A Study of Nuclear Structure of $^{122\text{-}128}$Te Even-Even Isotopes by the Interacting Boson Model-1

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Abstract

Feature of energy levels for even-even tellurium isotopes (122-128) and reduced electric transition probability B(E2) have been studied using the Interacting Boson Model-1 (IBM-1). The calculations of energy levels were carried out using the program package IBM-code, and the program IBMT-code for evaluating the electric transitions.

The present study assigned the spin/parity for some levels which have been assigned definitely, where the level (2.4 MeV) for ^{122}Te have been specified with a spin/parity 2_4^+, the levels (2.3, 2.2 and 2.6 MeV) for ^{124}Te by 5_1^+, 4_3^+ and 6_2^+ and the level (2.7 MeV) for ^{126}Te by 4_3^+ instead of (2^+, 3^+, 4^+) which is experimentally specified.

In addition to the determination of the spin/parity of other energy levels which are not assigned practically where assigned the spin/parity of the energy level (1.7 MeV) for ^{124}Te by the value 3_1^+ and the level (2.6 MeV) for the ^{126}Te by 5_1^+.

The present results suggest some new energy levels: (2.41 MeV) for ^{122}Te with spin/parity 5_1^+ (3.2 MeV) and (4.06 MeV) for ^{124}Te with spin/parity 8_2^+ and 10_2^+, (3.5 MeV) and (1.9 MeV) for ^{126}Te by 8_2^+ and 0_3^+ and the other levels (2.13 MeV, 2.19 MeV, 2.7 MeV, 2.84 MeV and 2.88 MeV) for ^{128}Te by 0_3^+, 4_2^+, 2_3^+, 2_4^+ and 4_3^+ respectively. The Te isotopes have shown their membership to the vibrational limit SU(5).

Introduction

Goldhaber and Weneser(1) and Wilest and Jean (2) Have been studied the first excited state for the $^{122\text{-}128}$Te with $J^\pi=2^+$ and the second excited state with $J^\pi=0^+$, 2^+, 4^+. Raganil and Walters (3) and Mariscatti (4) studied the $^{122\text{-}126}$ Te by some semiempirical models to determinate the Y band states and they founded the energy of four statuses only with (0^+, 2^+, 4^+, 6^+). Lopace [5] and Spis[6] used the vibrational model to study small and negative Q values. Sorensen(7) and Almoney and Borse (8) founded that Q_{21}^+ values for these isotopes are large and positive and decreases from ^{122}Te to ^{128}Te isotopes. Arima and Iachello [9] studied B(E2: $2^+ \rightarrow 0^+$) to the $^{122\text{-}130}$Te isotopes and founded a good agreement with the properties of the vibrational limit SU(5). Jackson and Meyer (10) studied $^{122\text{-}134}$Te and showed that the 122Te belong to the vibrational nucleus and 124-128 Te isotopes refers to the γ-softness nuclei. Robinson et al [11] and Park et al [12] and Subber et al[13] show that 12Te has different B(E2) properties from the nearest nuclei because of the difference in the nuclear structure. Schwengner et al [15] show that these nuclei belonging to the vibrational. Prochniak et al [16] studied the lower energy levels for these nuclei because of the collective excitations.

Interacting Boson Model-1 (IBM-1)

Arima and Iachello in 1974 were suggested [17] a new nuclear model called it the IBM which describe the collective structure for the nuclei of mass number A>100 except the nuclei near the closed shells. This model deals with the nucleons outside the closed shells for even-even nuclei as pairs of protons or neutrons which called bosons [18] that have not intrinsic spin but have an ability to be in tow levels one of spin (L=0) or ground state which called **s** boson and the other of (L=2) or excited state which called **d** boson.

The IBM-1 deals with bosons without considering it protons or neutrons while IBM-2 distinguishes between them, so the Hamiltonian operator for the IBM-1 system is[19];

$$\mathbf{H} = \boldsymbol{\varepsilon}_s (\mathbf{s}^\dagger \mathbf{s}) + \boldsymbol{\varepsilon}_d \Sigma_m \mathbf{d}_m^\dagger \mathbf{d}_m + \mathbf{V} \qquad (1)$$

Where $\mathbf{s}^\dagger$ ($\mathbf{s}$) and $\mathbf{d}_m^\dagger$ ($\mathbf{d}_m$) represent the creation and annihilation operators respectively.

$m = 0, \pm 1, \pm 2$ and ε_s and ε_d represent the boson energies s and de respectively. V is the interaction energy between the bosons. Also, the Hamiltonian operator can be written with another expression[20-23].

Dynamical Symmetries

We can divide the IBM-1 to three chains with three analytical solutions as follows:

The Vibrational Limit U(5)

In this limitation, the boson energy e is much greater than the interaction potential V, and the Hamiltonian operator is [24]

$$\mathbf{H}^{I} = \boldsymbol{\varepsilon}\, \Sigma_m \mathbf{d}_m^\dagger \mathbf{d}_m + \Sigma_L \tfrac{1}{2} (2L + 1)^{(1/2)}\, \mathbf{C}L[(\mathbf{d}^\dagger \mathbf{d}^\dagger)^{(2)} (\mathbf{dd})]^{(0)} \qquad (2)$$

The selection rules ($\Delta n_d = 0, \pm 1$) of the average transition for electric quadruple moment (E2) is given by the equation [25];

$$\mathbf{T}_m^{(E2)} = \boldsymbol{\alpha}_2 (d^\dagger s) + s^\dagger d) + \boldsymbol{\beta}_2 (d^\dagger d)_m^{(2)} \qquad (3)$$

Where $\boldsymbol{\alpha}_2$ and $\boldsymbol{\beta}_2$ factors for initial and final wave functions.

The Rotational Limit SU(3)

This limitation is dominant in the case of quadrupole interaction (Q.Q) between the bosons, also in the exist of dipole moment interaction (L.L), and the Hamiltonian operator in this chain is [26];

$$\mathbf{H}^{II} = \mathbf{a}_1\mathbf{L}^2 + \mathbf{a}_2\, \mathbf{Q}^2 \qquad (4)$$

Where $\mathbf{a}_1$ and $\mathbf{a}_2$ are constant.

To calculate E2 in this chain we must choose [26];

$$T_m^{E2} = \alpha_2\, [(d^\dagger s + sd)_m^{(2)} - \sqrt{7}/2\, (d^\dagger d)_m^{(2)} \qquad (5)$$

Where $\beta 2 = \sqrt{7}/2\, \alpha_2$

γ-Unstable limit O(6)

This limitation is taken in account when the pairing interaction is dominant on the interboson interaction, and the Hamiltonian operator is taken the form [27];

$$\mathbf{H}^{III} = \mathbf{AP}_6 + \mathbf{BC}_5 + \mathbf{CC}_3 \qquad (6)$$

Where $\mathbf{P.P}=\mathbf{P}_6$ and $\mathbf{C}_5$ and $\mathbf{C}_3$ represent Casimir operators for O(5) and O(3) respectively. The factor A, B, and C can be transform to equation (1) factor by the transformation as in references [27,28].

Transitional Regions in IBM-1

Most of nuclear spectra do not match with one of the previous limitation, so the Hamiltonian may be written in terms of two chains or more, therefore, we can classify the nuclei into four transitional types:

Type A:

The nuclei in this type have a properties intermediate between the chains I and II and the Hamiltonian is [23,29];

$$\mathbf{H}^{I+II} = \varepsilon n_d + \mathbf{a}_1\mathbf{L.L} + \mathbf{a}_2\mathbf{Q.Q} \qquad (7)$$

The properties of this type depends on ε/a_2 ratio that eigen functions of the Hamiltonian operator belong to chain I if ε/a_2 large, where it is belong to chain II if ε/a_2 is small.

Type B:

The properties of these nuclei are intermediate between II and III chains [23] and the Hamiltonian is

$$\mathbf{H}^{II+III} = \mathbf{a}_1\mathbf{L.L} + \mathbf{a}_2\mathbf{Q.Q} + \mathbf{a}_0\mathbf{P}^{\dagger}\mathbf{.P} \qquad (8)$$

The properties depend on a_0/a_2 ratio, that when this ratio is large than the egin functions for the Hamiltonian operator belong to III chain, and when the ratio is small the eigen functions belong to II chain.

Type C:

The nuclei are intermediate between I and III chains, and the Hamiltonian is;

$$\mathbf{H}^{I+III} = \varepsilon n_d + \mathbf{a}_0\ \mathbf{P}^{\dagger}\mathbf{.P} \qquad (9)$$

Type D:

These nuclei are intermediate between the three limits, and the Hamiltonian is [23, 29]

$$\mathbf{H} = \varepsilon n_d + \mathbf{a}_1\mathbf{L.L} + \mathbf{a}_0\mathbf{P}^{\dagger}\mathbf{.P} + \mathbf{a}_2\mathbf{Q.Q} \qquad (10)$$

Calculations and Discussion

1- Energy Levels

The vibrational limit U(5) has been used in IBM-1 to describe the $^{122\text{-}128}$Te presented. The coefficient values that give a good agreement with the experimental results are shown in Table (1) [10,13,15,30]. The present theoretical values of the energy levels are shown in the Figs. (1-4) which is compared with the experimental values [10, 13,15,30] and with theoretical values [13,30,31,32].

Table (1) : The Parameters which used in PHINT and the parameters in (MeV)

Parameters	^{122}Te	^{124}Te	^{126}Te	^{128}Te
HBR	0.56	0.6	0.666	0.74
C_o	0.934	0.17	0.223	0.287
C_2	0.0332	0.0546	0.046	0.021
C_4	0.047	0.049	0.037	0049

a-^{122}Te Nucleus

The energy level (2.4 MeV) is determined with spin/parity (2_4^+). Also, the energy level (2.4MeV) with (5_1^+) had been suggested as new.

b- ^{124}Te Nucleus

The present work show that the spin/parity of the energy level (2.7 MeV) with spin/parity 5_1^+ . Also, we suggest a new two energy levels (3.5 MeV) and (3.4 MeV) with spin/parity 8_2^+ and 10_1^+ , respectively.

c- ^{126}Te Nucleus

Two energy levels are suggested (3.57 MeV) and (2.13 MeV) with 8_2^+ and 0_3^+ respectively.

d- ^{128}Te Nucleus

The new energy level is suggested (3.17 MeV)with spin/parity 5_1^+.

2- B(E2) Values

Table (2) shows the B(E2; $2_1^+ \rightarrow 0_1^+$), E2SD and E2DD for $^{122-128}$Te isotopes which used in FBEM-code [21,33,34]. In the present work, the values of B(E2) have been presented in Table (3) for $^{122-128}$Te isotopes.

Table (2) : The parameters which used in FBEM-code for $^{122-128}$Te Isotopes

Isotopes	**B(E2;$2_1^+ \rightarrow 0_1^+$) (e^2b^2)**	**E2SD (e^2b^2)**	**E2DD (e^2b^2)**
^{122}Te	**0.1331**	**0.1379**	**-0.095**
^{124}Te	**0.1416**	**0.1536**	**-0.107**
^{126}Te	**0.0994**	**0.1410**	**-0.0988**
^{128}Te	**0.0770**	**0.1387**	**-0.097**

Conclusion

The phonon concept [35] is important in nuclear structure physics to describe the explanation of the excitation of the collective lower states in the vibrational nuclei near spherical nuclei. One can show the two phonon excitation by close the energy level 0_2^+ from the twice value of the energy level 2_1^+ and the closest of the energy levels (4_1^+, 2_2^+ and 0_2^+) [36].

Figure(5) show the good closest of the energy levels (2_2^+ and 4_1^+) and it is also close to twice value of the energy level 2_1^+ for $^{122-128}$Te isotopes, where the level 0_2^+ is far from this closest.

Table (3) : B(E2) values for $^{122-128}$Te (present work).

i → f	^{122}Te	^{124}Te	^{126}Te	^{128}Te
$2_1^+ \rightarrow 0_1^+$	0.1331	0.1416	0.0994	0.0770
$2_1^+ \rightarrow 0_2^+$	0.0456	0.0472	0.0318	0.0231
$2_1^+ \rightarrow 0_3^+$	0	0	0	0
$2_2^+ \rightarrow 0_1^+$	0	0	0	0
$2_2^+ \rightarrow 0_2^+$	0.0072	0.0092	0.0078	0.0075
$2_2^+ \rightarrow 0_3^+$	0.057	0.0566	0.0358	0.0231
$2_3^+ \rightarrow 0_1^+$	0	0	0	0
$2_3^+ \rightarrow 0_3^+$	0.1116	0.1036	0.0033	0.0032
$2_4^+ \rightarrow 0_1^+$	0	0	0	0
$2_1^+ \rightarrow 2_2^+$	0.2282	0.2359	0.1590	0.1154
$4_1^+ \rightarrow 2_1^+$	0.2282	0.2359	0.1590	0.1154
$4_1^+ \rightarrow 2_2^+$	0.0029	0.0037	0.0032	0.0031
$4_2^+ \rightarrow 2_3^+$	0	0	0.0341	0.0220
$4_2^+ \rightarrow 2_1^+$	0	0	0	0
$4_2^+ \rightarrow 2_2^+$	0.1494	0.1483	0.0937	0.605
$4_2^+ \rightarrow 2_3^+$	0.0296	0.0275	0.0088	0.0084

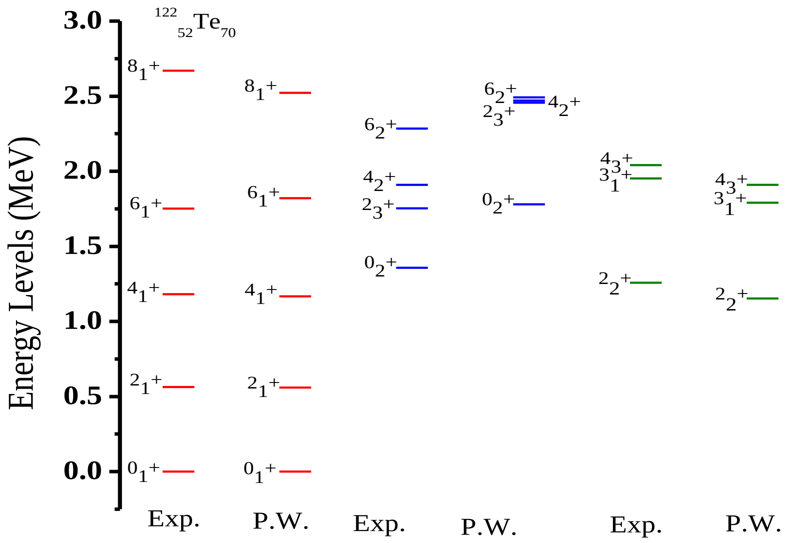

Fig.(1): Comparison of the experimental low-lying positive parity energy states of ^{122}Te with the predictions of the IBM-1. Experimental data were taken from Ref. [10].

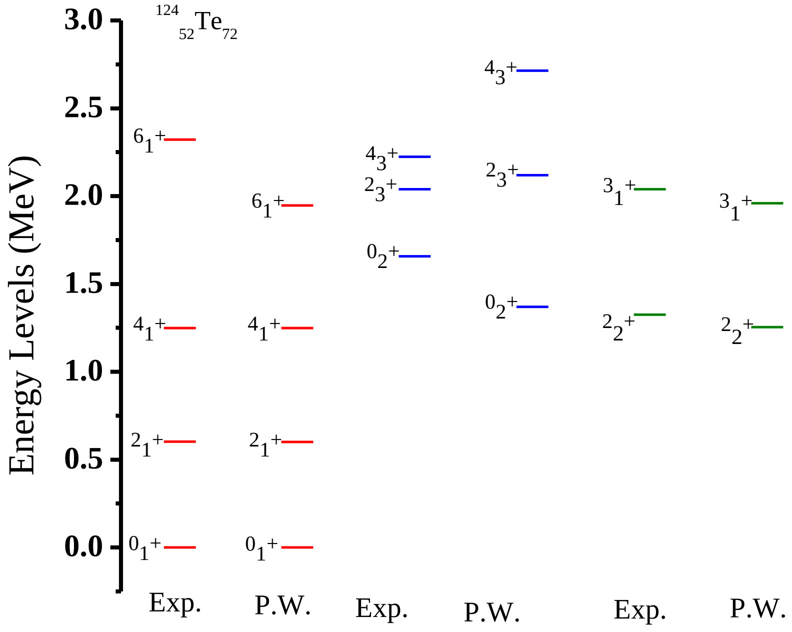

Fig.(2): Comparison of the experimental low-lying positive parity energy states of ^{124}Te with the predictions of the IBM-1. Experimental data were taken from Ref. [13].

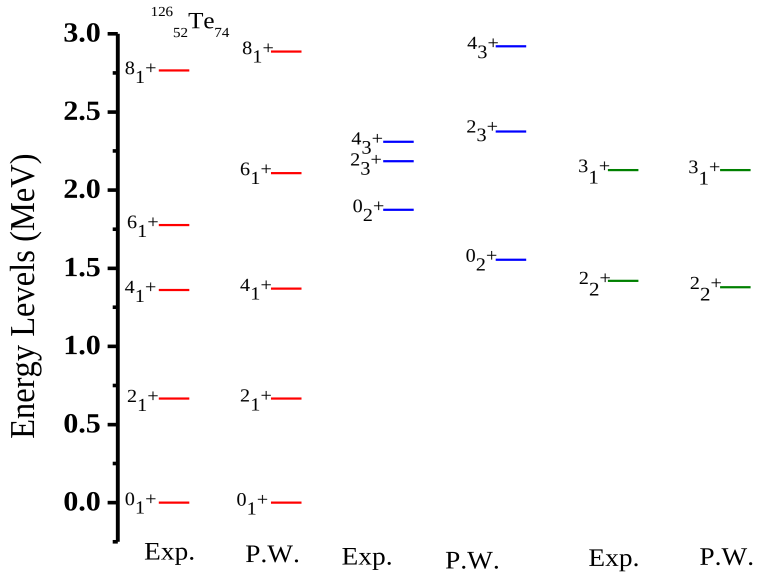

Fig.(3): Comparison of the experimental low-lying positive parity energy states of ^{126}Te with the predictions of the IBM-1. Experimental data were taken from Ref. [15].

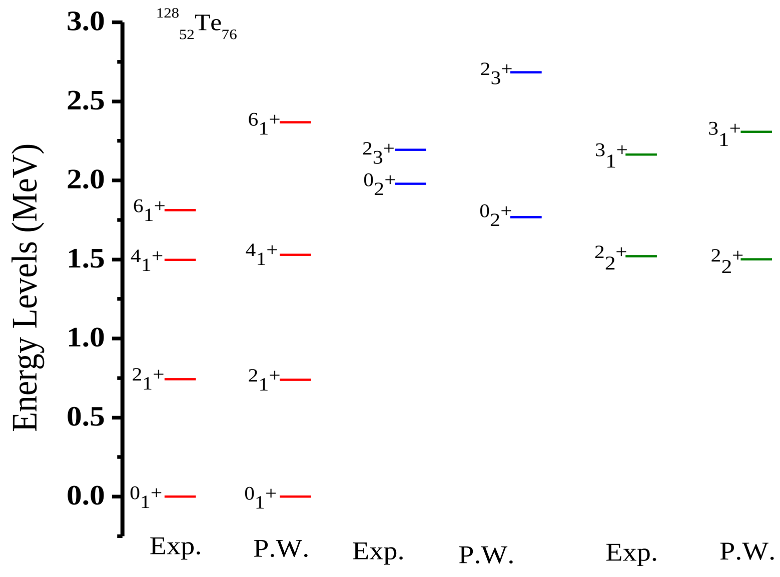

Fig.(4): Comparison of the experimental low-lying positive parity energy states of ^{128}Te with the predictions of the IBM-1. Experimental data were taken from Ref. [30].

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