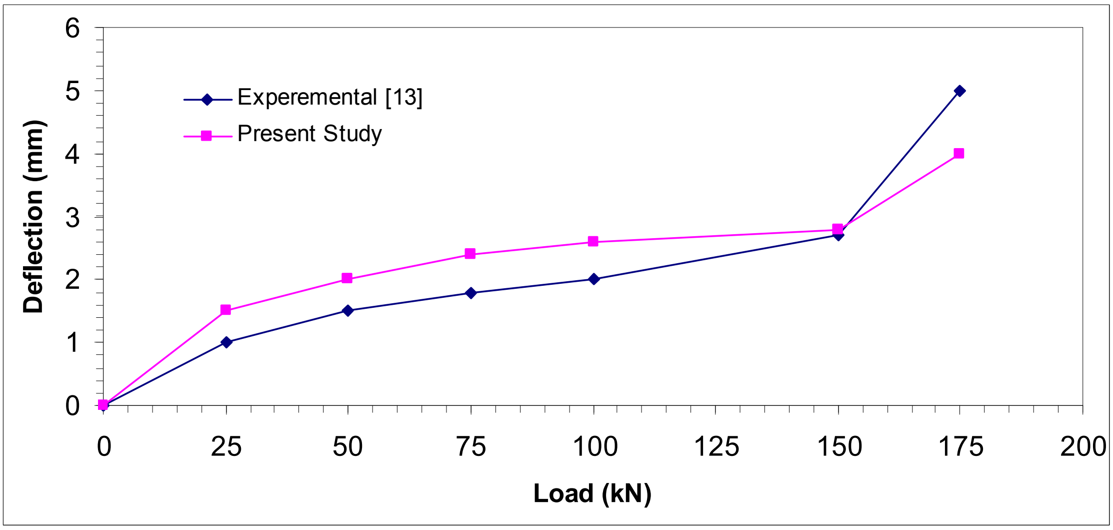


Fig. 8. Load deflection curve for beam No. 3.

7. Conclusions

The following conclusions regarding the strength of pretension partially prestressed concrete beams are drawn from the study:

1. Load deflection curves give good agreement with the experimental results.
2. The developed models concerning the plasticity theory proved to give satisfactory results for the analysis of pretension beams.

Good agreements have been noted between results, the present results and the experimental results especially in the context of ultimate load.

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