



Solving Multi-collinearity Problem by Ridge and Eigen value Regression with Simulation

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Abstract

In this paper, five new methods were proposed to estimate the ridge parameter by inserting the conditional number, which are used to estimate the parameters of the ridge regression model to deal with multicollinearity problem and then compare their efficiency with some classical methods that was studied by several researchers based on Mean square error and comparing them with Eigen value Regression through simulation study (MATLAB language program designed for this purpose), the research shows that the efficiency of the proposed methods in dealing with multicollinearity problem and the advantages of the proposed methods compared with the classical methods of Ridge and Eigen value Regression.

Keywords: Multiple Regression, Ridge parameter, Multicollinearity, Conditional number, Eigen value Regression.

1: Introduction:

Models miss-specification can be due to omission of one or several relevant variables, inclusion of unnecessary explanatory variables, wrong functional forms, autocorrelation etc. However, for modeling data, there is another problem that also might influence the results. This problem occurs in situations when explanatory variables are highly inter-correlated. In practice, there may be strong or near strong linear relationship exist among explanatory variables. Thus, independence assumption of explanatory variables is no longer valid, which causes multi-collinearity problem. In the presence of multicollinearity, the OLS estimator could become unstable due to their large variance, which leads to poor prediction and wrong inference about model parameters.

One of the popular numerical techniques to deal with multi-collinearity is the ridge regression due to Hoerl and Kennard (1970), Ridge regression approach has been studied by McDonald and Galarneau (1975), Swindel (1976), Lawless (1978), Singh and Chaubey (1987), Sarkar (1992), Saleh and Kibria (1993), Kibria (2003), Khalaf and Shukur (2005), Zhong and Yang (2007), Batah et al. (2008), Yan (2008), Yan and Zhao (2009), Muniz and Kibria (2009), Yang and Chang (2010), Khalaf (2012) and Dorugade (2014) and others. Ridge Regression estimator has been the benchmarked for almost all the estimators developed later in this context.

On the other hand (Webster et al., 1974), suggested a bias alternative method, namely the Eigen value Regression and can identify near singularities and determine whether or not these near singularities have predictive value. Those Eigen values and Eigen vectors that are non-predictive near singularities be removed. Subsequently, a stepwise backward elimination of variables is performed.



2: Methodology:

2.1: Regression Analysis:

So far, we have seen the concept of simple linear regression where a single predictor variable X was used to model the response variable Y. In many applications, there is more than one factor that influences the response. Multiple regression models thus describe how a single response variable Y depends linearly on a number of predictor variables. Consider following multiple linear regression model (Draper, N. R. and Smith, H., 1998):

$$y = \mathbf{X}\beta + \epsilon \quad \dots \quad (1)$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1k} \\ 1 & x_{21} & x_{22} & \dots & x_{2k} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nk} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

In linear algebra terms, the least-squares parameter estimates β are the vectors that minimize

$$\sum_{i=1}^n \epsilon_i^2 = \epsilon' \epsilon = (\mathbf{y} - \mathbf{X}\beta)'(\mathbf{y} - \mathbf{X}\beta)$$

Thus the least-squares estimator of β is (in vector form)

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T y \quad \dots \quad (2)$$

Here, $se(\hat{\beta}_j)$ is the square root of the j^{th} diagonal entry of the covariance matrix $\hat{\sigma}^2 (\mathbf{X}'\mathbf{X})^{-1}$ of the estimated parameter vector $\hat{\beta}$. And we have:

$$\hat{\sigma}^2 = \frac{(\mathbf{y} - \mathbf{X}\hat{\beta})'(\mathbf{y} - \mathbf{X}\hat{\beta})}{n}$$

2.2: Ridge Regression

Least squares regression isn't defined at all when the number of predictors exceeds the number of observations; It doesn't differentiate "important" from "less-important" predictors in a model, so it includes all of them. This leads to over fitting a model and failure to find unique solutions. Least squares also have issues dealing with multi-collinearity in data. Ridge regression avoids all of these problems. It works in part because it doesn't require unbiased estimators, while least squares produce unbiased estimates; variances can be so large that they may be wholly inaccurate. Ridge regression adds just enough bias to make the estimates reasonably reliable approximations to true population values.

Add KI a "ridge" of size to the diagonal of $\mathbf{X}^T \mathbf{X}$, to stabilize the matrix inverse $\mathbf{X}^T \mathbf{X}$ and we get the following formula (Dorugade, A.V., 2014):

$$\hat{\beta}_{ridge} = (\mathbf{X}^T \mathbf{X} + \mathbf{KI})^{-1} \mathbf{X}^T y \quad \dots \quad (3)$$

Where: I = unity matrix, and K = small constant.

By doing this we avoid the numerical problems we will get when trying to invert an (almost) singular matrix. But we are paying a price for doing this. By doing this we have biased the prediction and hence we are solving the solution to a slightly different problem. As long as the error due to the bias is smaller than the error we would have got from having a (nearly) singular $\mathbf{X}^T \mathbf{X}$, we will end up getting a smaller mean square error and hence ridge regression is desirable.

Another view: penalized likelihood

$$\hat{\beta}_{ridge} : \arg \min_{\beta} \left\{ (\mathbf{y} - \mathbf{X}\beta)^T (\mathbf{y} - \mathbf{X}\beta) + \lambda \|\beta\|^2 \right\} \quad \dots \quad (4)$$



Ridge estimator is defined as a family of estimators parameterized by the biasing parameter $K > 0$ and λ_i are Eigen values for $X^T X$ matrix. Efficiency Criteria: to compare the efficiency of estimated regression models, we can use Mean Square Error (MSE), which we obtain from the following formula:

$$MSE_{ridge} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n - p} \quad \dots (5)$$

Also we can use the mean square error of the estimated parameters of the following:

$$MSE_{ridge}(\hat{\beta}_i) = \frac{\sum_{i=1}^h (\beta_i - \hat{\beta}_i)^2}{h} \quad \dots (6)$$

Where h represents the number of iterations. The smallest values of the two criteria will have the best estimated regression models.

2.2.1: Condition Number:

According to [Belsley et al, 1980] suggested the combined use of two diagnostic tools to detect the coefficients which are most likely to be affected by the collinearity. Their examination of the eigen values and eigenvectors of the correlation matrix yield most of the required information when investigating multi-collinearity. The first statistic is the ‘condition number’ and associated ‘condition index’ of the $(X^T X)$ matrix. The condition number is:

$$\kappa = \sqrt{\frac{\max \lambda_i}{\min \lambda_i}} \quad \dots (7)$$

Where κ is between 10 and 30, there is moderate to strong multi-collinearity and if it exceeds 30 there is severe multi-collinearity. However, if κ is less than 10, this means there is no multi-collinearity problem.

2.2.2: Classical formulas to estimate a Ridge parameter:

From the Some Ridge Regression Estimators we will review available methods in literature to estimate the value of k . Hoerl and Kennard (1970) suggested k to be:

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2} \quad \dots (8)$$

Where $\alpha = D^T \beta$ and $\hat{\alpha} = D^T \hat{\beta}$, D is the Eigen vector of matrix $X^T X$ and $\hat{\beta}$ is OLS estimator, $\hat{\alpha}_{\max}$ is the maximum element of $\hat{\alpha}$.

We have $\hat{\sigma}^2 = \frac{\sum_{i=1}^n \hat{e}_i^2}{n - p}$ and $\hat{e}_i^2 = y_i - X_j^T \hat{\alpha}_i$

Hoerl and Kennard claimed that (8) gives smaller MSE then the OLS Method. Horel in (1975) Proposed k to be denoted by \hat{k}_{HKB}

$$\hat{k}_{HKB} = \frac{p \hat{\sigma}^2}{\hat{\alpha}^T \hat{\alpha}} \quad \dots (9)$$



Where p = no. of parameters

Hocking, Speed, and Lynn (1979) suggested k to be:

$$\hat{k}_{HSL} = \hat{\sigma}^2 \frac{\sum_{i=1}^p (\lambda_i \hat{\alpha}_i)^2}{\sum_{i=1}^p (\lambda_i \hat{\alpha}_i^2)^2} \quad \dots (10)$$

Kibria (2003) proposed the following estimators for k based on arithmetic mean (AM), geometric mean (GM), and median of $\hat{\sigma}^2 / \hat{\alpha}_i^2$ and the estimator based on AM (denoted by \hat{k}_{AM}) defined as follows :

$$\hat{k}_{AM} = \frac{1}{p} \sum_{i=1}^p \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} \quad \dots (11)$$

The estimator based on GM (denoted by \hat{k}_{GM})

$$\hat{k}_{GM} = \frac{\hat{\sigma}^2}{(\prod_{i=1}^p \hat{\alpha}_i^2)^{\frac{1}{p}}} \quad \dots (12)$$

The estimator based on median (denoted by \hat{k}_{med})

$$\hat{k}_{med} = Median \left\{ \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} \right\}, \quad i = 1, 2, 3, \dots, p \quad \dots (13)$$

Based on modification of \hat{k}_{HK} , Khalaf and Shukur (2005) suggested k to be (denoted by \hat{k}_{KS})

$$\hat{k}_{KS} = \frac{\lambda_{max} \hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max} \hat{\alpha}_{max}^2} \quad \dots (14)$$

Applying algorithm of GM and square root to Khalaf and Shukur (2005), Muniz and Kibria (2009) proposed the following estimator of k :

$$\hat{k}_{KM3} = \max \left[\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}} \right] \quad \dots (15)$$

From the based square root transformations, Muniz et al. (2012) proposed the following estimator of k :

$$\hat{k}_{KM9} = \max(q_i) \quad \dots (16)$$

Khalaf (2012), based on modification of k_{HK} Proposed k to be (denoted by \hat{k}_{GK})

$$\hat{k}_{GK} = \hat{k}_{HK} + \frac{2}{(\lambda_{max} + \lambda_{min})^T} \quad \dots (17)$$

2.2.3: Proposed formulas to estimate a Ridge parameter:

The proposal to involve the conditional number in the modified formulas to estimate the parameter of the ridge and through experimental simulations was as follows:



$$k_{T1} = \frac{\text{kappa}^{(2p)}}{\hat{\sigma}^2 | \text{mean}(a_i^2) |} \dots (18)$$

$$k_{T2} = \frac{\text{kappa}^{(1/(n-p))} \cdot \hat{\sigma}^2}{\max(a_i^2)} \dots (19)$$

$$k_{T3} = \text{kappa}^{(1/p)} \max\left(\frac{\max(\lambda_i) \cdot \hat{\sigma}^2}{n^2 \hat{\sigma}^2 + \max(\lambda_i) \cdot a_i^2}\right) \dots (20)$$

$$k_{T4} = \frac{p | \text{mean}(a_i) |}{\text{kappa}^2} \dots (21)$$

$$k_{T5} = \frac{n \hat{\sigma}^2}{\text{kappa}^2 (\min(\lambda_i) + \min(a_i))} \dots (22)$$

2.3: Eigen value Regression:

Multi-collinearity occurs when there is an almost exact linear dependency among the independent variables, where the coefficient of determination (R^2) will be very close to one. This type of ill-conditioning among the independent variables is referred to as near singularity and the OLS estimators can be very poor in this situation.

Let Ω (by dimension $n \times k + 1$) be an augment matrix of the standardized dependent variable, and standardized variables X.

$$\Omega = \begin{bmatrix} y_1^* & x_{11}^* & x_{12}^* & \dots & x_{1k}^* \\ y_2^* & x_{21}^* & x_{22}^* & \dots & x_{2k}^* \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_n^* & x_{n1}^* & x_{n2}^* & \dots & x_{nk}^* \end{bmatrix} \dots (23)$$

Where

$$y_i^* = (y_i - \bar{y}) / \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}$$

$$x_i^* = (x_i - \bar{x}) / \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}$$

The $(\Omega^T \Omega)$ is the correlation matrix and it has Eigen values and Eigen vectors defined by: $|\Omega^T \Omega - \lambda_j I| = 0$ and $(\Omega^T \Omega - \lambda_j I) \cdot \gamma_j = 0$ for $(j = 0, 1, \dots, k)$.

Let $\gamma_j^T = (\gamma_{0j}, \gamma_{1j}, \dots, \gamma_{kj})$ be the elements of j^{th} Eigen vectors and $\gamma_j^{OT} = (\gamma_{1j}, \gamma_{2j}, \dots, \gamma_{kj})$. Assume the $\lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_k$ be the ordered Eigen values and corresponding Eigen vectors. The augment matrix for Eigen values and Eigen vectors for dependent and independent variables is:

$$\Gamma(\lambda, \gamma) = \begin{bmatrix} \lambda_0 & \gamma_{00} & \gamma_{01} & \dots & \gamma_{0k} \\ \lambda_1 & \gamma_{10} & \gamma_{11} & \dots & \gamma_{1k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_k & \gamma_{k0} & \gamma_{k1} & \dots & \gamma_{kk} \end{bmatrix} \dots (24)$$

Webster et al., (1974), Lawrence and Arthur, (1989) pointed out that small Eigen values correspond to non-predictive near singularities and they suggested a cut-off value in



which $\lambda_j < 1$ and $|\gamma_{0j}| \leq 0.5$. The least squares estimator of coefficients can be written as a form of Eigen values and Eigen vectors $\Gamma(\lambda, \gamma)$ as follows:

$$\hat{\beta}_{OLS} = -\eta \sum_{j=0}^k \alpha_j \gamma_{0j} \quad \dots \quad (25)$$

Where

$$\alpha_j = \gamma_{0j} \lambda_j^{-1} \left(\sum_{t=0}^k \gamma_{0t}^2 / \lambda_t \right)^{-1} \quad \text{and} \quad \eta^2 = \sum_{i=1}^n (y_i - \bar{y})^2$$

With sum squares of residuals

$$SSE_{OLS} = \eta^2 \left(\sum_{j=0}^k \gamma_{0j}^2 / \lambda_j \right)^{-1} \quad \dots \quad (26)$$

In the presence of multicollinearity in the data set, some values of α become sufficiently large relative to the other values of α_j , that leads to distortion of some of the coefficients due to large term $\alpha_0 \gamma_{00}$ in (23). Let $\gamma_0, \gamma_1, \dots, \gamma_{r-1}$ related with non-predictive near singularities, then the non-predictive values are eliminated and only the predictive values are retained. Webster et al., (1974) proposed modified OLS estimator by setting $\alpha_0 = \alpha_1 = \dots = \alpha_{r-1} = 0$, so the adjusted least squares estimator, namely Eigen values Regression (EVR) is defined as:

$$\hat{\beta}_{EVR} = -\eta \sum_{j=r}^k \alpha_j \gamma_{0j} \quad \dots \quad (27)$$

Where

$$\alpha_j = \gamma_{0j} \lambda_j^{-1} \left(\sum_{t=r}^k \gamma_{0t}^2 / \lambda_t \right)^{-1} ; \quad j = r, r+1, \dots, k$$

The residuals sum of squares for Eigen value is:

$$SSE_{EVR} = \eta^2 \left(\sum_{j=r}^k \gamma_{0j}^2 / \lambda_j \right)^{-1} \quad \dots \quad (28)$$

If all near singularities have predictive value, none of the α_j s equal to zero and then the least squares coefficients and the Eigen value coefficient will be equivalent.

The mean square errors for $\hat{\beta}_{EVR}$ is not known exactly but it can be approximately computed by similar way as the mean square error of the Principal Components Regression on the Eigen vectors $(X^T X)$, then:

$$MSE(\hat{\beta}_{EVR}) \approx \hat{\sigma}^2 \sum_{i=1}^k \ell_i^{-1} + (\alpha_i^T \hat{\beta}_{EVR})^2 \quad \dots \quad (29)$$

Where $\ell_1 < \ell_2 < \dots < \ell_k$ are the Eigen values of the design matrix $(X^T X)$.

3: Application:

The simulation was applied on Ridge and Eigen value regression follows:

3.1: Ridge Regression:



To determine the efficiency of the proposed regression parameter estimation methods, a simulation was performed using a MATLAB language program designed for this purpose (Appendix), assuming we have a multiple regression model with two independent variables and their linear relationship (multi-collinearity) Measured by a conditional number greater than (10) when var-covariance matrix is:

$$\text{var - covariance} = \begin{bmatrix} 25 & 31 \\ 31 & 40 \end{bmatrix}$$

To deal with multi-collinearity problem the conditional number must be less than (10), several combinations of different values were selected to (sample size, covariance and slop regression), where the random error are i.i.d. $N(0, \sigma^2)$ and, without loss of any generality, we will assume $(\hat{\beta}_0 = 2)$ for Regression model, the experiment was repeated (1000) times and the averages calculation (regression parameters, Conditional number and MSE, it was summarized in the following tables according to the different cases assumed:

Case (1): When $n=20$, $\sigma^2 = 20$ and $\hat{\beta} = [2 \ 5 \ 9]^T$

Table (1): Average of estimated regression parameters, conditional number and MSE (case-1)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	4.8230	4.746479	9.1497	16.088	0.008056
Hoerl and Kennard (1970)	2.1728	5.153958	8.7832	9.3462	0.000101
Hoerl and Kennard (1975)	0.8501	5.47001	8.4494	7.5946	0.001846
Hocking, Speed, and Lynn (1979)	2.1734	5.153834	8.7833	9.3468	0.000101
Kibria (2003)1	5.9288	5.69187	7.8112	4.6121	0.017327
Kibria (2003)2	7.0592	5.6946	7.7320	4.3145	0.027686
Kibria (2003)3	10.3815	5.63917	7.5827	3.93378	0.072668
Khalaf and Shukur (2005)	2.2552	5.1384	8.7984	9.4192	0.000125
Muniz and Kibria (2009)	0.7705	5.5003	8.4141	7.3707	0.002105
Muniz et al. (2012)	2.1721	5.15408	8.7830	9.3456	0.000101
Khalaf (2012)	3.9985	5.7352	7.8831	4.6804	0.005782
Proposed-1	20.4886	5.3097	7.3427	3.8690	0.344670
Proposed-2	1.9501	5.1932	8.74561	9.1505	0.000100
Proposed-3	1.2053	5.32028	8.6259	8.39656	0.000874
Proposed-4	4.6732	4.76881	9.1298	11.0063	0.007216
Proposed-5	1.7899	5.26178	8.6645	8.83945	0.000225

Table (1) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fourth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (2): When $n=20$, $\sigma^2 = 50$ and $\hat{\beta} = [2 \ 5 \ 9]^T$

Table (2): Average of estimated regression parameters, conditional number and MSE (case-2)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	0.2059	5.1075	8.9631	15.909	0.003232
Hoerl and Kennard (1970)	-0.6522	5.7569	8.1618	6.0641	0.008310
Hoerl and Kennard (1975)	6.9939	5.7354	7.6846	4.0896	0.027211
Hocking, Speed, and Lynn (1979)	-0.6540	5.7568	8.1620	6.065	0.008319



Kibria (2003)1	46.028	4.8574	6.2674	2.233	1.9459
Kibria (2003)2	52.406	4.6981	6.0542	2.0817	2.5495
Kibria (2003)3	63.886	4.4173	5.6692	1.9257	3.8413
Khalaf and Shukur (2005)	-1.6747	5.7278	8.2679	6.6456	0.014569
Muniz and Kibria (2009)	0.1344	5.7993	8.0538	5.5126	0.005015
Muniz et al. (2012)	-0.6520	5.7569	8.1618	6.0639	0.008309
Khalaf (2012)	4.9352	5.7975	7.7390	4.0715	0.010842
Proposed-1	0.1766	5.6535	8.2407	6.6289	0.004329
Proposed-2	-0.1769	5.7716	8.1110	5.7840	0.006125
Proposed-3	-2.3062	5.5645	8.5249	8.2880	0.019088
Proposed-4	0.0968	5.1259	8.9459	9.828	0.003641
Proposed-5	2.4919	5.7566	7.9534	5.3866	0.001910

Table (2) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fifth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (3): When $n=50$, $\sigma^2 = 20$ and $\hat{\beta} = [2 \ 5 \ 9]^T$

Table (3): Average of estimated regression parameters, conditional number and MSE (case-3)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	1.7644	5.0252	8.9822	15.467	0.000560
Hoerl and Kennard (1970)	0.8411	5.1700	8.8509	9.8066	0.001394
Hoerl and Kennard (1975)	-0.2233	5.3606	8.6679	8.8417	0.005183
Hocking, Speed, and Lynn (1979)	0.8414	5.1699	8.8510	9.8069	0.001393
Kibria (2003)1	-0.5059	5.7119	8.2188	6.3159	0.007397
Kibria (2003)2	-0.2055	5.7506	8.1473	5.9583	0.006155
Kibria (2003)3	0.6348	5.7700	8.0653	5.5325	0.0033300
Khalaf and Shukur (2005)	0.8857	5.1626	8.8578	9.8412	0.001288
Muniz and Kibria (2009)	-0.2889	5.3746	8.6537	8.7410	0.005499
Muniz et al. (2012)	0.8409	5.1700	8.8509	9.8065	0.001395
Khalaf (2012)	-0.9218	5.7011	8.2609	6.5142	0.009574
Proposed-1	1.4917	5.6730	8.1377	5.7770	0.001455
Proposed-2	0.8019	5.1764	8.8450	9.7759	0.001491
Proposed-3	1.0673	5.1317	8.8869	9.9728	0.000900
Proposed-4	1.7197	5.0322	8.9759	5.438	0.000080
Proposed-5	0.4639	5.2381	8.7853	9.4834	0.002462

Table (3) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fourth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (4): When $n=50$, $\sigma^2 = 50$ and $\hat{\beta} = [2 \ 5 \ 9]^T$

Table (4): Average of estimated regression parameters, conditional number and some efficiency criteria (case-4)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	3.9751	4.7104	9.231	15.556	0.004038
Hoerl and Kennard (1970)	0.1585	5.3718	8.6017	7.7611	0.003688
Hoerl and Kennard (1975)	0.6088	5.6739	8.1687	5.7152	0.003081
Hocking, Speed, and Lynn	0.1592	5.3716	8.6019	7.7619	0.003685



(1979)					
Kibria (2003)1	16.026	5.5386	7.3257	3.1896	0.199820
Kibria (2003)2	19.680	5.4658	7.1804	2.9542	0.316120
Kibria (2003)3	25.730	5.3272	6.9635	2.7039	0.567350
Khalaf and Shukur (2005)	0.6061	5.2627	8.7176	8.2795	0.002092
Muniz and Kibria (2009)	0.0016	5.4395	8.5218	7.3191	0.004416
Muniz et al. (2012)	0.1585	5.3718	8.6017	7.7610	0.003688
Khalaf (2012)	0.2365	5.6381	8.2411	5.9176	0.004093
Proposed-1	1.7123	5.0969	8.8653	8.5693	0.000110
Proposed-2	0.1026	5.3883	8.5835	7.6724	0.003924
Proposed-3	3.0991	4.8396	9.1167	9.0460	0.001247
Proposed-4	3.9128	4.7195	9.2230	9.5270	0.003787
Proposed-5	-0.0425	5.5244	8.4116	6.9567	0.004793

Table (4) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the first proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (5): When $n=20$, $\sigma^2 = 20$ and $\hat{\beta} = [2 \ 3 \ -6]^T$

Table (5): Average of estimated regression parameters, conditional number and MSE (case-5)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	4.8230	2.7465	-5.8503	16.088	0.008056
Hoerl and Kennard (1970)	22.740	0.4995	-4.0492	7.6703	0.440210
Hoerl and Kennard (1975)	29.242	-0.3864	-3.3015	5.8914	0.760850
Hocking, Speed, and Lynn (1979)	31.594	-0.7775	-2.9365	4.8754	0.899450
Kibria (2003)1	27.315	-0.1183	-3.5305	6.4763	0.656650
Kibria (2003)2	32.358	-0.9494	-2.7581	4.3792	0.947740
Kibria (2003)3	29.831	-0.4816	-3.2138	5.6659	0.794430
Khalaf and Shukur (2005)	21.523	-0.6591	-4.1808	7.9442	0.389950
Muniz and Kibria (2009)	19.315	0.9368	-4.4037	8.4400	0.306610
Muniz et al. (2012)	22.742	0.4993	-4.0490	7.6700	0.440280
Khalaf (2012)	29.826	-0.4585	-3.2443	5.7087	0.793870
Proposed-1	24.647	-0.9247	-2.2724	2.8808	0.542170
Proposed-2	23.867	0.3528	-3.9288	7.3942	0.489480
Proposed-3	18.562	1.0410	-4.4925	8.4790	0.280410
Proposed-4	5.2563	2.6928	-5.8076	8.0260	0.010735
Proposed-5	54.565	-1.077	-4.1349	5.9359	2.783200

Table (5) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fourth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (6): When $n=20$, $\sigma^2 = 50$ and $\hat{\beta} = [2 \ 3 \ -6]^T$

Table (6): Average of estimated regression parameters, conditional number and MSE (case-6)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	3.1707	2.7280	-5.7202	16.803	0.001523
Hoerl and Kennard (1970)	30.315	-0.9662	-2.6051	4.0992	0.828990



Hoerl and Kennard (1975)	28.418	-1.2045	-2.1612	2.8496	0.730330
Hocking, Speed, and Lynn (1979)	24.791	-1.2006	-1.9244	2.3153	0.553690
Kibria (2003)1	29.599	-1.1608	-2.2983	3.2068	0.792720
Kibria (2003)2	21.873	-1.1642	-1.7783	2.0807	0.430110
Kibria (2003)3	27.754	-1.2102	-2.1095	2.7203	0.696140
Khalaf and Shukur (2005)	28.525	-0.5458	-3.0455	5.2763	0.724860
Muniz and Kibria (2009)	24.489	0.0201	-3.5308	6.5788	0.520720
Muniz et al. (2012)	30.315	-0.9662	-2.6051	4.0992	0.828990
Khalaf (2012)	30.372	-0.8824	-2.7200	4.3096	0.830790
Proposed-1	23.201	0.0114	-3.4301	5.3911	0.465030
Proposed-2	30.372	-1.0241	-2.5317	3.8752	0.833180
Proposed-3	16.463	1.0766	-4.4029	8.2627	0.215420
Proposed-4	3.6077	2.6735	-5.6769	9.7410	0.002796
Proposed-5	29.938	-1.0937	-2.4087	3.2443	0.810210

Table (6) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fourth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (7): When $n=50$, $\sigma^2 = 20$ and $\underline{\hat{\beta}} = [2 \ 3 \ -6]^T$

Table (7): Average of estimated regression parameters, conditional number and MSE (case-7)

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	1.3612	3.0220	-5.9985	16.566	0.000409
Hoerl and Kennard (1970)	11.978	1.7013	-4.9460	8.9664	0.102350
Hoerl and Kennard (1975)	19.575	0.7381	-4.1688	7.5940	0.317360
Hocking, Speed, and Lynn (1979)	24.092	0.1450	-3.6796	6.5994	0.501580
Kibria (2003)1	16.970	1.0689	-4.4361	8.0912	0.230280
Kibria (2003)2	26.284	-0.1526	-3.4292	6.0527	0.606270
Kibria (2003)3	20.590	0.6050	-4.0592	7.3844	0.355100
Khalaf and Shukur (2005)	10.834	1.8446	-5.0608	9.1486	0.080251
Muniz and Kibria (2009)	8.9959	2.0730	-5.2427	9.4419	0.050376
Muniz et al. (2012)	11.978	1.7012	-4.9460	8.9663	0.102360
Khalaf (2012)	19.991	0.6845	-4.1252	7.4901	0.332570
Proposed-1	28.297	-0.6657	-2.8790	4.3076	0.714720
Proposed-2	12.356	1.6539	-4.9081	8.9030	0.110240
Proposed-3	4.7691	2.5998	-5.6630	9.0630	0.007941
Proposed-4	1.5309	3.0010	-5.9817	9.5430	0.000220
Proposed-5	10.214	1.9210	-5.1213	9.2656	0.069407

Table (7) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fourth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

Case (8): When $n=50$, $\sigma^2 = 50$ and $\underline{\hat{\beta}} = [2 \ 3 \ -6]^T$

Table (8): Average of estimated regression parameters, conditional number and MSE (case-8)



Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
OLS	1.7886	2.9687	-5.9318	15.507	0.000050
Hoerl and Kennard (1970)	27.015	-0.2738	-3.2879	5.7815	0.643830
Hoerl and Kennard (1975)	30.768	-0.9333	-2.6581	4.1441	0.854210
Hocking, Speed, and Lynn (1979)	30.705	-1.1403	-2.3776	3.3482	0.854240
Kibria (2003)1	30.078	-0.7679	-2.8329	4.6381	0.812600
Kibria (2003)2	29.870	-1.1987	-2.2439	2.9854	0.808500
Kibria (2003)3	30.915	-0.9925	-2.5890	3.9550	0.863660
Khalaf and Shukur (2005)	21.636	0.4619	-3.9107	7.1379	0.396390
Muniz and Kibria (2009)	15.975	1.1867	-4.5003	8.2568	0.200840
Muniz et al. (2012)	27.015	-0.2738	-3.2879	5.7815	0.643840
Khalaf (2012)	25.717	-0.0822	-3.4570	6.1708	0.578470
Proposed-1	19.033	0.7855	-4.1686	7.3767	0.298370
Proposed-2	27.335	-0.3202	-3.2473	5.6840	0.660480
Proposed-3	5.2457	2.5385	-5.5883	9.0050	0.010917
Proposed-4	1.9503	2.9485	-5.9157	9.4850	0.000012
Proposed-5	24.773	0.0258	-3.5380	6.3341	0.533530

Table (8) shows that all classical and proposed methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)). While the fourth proposed method is the best depending on the criteria MSE because they has less value compared with other methods.

3.2: Eigen value Regression:

The same simulation data was re-applied to Eigen value Regression based on a program designed for this purpose in MATLAB. The first case and the first experiment were as follows:

The simple correlation coefficient between the independent variables and the dependent variable was as follows:

Table (9): Correlation matrix

	Y	X ₁	X ₂
Y	1	0.85011	0.84768
X ₁	0.85011	1	0.97668
X ₂	0.84768	0.97668	1

The augment matrix for Eigen values and Eigen vectors for dependent and independent variables is:

$$\Gamma(\lambda, \gamma) = \begin{bmatrix} 2.7844 & 0.5582 & 0.8297 & 0.0072 \\ 0.1923 & 0.5869 & -0.3888 & -0.7102 \\ 0.0233 & 0.5864 & -0.4007 & 0.7039 \end{bmatrix}$$

From $\Gamma(\lambda, \gamma)$ matrix, we note that the deletion condition applies to the $\lambda_3 < 1$ and $|\gamma_{03}| \leq 0.5$ will therefore be omitted when estimating the Eigen values regression parameters and compute Conditional number, MSE of model and $MSE(\hat{\beta})$, it was summarized with (OLS) method in the following tables:

Table (10): Comparison of OLS and EVR

Method	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE	$MSE(\hat{\beta}_i)$	R ²
OLS	0.48149	0.37742	10.93	0.00564	0.24994	0.7293
EVR	0.42248	0.43643	3.805	0.00564	0.25008	0.7291



Table (10) shows that EVR methods have deal with multi-collinearity problem (because all Conditional number values were less than the number (10)), while maintaining the values of almost equal criteria for the (OLS) method.

The same simulation data (for Ridge Regression) was re-applied and repeated (1000) times and the averages calculation (regression parameters, Conditional number MSE, it was summarized in the following tables according to the different cases assumed:

Table (11): Comparison of OLS and EVR for 8-Cases

Case	Method	$\hat{\beta}_1$	$\hat{\beta}_2$	kappa	MSE
Case-1: n=20 $\sigma^2 = 20$ $\hat{\beta} = [2 \ 5 \ 9]^T$	OLS	0.3882	0.6072	15.605	0.0546
	EVR	0.2307	0.7640	9.7611	0.0542
Case-2: n=20 $\sigma^2 = 50$ $\hat{\beta} = [2 \ 5 \ 9]^T$	OLS	-0.4759	1.2240	11.468	0.7142
	EVR	0.4623	0.2842	2.7814	0.8119
Case-3: n=50 $\sigma^2 = 20$ $\hat{\beta} = [2 \ 5 \ 9]^T$	OLS	0.1715	0.8013	14.005	0.0755
	EVR	0.6824	0.2493	8.3007	0.0838
Case-4: n=50 $\sigma^2 = 50$ $\hat{\beta} = [2 \ 5 \ 9]^T$	OLS	0.6363	0.2564	14.615	0.3332
	EVR	0.4276	0.4650	4.4142	0.3349
Case-5: n=20 $\sigma^2 = 20$ $\hat{\beta} = [2 \ 3 \ -6]^T$	OLS	0.4993	-1.3663	14.726	0.9351
	EVR	-0.5120	-0.3542	3.9359	1.0495
Case-6: n=20 $\sigma^2 = 50$ $\hat{\beta} = [2 \ 3 \ -6]^T$	OLS	1.0635	-1.2494	10.814	2.3765
	EVR	-0.1211	-0.0646	1.4766	2.5396
Case-7: n=50 $\sigma^2 = 20$ $\hat{\beta} = [2 \ 3 \ -6]^T$	OLS	-0.3858	-0.3965	12.851	0.5149
	EVR	-0.3914	-0.3908	3.0512	0.5149
Case-8: n=50 $\sigma^2 = 50$ $\hat{\beta} = [2 \ 3 \ -6]^T$	OLS	0.1829	-0.5749	10.611	0.7471
	EVR	-0.2080	-0.1839	1.7157	0.7535

Table (11) shows that EVR method has deal with multi-collinearity problem (because all Conditional number values were less than the number (10) for all cases), while maintaining the values of almost equal criteria for the (OLS) method.

3.3: Comparison of Ridge and Eigen Value Regression:

To compare the best results of the Ridge Regression (RR) method with the Eigen Value Regression method of simulations that were performed and repeated (1000) times and to (8) cases, the following table is arranged:

Table (12): Comparison of Ridge and Eigen Value Regression

Case	Method	kappa	MSE
Case-1: n=20 $\sigma^2 = 20$ $\hat{\beta} = [2 \ 5 \ 9]^T$	RR	9.1505	0.000100
	EVR	9.7611	0.0542
Case-2: n=20 $\sigma^2 = 50$ $\hat{\beta} = [2 \ 5 \ 9]^T$	RR	5.3866	0.001910
	EVR	2.7814	0.8119
Case-3: n=50 $\sigma^2 = 20$ $\hat{\beta} = [2 \ 5 \ 9]^T$	RR	5.438	0.000080
	EVR	8.3007	0.0838
Case-4: n=50 $\sigma^2 = 50$ $\hat{\beta} = [2 \ 5 \ 9]^T$	RR	8.5693	0.000110



	EVR	4.4142	0.3349
Case-5: $n=20 \sigma^2 = 20 \hat{\beta} = [2 \ 3 \ -6]^T$	RR	8.0260	0.010735
	EVR	3.9359	1.0495
Case-6: $n=20 \sigma^2 = 50 \hat{\beta} = [2 \ 3 \ -6]^T$	RR	9.7410	0.002796
	EVR	1.4766	2.5396
Case-7: $n=50 \sigma^2 = 20 \hat{\beta} = [2 \ 3 \ -6]^T$	RR	9.5430	0.000220
	EVR	3.0512	0.5149
Case-8: $n=50 \sigma^2 = 50 \hat{\beta} = [2 \ 3 \ -6]^T$	RR	9.4850	0.000012
	EVR	1.7157	0.7535

Table (12) shows that the EVR method is better than the Ridge regression method to deal with multi-collinearity problem because it has lower conditional number values, except in some cases (case-1 and 2) The situation is contradict. While the proposed method of Ridge regression is the most efficient depending on the MSE because it has lower values for all cases.

4. Discussion and Conclusions

Through simulation results we note that all the proposed methods handle the multi-collinearity problem and in all simulation scenarios, the proposed methods perform better than classical methods, depending on MSE because they have less value compared with classical methods. The fourth proposed method was best in most different cases of sample size and variance values, especially when the slope of regression is negative.

The results of MSE values fluctuate as a result of the efficiency of the estimated model parameters, which depend on the different methods of ridge parameter estimation.

The EVR method is better than Ridge regression method for handling multi-collinearity problem, except in some cases (case-1 and 2) the situation is contradict.

The proposed method for Ridge regression is more efficient than EVR method with respect to MSE for all cases.

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ملخص

تم في هذا البحث إقتراح خمسة طرائق جديدة لتقدير معلمة الحرف من خلال إقحام العدد الشرطي، وإستخدامه في تقدير معالمات أنموذج إنحدار الحرف لمعالجة مشكلة تعدد العلاقات الخطية ومن ثم مقارنة كفاءتها مع بعض الطرائق التقليدية المقدمة من قبل عدة باحثين بالإعتماد على المعيار متوسط الخطأ التربيعي ومن ثم مقارنتها مع طريقة إنحدار الجذور المميزة، وذلك من خلال المحاكاة (بواسطة برنامج بلغة ماتلاب مصمم لهذا الغرض)، وإستنتاج البحث كفاءة الطرائق المقترحة في معالجة مشكلة تعدد العلاقات الخطية، وأفضليتها على الطرائق التقليدية لإنحدار الحرف وإنحدار الجذور المميزة.

Appendix

Part of Program (1) for Ridge Regression

```

clc
q=1; N=1000;
while q < N+1
% simulation
p=3; n=50; B=[3 -6]'; z=randn(2,n); sigma=[25 31;31 40]; mu=[ones(1,n)*20;ones(1,n)*15]; T=chol(sigma);
x=(mu+T'*z); r=corr(x); beta=[2 3 -6]'; error=normrnd(0,50,n,1); cov(error); E=[ones(size(error)) x]; y=E*beta+error;
Y1=y-mean(y); X1=[x(:,1)-mean(x(:,1)) x(:,2)-mean(x(:,2))]; C1=X1*X1'; EIG=eig(C1); kappa=sqrt(max(EIG)/min(EIG));
% OLS estimator
betaols=E\y; Y=E*betaols; e=y-Y; MSE=e'*e/(n-p); C=E'*E;
MSEbols=MSE*trace(inv(C1)); kappam(q)=kappa; betaolsm(q,:)=betaols;
MSEbolsm(q)=MSEbols; SU(q,:)=(betaols-beta).^2;
% Estimation of ridge parameter
[D, EV]=eig(C1); X=X1*D; a=D'*B; k=MSE./(a.^2); l=eig(C1);
% method 11
kT1=kappa^(2*p)/(MSE*abs(mean(a.^2)));
W=inv(eye(p-1) +kT1*(inv(C1))); CK=(C1+(kT1*eye(p-1)));
beta11=(inv(C1+kT1*eye(p-1)))*X1'*Y1;
beta0=mean(y)-beta11(1)*mean(x(:,1))-beta11(2)*mean(x(:,2));
b1=[beta0 beta11']; Y=E*b1; e11=y-Y; MSE11=e11'*e11/(n-p);
MSEbeta11=MSE11*trace(W*(inv(C1))*W)+(kT1^2)*(beta11'*CK^(-2)*beta11); EIG=eig(CK);
kappa11=sqrt(max(EIG)/min(EIG)); kappam11(q)=kappa11; betam11(q,:)=b1; MSEm11(q)=MSEbeta11; SU11(q,:)=(b1-
beta).^2;
% method 12
kT2=kappa^(1/(n-p))*MSE/(max(a.^2));
W=inv(eye(p-1) +kT2*(inv(C1))); CK=(C1+(kT2*eye(p-1)));
beta12=(inv(C1+kT2*eye(p-1)))*X1'*Y1;
beta0=mean(y)-beta12(1)*mean(x(:,1))-beta12(2)*mean(x(:,2));
b1=[beta0 beta12']; Y=E*b1; e12=y-Y; MSE12=e12'*e12/(n-p);
MSEbeta12=MSE12*trace(W*(inv(C1))*W)+(kT2^2)*(beta12'*CK^(-2)*beta12); EIG=eig(CK);
kappa12=sqrt(max(EIG)/min(EIG)); kappam12(q)=kappa12; betam12(q,:)=b1; MSEm12(q)=MSEbeta12; SU12(q,:)=(b1-
beta).^2;
% method 13
kT3=kappa^(1/p)*max((max(l)*MSE)./(n^2*MSE+max(l)*a2)); W=inv(eye(p-1) +kT3*(inv(C1))); CK=(C1+(kT3*eye(p-
1))); beta13=(inv(C1+kT3*eye(p-1)))*X1'*Y1;

```



```

beta0=mean(y)-beta13(1)*mean(x(:,1))-beta13(2)*mean(x(:,2)); b1=[beta0 beta13']; Y=E*b1; e13=y-Y;
MSE13=e13'*e13/(n-p); MSEbeta13=MSE13*trace(W*(inv(C1))*W)+(kT3^2)*(beta13'*CK^(-2)*beta13); EIG=eig(CK);
kappa13=sqrt(max(EIG)/min(EIG)); kappam13(q)=kappa13; betam13(q,:)=b1; MSEm13(q)=MSEbeta13; SU13(q,:)=(b1-
beta).^2;
% method 14
kT4=p*abs(mean(a))/kappa^2;
W=inv(eye(p-1)+kT4*(inv(C1))); CK=(C1+(kT4*eye(p-1)));
beta14=(inv(C1+kT4*eye(p-1)))*X1'*Y1;
beta0=mean(y)-beta14(1)*mean(x(:,1))-beta14(2)*mean(x(:,2));
b1=[beta0 beta14']; Y=E*b1; e14=y-Y; MSE14=e14'*e14/(n-p);
MSEbeta14=MSE14*trace(W*(inv(C1))*W)+(kT4^2)*(beta14'*CK^(-2)*beta14); EIG=eig(CK);
kappa14=sqrt(max(EIG)/min(EIG)); kappam14(q)=kappa14; betam14(q,:)=b1; MSEm14(q)=MSEbeta14; SU14(q,:)=(b1-
beta).^2;
% method 15
kT5=n*MSE/(kappa^2*(min(l)+min(a)));
W=inv(eye(p-1)+kT5*(inv(C1))); CK=(C1+(kT5*eye(p-1)));
beta15=(inv(C1+kT5*eye(p-1)))*X1'*Y1;
beta0=mean(y)-beta15(1)*mean(x(:,1))-beta15(2)*mean(x(:,2));
b1=[beta0 beta15']; Y=E*b1; e15=y-Y; MSE15=e15'*e15/(n-p);
MSEbeta15=MSE15*trace(W*(inv(C1))*W)+(kT5^2)*(beta15'*CK^(-2)*beta15); EIG=eig(CK);
kappa15=sqrt(max(EIG)/min(EIG)); kappam15(q)=kappa15; betam15(q,:)=b1; MSEm15(q)=MSEbeta15; SU15(q,:)=(b1-
beta).^2; q=q+1;
end

```

Program (2) for Eigen Value Regression

```

clc
p=2;input data;y=data(:,1);x1=data(:,2); x2=data(:,3);n=length(y); R=corr(data);[G I]=eig(R);
ys=(y-mean(y))/sqrt((n-1)*var(y)); x1s=(x1-mean(x1))/sqrt
((n-1)*var(x1)); x2s=(x2-mean(x2))/sqrt((n-1)*var(x2));
datas=[ys x1s x2s]; xs=[x1s x2s];
beta=(inv(xs'*xs))*xs'*ys; SSE=(ys'*ys)-beta*(xs'*ys);
SST=(n-1)*var(ys); SSR=SST-SSE;
RS=SSR/SST;SIGMA=sqrt(SSE/(n-p-1)); V=SIGMA^2*(xs'*xs)^(-1);
MSEolsbeta=trace(V); xr=[x1s x2s ]; yh=xr*beta; e=yh-ys;MSEols=e'*e/(n-2)
Y=SST*yh+mean(y);er=Y-y; MSE1=er'*er/(n-2); a=diag(I);
for i=1:p
    if abs(G(1,i))< 0.5 & a(i)< 1
        G(:,i)=[];a(i)=[];
    end
end
a, G, w=0, d=0
for i=1:2
    w=w+(G(1,i)/a(i))*G(:,i);d=d+G(1,i)^2/a(i)
end
w(1)=[]; betaridge=-w/d; SSEr=(ys'*ys)-betaridge*(xs'*ys);
SSTr=(n-1)*var(ys); SSRr=SSTr-SSEr;
RSr=SSRr/SSTr;SIGMAr=sqrt(SSEr/(n-p-1)); Vr=SIGMAr^2*(xs'*xs)^(-1);
MSERbeta=trace(Vr);yh=xr*betaridge; e=yh-ys; MSERID=e'*e/(n-2)

```