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CORE-POLARIZATION EFFECTS ON E2 TRANSITIONS IN EXOTIC NUCLEUS ^{70}Ni

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ABSTRACT

The structure of ^{70}Ni nucleus is studied by using Particle-Vibration Coupling Model (PVCM), which separates the nucleus into three systems, vibrating core and two interacting neutrons. The vibration of the core is represented by single quadrupole phonon, and the two neutrons are assumed to occupy the state $1g_{9/2}$ and interacted by MSDI. A very well reproduction for the positive parity states of ^{70}Ni is obtained by applying the PVCM. The core polarization effect that includes particle-hole excitation from the proton states of the core, to the states outside the model space with $2\hbar\omega$, is very necessary to reproduce the reduced E2 transition probability.

Keywords : *Core-Polarization Effect, Particle-Vibration Coupling Model, ^{70}Ni Nucleus.*

1. INTRODUCTION

The study of exotic nuclei is one of the current topics in nuclear structure physics. The structure of nuclei in the medium and heavy mass regions can be studied by using large basis shell model calculations. To investigate the structure of exotic nucleus ^{70}Ni in the framework of shell model, Chiara *et al.* [1] used the effective interaction of 14 neutrons distributed over the model space $1f_{5/2}$ $1p_{3/2}$ $1p_{1/2}$ and $1g_{9/2}$ outside the inert core ^{56}Ni, which is a double magic nucleus. The comparison of experimental properties of nuclei around ^{68}Ni, leads to the conclusion of the double magicity of Z=28 and N=40 [2,3]. This conclusion candidates ^{68}Ni as inert core to study the shell structure for nuclei with Z>28 and N>40. Van Roosbroeck *et al.*[4] discussed

the ^{70}Ni β-decay to the isomeric states in ^{70}Cu and their subsequent isomeric decay and β-decay to ^{70}Zn within two nucleons outside ^{68}Ni inert core, and they yield satisfactory results.

In another point of view, the inert core may be activated through surface vibration, which represented by phonons creation and annihilation. This approach is called Particle Vibration Coupling Model (PVCM). Oros-Peusquens and Mantica[5] investigated the odd-mass Ni and Cu isotopes around ^{68}Ni core nucleus by using PVCM. Their calculated properties are in good agreement with data.

In the present work, the structure of even-mass ^{70}Ni nucleus is studied by using PVCM. The model space is restricted by $1g_{9/2}$ for particles, and from 0 to 3 quadrupole phonons for surface vibrations of ^{68}Ni core states. In the calculations of reduced transition probability, the core polarization effects are taken into account through admixture of higher orbits which include 1p-1h excitations up to $2\hbar\omega$.

2. THEORY

2.1 Hamiltonian and Basis Function

The total Hamiltonian operator of the three body system which consists of core plus two nucleons is given by[6,7,8]:

$$\hat{H} = \hat{H}_{coll} + \sum_{i=1}^{2} \hat{H}_{s.p.}(i) + \sum_{i=1}^{2} \hat{H}_{int}(i) + \hat{V}(1,2) \quad (1),$$

where the collective Hamiltonian of the vibrating core is given in terms of boson creation and annihilation operators by[6,8],

$$\hat{H}_{coll} = \sum_{\mu} \hbar\omega_2 \left[b^{\dagger}_{2\mu} b_{2\mu} + \frac{1}{2} \right] \quad (2),$$

with $\hbar\omega_2$ represents the quadrupole phonon energy. $\hat{H}_{s.p.}(i)$, is that for a particle in a harmonic oscillator potential well, and the particle-core interaction Hamiltonian, $\hat{H}_{int}(i)$, is that for the *i*th particle in the field is caused by the vibrating core which is supposed to follow the density vibrations adiabatically. This coupling Hamiltonian is given in terms of the scalar products of boson creation and annihilation operators and spherical harmonics, by[9],

$$\hat{H}_{int}(i) = \sqrt{\frac{\pi}{5}} \xi_2^{(i)} \hbar\omega_2 \left[b_2^{\dagger} \cdot Y_2(\hat{r}_i) + b_2 \cdot Y_2^*(\hat{r}_i) \right] \quad (3),$$

where the strength of this interaction is given by the parameters $\xi_2^{(i)}$. The last term of $\hat{H}$ is the two-particle potential, $V(1,2)$.

The space of the PVCM is defined by the basis function $|N_2 R_2, (j_1 j_2) J; IM\rangle$. Where N_2 represents the number of quadrupole phonons coupled to angular momentum R_2. The single particle states j_1 and j_2 are coupled to J, then both R_2 and J are coupled to total angular momentum I in the order $\vec{I} = \vec{R}_2 + \vec{J}$, with $|R_2 - J| \leq I \leq R_2 + J$.

The diagonalization of the Hamiltonian in the configuration mixing space of the PVCM

states $|N_2R_2,(j_1j_2)J;IM\rangle$, yields energy eigenvalues, $E^{(\alpha)}$, and eigenvectors, $C_\alpha(N_2R_2,(j_1j_2)J;I)$.

2.2 Electromagnetic Transitions

The electromagnetic transition operator can be written in the framework of PVCM as:

$$\hat{O}^{\eta}_{LM} = \hat{O}^{\eta}_{LM}(core) + \sum_{i=1}^{2}\hat{O}^{\eta}_{LM}(i) \qquad (4),$$

where $\hat{O}^{\eta}_{LM}(i)$ and $\hat{O}^{\eta}_{LM}(core)$ are the single –particle and the core parts, respectively, and

$$\left\langle N_2'R_2',(j'_1j'_2)J',I'\right|\left|\hat{O}^{\eta}_{L}\right|\left|N_2R_2,(j_1j_2)J,I\right\rangle =$$

$$(-1)^{I+L}\sqrt{(2I+1)(2I'+1)}\times\left[(-1)^{R_2+J}\begin{Bmatrix} J' & J & L \\ I & I' & R'_2\end{Bmatrix}\sum_{i=1}^{2}\left\langle J'\right|\left|\hat{O}^{\eta}_{L}(i)\right|\left|J\right\rangle\delta_{N_2N_2'}\delta_{R_2R_2'} +\right.$$

$$\left.(-1)^{R'_2+J'}\begin{Bmatrix} R'_2 & R_2 & L \\ I & I' & J\end{Bmatrix}\left\langle N_2'R_2'\right|\left|\hat{O}^{\eta}_{L}(core)\right|\left|N_2R_2\right\rangle\delta_{JJ'}\,\delta_{j_1j'_1}\delta_{j_2j'_2}\right] \qquad (5)$$

where the states $|J\rangle$ and $|R_2\rangle$ represent, respectively, the two- particle and collective model space wave functions. The operator $\hat{O}^{\eta}_{LM}(i)$ acts on a single –particle initial state $|j\rangle$ and transit it to final state $|j'\rangle$ within the single- particle model space.

Introducing the core polarization (CP) effects based on a microscopic theory that combines shell model wave functions and configurations with higher energy as first order perturbations, on the single particle operator of $\hat{O}^{\eta}_{LM}(i)$ which corrects its reduced matrix elements as follow [6]:

η stands for the type of the operator (η = M or E for magnetic or electric operator, respectively,), L is the transition multipolarity.

By using the standard Racah Algebra, the reduced matrix element of the electromagnetic transition operator between the PVCM states can be written as:-

$$\left(j'\left\|\hat{O}^{\eta}_{L}(i)\right\|j\right) = \left\langle j'\right|\left|\hat{O}^{\eta}_{L}(i)\right|\left|j\right\rangle +$$

$$\left\langle j'\right|\left|\hat{O}^{\eta}_{L}(i)\frac{\hat{Q}}{E_j-H^{(0)}}V_{res}\right|\left|j\right\rangle +$$

$$\left\langle j'\right|\left|V_{res}\frac{\hat{Q}}{E_{j'}-H^{(0)}}\hat{O}^{\eta}_{L}(i)\right|\left|j\right\rangle \qquad (6),$$

The first term is the model space contribution, while the second and third terms appear due to the core-polarization effect, i.e., outside the model space. The operator $\hat{Q}$ is the projection operator onto the space outside the model space. E_j and $E_{j'}$ are the energies of the initial and final single-particle states, respectively, and $H^{(0)}$ is the unperturbed Hamiltonian.

The CP terms are given by[6]:

$$\sum_{j_p j_h J}(2J+1)(-1)^{j+j_h+J}\begin{Bmatrix} j' & j & L \\ j_h & j_p & J \end{Bmatrix}\sqrt{\left(1+\delta_{j_p j'}\right)\left(1+\delta_{j_h j}\right)}\frac{\left\langle j'\left|\left|\hat{O}_L^{\eta}(i)\right|\right|j\right\rangle}{e_j - e_{j'} - e_{j_p} + e_{j_h}}$$

$$\langle j' j_p | V_{res} | j j_h \rangle_J + terms\ with\ j_p\ and\ j_h\ exchanged\ with\ an\ overall\ minus\ sign \quad (7),$$

Where, V_{res} the residual interaction , j_p and j_h run over particle and hole states outside the model space .In proton-neutron formalism, the sum will be over all protons and neutrons particle-hole pairs. The single-particle energy $e_{n\ell j}$ can be calculated from [6,10]:

$$e_{n\ell j} = \left(2n + \ell - \frac{1}{2}\right)\hbar\omega + \begin{cases} -\frac{1}{2}(\ell+1)\langle f(r)\rangle & \text{for } j = \ell - \frac{1}{2} \\ \frac{1}{2}\ell\langle f(r)\rangle & \text{for } j = \ell + \frac{1}{2} \end{cases} \quad (8),$$

Where, $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ MeV, and $\langle f(r)\rangle_{n\ell} = -20\ A^{-2/3}$.

The reduced transition probability for electromagnetic transition in the framework of PVCM is[6]:

$$B(\eta L, I \to I') = \frac{1}{2I+1}\left|\sum_{\alpha\beta} C_\alpha C_\beta \left\langle \alpha; I'\left|\left|\hat{O}_L^{\eta}\right|\right|\beta; I\right\rangle\right|^2 \quad (9),$$

with $\alpha \equiv N_2' R_2' j'_1 j'_2 J'$ and $\beta \equiv N_2 R_2 j_1 j_2 J$. The eigenvectors C_α and C_β are obtained from the diagonalization of the Hamiltonian within configuration mixing of PVCM states, while the reduced matrix elements of $\hat{O}_L^{\eta}$ are separated in eq.(6) in terms of the reduced matrix elements of single-particle and collective operators.

The single particle contribution in the electric quadrupole operators is given by[6]:

$$\hat{O}_2^{E}(i) = er^2\, Y_2(\hat{r}_i) \quad (10),$$

and the core contribution is given by[6]:

$$\hat{O}_2^{E}(core) = \frac{3}{4\pi} Z\, e\, R_0^2\, \alpha_2 \quad (11),$$

where $Y_2(\hat{r}_i)$ represents the spherical harmonics, Z is the atomic number, and R_0 is the equilibrium radius of the core nucleus, which is given in term of the mass number A by $R_0 = 1.2\, A^{1/3}$ fm . And the deformation parameter $\hat{\alpha}_{\lambda\mu}$ is given in terms of boson creation and annihilation operators by[11]:

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}\left[\hat{b}_{\lambda\mu} + (-1)^{\mu}\,\hat{b}^{\dagger}_{\lambda-\mu}\right] \quad (12),$$

3. RESULTS AND DISCUSSION

The nucleus ^{70}Ni is treated as three-body system consisting of two neutrons outside ^{68}Ni core. Both neutrons are assumed to occupy the

single particle state, $1g_{9/2}$. The unperturbed energy of each neutron is $e_{1g9/2} = -4.6\,MeV$[2]. The unperturbed energy of the quadrupole phonons is $\hbar\omega_2 = 2.034\,MeV$[12], which represents the energy of the first 2^+state of the core nucleus ^{68}Ni. The mutual interaction between neutrons is assumed to be Modified Surface Delta Interaction (MSDI). The strength parameters of this interaction are $A_0 \simeq A_1 \simeq B \simeq \frac{25}{A}\,MeV = 0.357\,MeV$, and $C\simeq 0$ [6], where A represents the atomic mass. The strength parameters of the particle-core interaction are taken as adjustable parameters that reproduce the best energy levels of ^{70}Ni, which found in the present work, $\xi_2^{(1)} = \xi_2^{(2)} = 0.1$. Diagonalization of the Hamiltonian matrix gives eigenvalues (energy levels) and eigenvectors (configuration mixing wave functions). Fig.(1) shows the experimental [12] and calculated (represented as PVCM1)the energy

eigenvalues of positive parity states in ^{70}Ni. The weak coupling of the core nucleus ^{68}Ni with the two extra neutrons appears very important to reproduce the I^π values. The calculated energy levels of ^{70}Ni are reasonable as compared with the experimental values. However, the second 2^+ state appears higher than 4^+state, which is similar to the result obtained in ref. [1]. Chiara *et al.* [1] ascribed this result to the lack of proton excitations across the Z=28 shell gap, which are necessary to be included.

In our somewhat simple model, the results are very well enhanced by increasing the strength parameters of MSDI slightly to be A_0=A_1=B=0.45 MeV and C=0.3 MeV. The results are given as PVCM2 in Fig. (1). It appears that this justified modification in MSDI parameters moves the energy levels toward their corresponding experimental one. Recently Prokop *et al.*[13] discovered a new 0_2^+ state at 1.567 MeV in ^{70}Ni, through γ-ray spectroscopy following the β-decay of ^{70}Co. In our present work, a 0_2^+ state appears at 4.671 MeV.

The eigenvectors of each nuclear state in ^{70}Ni are given in table (1). The table shows the predominance of the two particle contribution in the PVCM state. However, there is a non-negligible contribution of the collective states, especially for the 2^+states. The eigenvectors are important in the calculation of the following reduced electric quadrupole transition probability, B(E2), as well as moment, in ^{70}Ni.

The calculated and experimental values of reduced transition probability, B(E2), in ^{70}Ni are given in table (2). Our bare PVCM calculations are far from the available experimental data, as appear in the 3rd column of table (2).

Since the extra two neutrons give no contribution to the electric quadrupole transition, these results are due to quadrupole vibrations of the core only. However, the

unexpectedly large experimental values of B(E2) in ^{70}Ni, make Perru *et al.* [14] to indicate that neutrons added above N=40 strongly polarize the Z=28 proton core. However, the nucleons of the core, in the present work, are partially excited through collective surface oscillation. So, we can extend the shell model space by including core polarization effect in the calculation of B(E2), which excites protons from the core orbits 1p-2s1d-1$f_{7/2}$ to the higher allowed orbits outside the core shells of protons, with $2\hbar\omega$ excitations. Two potentials are assumed as residual interactions between the intermediate 1p-1h states, which are MSDI [6] and the Michigan three range Yukawa interaction M3Y [11]. The parameters of M3Y used in this work were those of Reid [15]. The enhancement in the calculations of B(E2) by including core polarization effects clearly appears in 4th and 5th columns of table (2). Both $B(E2, 6_1^+ \rightarrow 4_1^+)$ and $B(E2, 8_1^+ \rightarrow 6_1^+)$ are reproduced excellently by PVCM+CP calculations. However, adopting M3Y as a residual interaction increases the results slightly as compared with the experimental one, but it enhances the calculated $B(E2, 2_1^+ \rightarrow 0_1^+)$ value as compared with that obtained from PVCM+CP with MSDI as residual interaction.

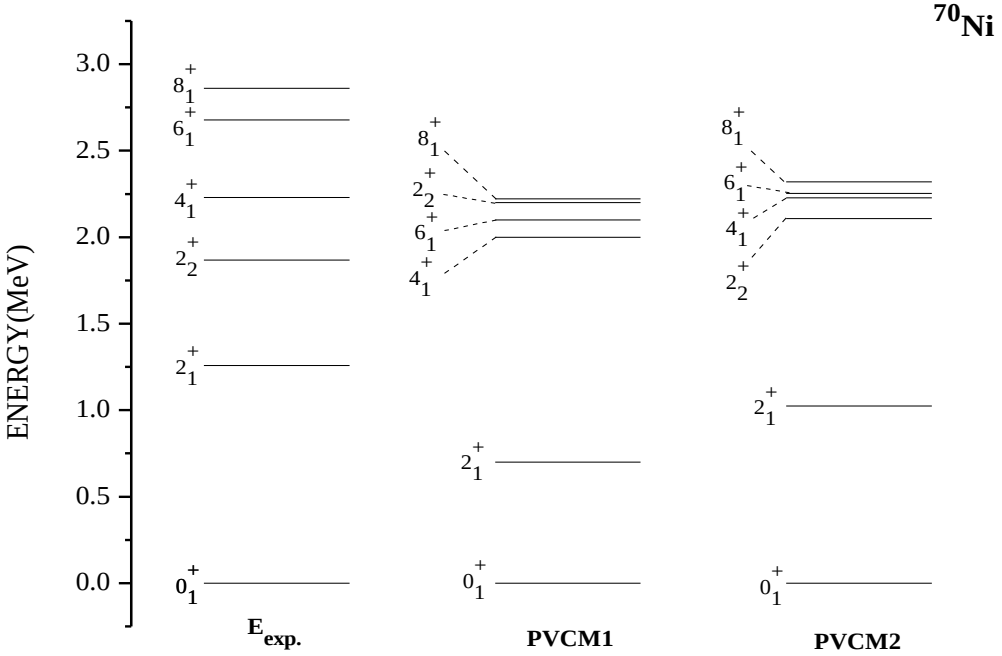

Figure 1:The calculated energy levels of ^{70}Ni nucleus in the framework of PVCM as compared with the experimental values.

Table 1: The eigenvectors $\times 10^{-4}$ for the ground and excited states in ^{70}Ni.

N_2	R_2	j_1	j_2	J	0_1^+	2_1^+	2_2^+	4_1^+	6_1^+	8_1^+
0	0	9/2	9/2	0	9488					
1	2	9/2	9/2	2	3159	6766	6205	-2519		
0	0	9/2	9/2	2		4678	-7778			
1	2	9/2	9/2	4		5570	-994.0			
1	2	9/2	9/2	2		-1138	53.15			
0	0	9/2	9/2	4				9635		
1	2	9/2	9/2	4				-2096	-1668	
1	2	9/2	9/2	6				-1641	-1152	-2018
0	0	9/2	9/2	6					9583	
1	2	9/2	9/2	8					-2012	-2357
0	0	9/2	9/2	8						9506

Table 2: Experimental and calculated values of reduced electric quadrupole transitions in ^{70}Ni nucleus to one phonon .

Transitions	$B(E2)_{exp.}$ (W.u.)	$B(E2)_{theo.}$ (W.u.)		
		PVCM	PVCM+CP	
			MSDI	M3Y
$2_1^+ \rightarrow 0_1^+$	2.0[16]	0.00393	0.5915	0.7449
$2_2^+ \rightarrow 2_1^+$	---	0.00105	1.6827	2.0739
$2_2^+ \rightarrow 0_1^+$	---	0.00571	0.5723	0.6947
$4_1^+ \rightarrow 2_1^+$	---	0.00068	0.8206	1.0106
$4_1^+ \rightarrow 2_2^+$	---	0.00022	1.244	1.5368
$6_1^+ \rightarrow 4_1^+$	1.73[12]	0.00142	1.5812	1.9467
$8_1^+ \rightarrow 6_1^+$	0.655[12]	0.0018	0.6868	0.8419

4. CONCLUSION

The coupling of core excited through one quadrupole phonon with the simple shell model space, $1g_{9/2}$, is necessary to reproduce the I^{π}values, as well as the energy levels of ^{70}Ni. Introducing another

manner of core excitation through core polarization effect is found essential in the calculation of reduced transition probability B(E2) between nuclear states of ^{70}Ni. The choice of the form of particle-hole residual interaction is very important in the calculation of core polarization parts of the reduced electric matrix elements. A more realistic interaction may reproduce the value of the transition strength $B(E2, 2_1^+ \rightarrow 0_1^+)$.

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تأثير استقطاب اللب على انتقالات E2 في النواة الغريبة ^{70}Ni

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الخلاصة:

درس تركيب نواة ^{70}Ni والتي تتكون من نيوترونين خارج قلب مغلق هو ^{68}Ni باستخدام أنموذج اقتران جسيم – ذبذبه (PVCM) والذي يقسم النواة إلى ثلاثة منظومات , لب متذبذب ونيوترونين متفاعلين, يمثّل تذبذب اللب بفونونات رباعية القطب أما النيوترونين فيشغلان الحالة $1g_{9/2}$ ويتفاعلان بتفاعل أل MSDI. تم إعادة إنتاج الحالات ذات التماثل الموجب لل ^{70} Ni بصوره جيده جدا, وباستخدام PVCM .كذلك أضيف تأثير استقطاب اللب CPE_s والذي يتضمن تهيجات جسيم – فجوه من اعتماد حالات البروتونات في اللب إلى حالات خارج فضاء النموذج ب $2\hbar\omega$ والتي وجد أنها ضرورية لإعادة إنتاج احتماليه الانتقال المختزلة B(E2).