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7-Modular Character of The Covering group $\bar{S}_{23}$

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Abstract:
In this paper we find the modular characters of the covering group $\bar{S}_{23}$ modulo p=7 which can give the irreducible modular spin characters for S_{23} modulo p=7, also we give the 7 −decomposition matrix of $\bar{S}_{23}$

Section (1)

Introduction(1.1):

The Symmetric group S_n has a covering group denoted by $\overline{S_n}$ of order $2(n!)$, the projective characters of S_n is called the spin characters of S_n, which are the ordinary characters of $\overline{S_n}$ indexed by the partitions of n with distinct parts.[I. Schur1911],[A.O. Morris1962].

For $p = 7$ Yaseen [A.K.Yaseen 1987] found the modular irreducible spin character of S_n,and $7 \le n \le 13$, for $n = 14$ are found by Yaseen and Taban[A.K.Yaseen and S.A.Taban 1995], for $n = 16, 17$ and 18 are found by Taban[S.A.Taban 1998, 2001 and 2004 respectively], for $n = 19$ by Najla'a[N.S.Abdullah 2009] for $n = 20$ by Jenan [J. A. Resan 2010] and $n = 21$ founded by A. H. Jassim [A. H. Jassim 2011]. Finally Nizar [N. M. Yacoob 2014] founded $n = 22$.

Preliminaries(1.2):

For any group there are three kinds of characters ordinary, modular (for a given prime p), and projective (for S_n called spin). The decomposition matrix is the relation between the ordinary and modular characters for a given prime p.
The characters of $\overline{S_n}$ fall into two classes

1) The characters indexed by the partition of n
2) The characters indexed by the partition of n with distinct parts spin (modular).

Character of S_n can be written as a linear combination, with non-negative integer coefficients, of the irreducible spin (modular) characters [L. Dornhoff 1972]. Below some theorems we need to evaluate the decomposition matrix and modular spin characters for S_n:

1. Degree of the spin character $\langle\alpha\rangle = \langle\alpha_1, \dots, \alpha_m\rangle$ is:

   $deg\langle\alpha\rangle = 2^{[\frac{n-m}{2}]} \frac{n!}{\prod_{i=1}^{m}(\alpha_i!)} \prod_{1\le i<j\le m}(\alpha_i - \alpha_j)/(\alpha_i + \alpha_j)$ [A.O.Morris 1962], [A.O.Morris and A. K. Yaseen 1988].

2. Let B be the block of defect one and let b the number of p^a −conjugate characters to the irreducible ordinary character χ of G.Then [B. M. Puttaswamaiah and J. D. Dixon 1977]:
   a) There exists a positive integer number N such that the irreducible ordinary characters of G are lying in the block B divided into two disjoint classes: B_1={$\chi \in B$ $|b \deg x \equiv N \; mod \; p^a$}, B_2={$\chi \in B$ $|b \deg x \equiv -N \; mod \; p^a$}.
   b) Each coefficient of the decomposition matrix of the block B is 0 or 1.
   c) If $\alpha_1$and α_2 are not p −conjugate characters and belong to the classes(B_1 and B_2)respectively, then they have no irreducible modular character in common
   d) For every irreducible ordinary character χ in B_1, there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with one multiplicity .
3. If C is a principal character of G for a prime p, then $\deg C \equiv 0 \; mod \; p^A$, where $o(G) = p^A m \, , (p,m) = 1$ [S.A.Taban 1989],[J.F. Humphreys 1977].
4. If the decomposition matrix $D_{n-1,p} = \left(d_{ij}\right)$ for S_{n-1} is known, then we can induce columns $\left(\psi_j \uparrow^{(r,\bar{r})} S_n\right)$ for S_n [A.K.Yaseen 1987], these columns are a linear combination with non-negative coefficients from the columns of $D_{n,p}$ [G.D.James and A.Kerber 1981].

Notation*(1.3)*:

$(<\lambda>)^{no}$	(no) mean the number of i.m.s. in $<\lambda>$
i.m.s.	Irreducible modular spin character.
m.s.	Modular spin character.
p.i.s.	Principle indecomposable spin character.
p.s.	Principle spin character.

Decomposition matrix for S_{23} modulo p=7

The decomposition matrix for S_{23} modulo $p = 7$ of degree (156,105) [A.O.Morris 1962], [A.O.Morris and A.K.Yaseen 1988].There are 13 blocks, the block B_1 of defect three, the blocks B_2 B_3 ,B_4, and B_5 are of defect two , B_6 , B_7, B_8 , are of defect one and B_9, B_{10},… ,B_{13} are of defect zero.

Section (2) blocks of defect one

In this section, we determine the Brauer trees of the blocks B_6 ,B_8, all i.m.s. are associate, in B_7 all i.m.s. of the decomposition matrix for this block is double [A.K.Yaseen 1987].

Lemma (2 . 1):

Brauer tree for the block B_8 is:

$$\begin{array}{l} \langle 16,4,2,1\rangle\_\_\langle 11,9,2,1\rangle\_\_\langle 9,8,4,2\rangle \\ \langle 16,4,2,1\rangle'\_\_\langle 11,9,2,1\rangle'\_\_\langle 9,8,4,2\rangle' \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \langle 9,7,4,2,1\rangle^{*}$$

Proof:

- $\deg(\langle 11,9,2,1\rangle, \langle 11,9,2,1\rangle', \langle 9,7,2,1\rangle^*) \equiv 294 \; mod \; 7^3$,
  $\deg(\langle 16,4,2,1\rangle, \langle 16,4,2,1\rangle', \langle 9,8,4,2\rangle, \langle 9,8,4,2\rangle') \equiv -294 \; mod \; 7^3$.
- By using inducing of p.i.s. for S_{22} to S_{23} we have on p.i.s. we have:

$D_{72} \uparrow^{(1,0)} S_{23} = d_{95}$, $D_{73} \uparrow^{(1,0)} S_{23} = d_{96}$ (no sub sum of them $\equiv 0 \; mod \; 7^3$),

and p.s.

$D_{74} \uparrow^{(1,0)} S_{23} = k_1, D_{75} \uparrow^{(1,0)} S_{23} = k_2$, $D_{79} \uparrow^{(4,4)} S_{23} = k_3$.

$\langle 9,8,4,2,1\rangle$ and $\langle 9,8,4,2,1\rangle'$ are p.i.s. of S_{24} (of defect 0 in $S_{24}, p = 7$) we have:

$\langle 9,8,4,2,1\rangle \downarrow_{(1,0)} S_{23} = \langle 9,8,4,2\rangle + \langle 9,7,4,2,1\rangle^* = d_{99}$

$\langle 9,8,4,2,1\rangle' \downarrow_{(1,0)} S_{23} = \langle 9,8,4,2\rangle' + \langle 9,7,4,2,1\rangle^* = d_{100}$

Since $k_3 = k_1 + k_2 - d_{99} - d_{100}$, either ($k_1 - d_{99} \; and \; k_2 - d_{100}$) or ($k_1 - d_{100} \; and$ $k_2 - d_{99}$)are p.s. In any case we have k_2, k_3 are not p.i.s. so we take $d_{97} = k_1 - d_{100}$, $d_{98} = k_2 - d_{99}$. Hence, we have the Braure tree for this block B_8 ■.

***Lemma* (2 . 2):**

Brauer tree for the block B_7 is:

$\langle 17,3,2,1\rangle$__$\langle 10,9,3,1\rangle$__$\langle 10,8,3,2\rangle$ \
                                                                                      $\langle 10,7,3,2,1\rangle^*$
$\langle 17,3,2,1\rangle'$__$\langle 10,9,3,1\rangle'$__$\langle 10,8,3,2\rangle'$ /

Proof:

- $\deg(\langle 10,9,3,1\rangle, \langle 10,9,3,1\rangle', \langle 10,7,3,2,1\rangle^*) \equiv 249 \; mod \; 7^3$,
  $\deg(\langle 17,3,2,1\rangle, \langle 17,3,2,1\rangle', \langle 10,8,3,2\rangle, \langle 10,8,3,2\rangle') \equiv -294 \; mod \; 7^3$.
- By using inducing of p.i.s. for S_{22} to S_{23} we have on p.i.s.:

$D_{66} \uparrow^{(1,0)} S_{23} = d_{89}$, $D_{97} \uparrow^{(1,0)} S_{23} = d_{90}$ (no sub sum of them $\equiv 0 \; mod \; 7^3$),

and p.s.

$D_{68} \uparrow^{(1,0)} S_{23} = k_1, D_{69} \uparrow^{(1,0)} S_{23} = k_2$, $D_{70} \uparrow^{(1,0)} S_{23} = k_3, D_{93} \uparrow^{(2,6)} S_{23} = k_4$.

Since $\langle 10,8,3,2,1\rangle$ and $\langle 10,8,3,2,1\rangle'$ are p.i.s. of S_{24} (of defect 0 in $S_{24}, p = 7$) and:

$\langle 10,8,3,2,1\rangle \downarrow_{(1,0)} S_{23} = \langle 10,8,3,2\rangle + \langle 10,7,3,2,1\rangle^* = d_{93}$

$\langle 10,8,3,2,1\rangle' \downarrow_{(1,0)} S_{23} = \langle 10,8,3,2\rangle' + \langle 10,7,3,2,1\rangle^* = d_{94}$

then $k_3 = d_{93} + d_{94}$, and since $k_4 = k_1 + k_2 - d_{93} - d_{94}$, either $(k_1 - d_{93}\ and\ k_2 - d_{94})$ or $(k_1 - d_{94}\ and\ k_2 - d_{93})$are p.s. In any case, we have k_2, k_3 are not p.i.s. so we take $d_{91} = k_1 - d_{94},\ d_{92} = k_2 - d_{93}$. Hence, we have the Braure tree for this block B_7 ■.

***Lemma* (2 . 3)**:

Brauer tree for the block B_6 is:

$$\langle 18,4,1\rangle^{*}\_\_\_\langle 11,8,4\rangle^{*}\_\_\_\langle 11,7,4,1\rangle = \langle 11,7,4,1\rangle'\_\_\_\langle 11,5,4,2,1\rangle^{*}.$$

Proof:

by(4,4)-inducing of p.i.s D_9, D_{17}, D_{23} of S_{22} to S_{23} we get on the Brauer tree of the block B_6. ■

Section (3) block of defect two

In this section, the decomposition matrices for blocks B_2, B_3, and B_4 all i.m.s. are associate, and B_5 all i.m.s. of the decomposition matrix is double [A.K.Yaseen 1987].

***Lemma*(3. 1)**:

The decomposition matrix for the block B_5 is $D_{23,7}{}^{5}$ (as in appendix 2).

Proof:

By using $(r, \bar{r})$-inducing of p.i.s. for S_{22} to S_{23} we get:

$D_8 \uparrow^{(3,5)} S_{23} = c_1,\ D_{11} \uparrow^{(3,5)} S_{23} = c_2,\ D_{15} \uparrow^{(3,5)} S_{23} = c_3,\ D_{48} \uparrow^{(1,0)} S_{23} = c_4,\ D_{25} \uparrow^{(3,5)} S_{23} = c_5,$ $D_{33} \uparrow^{(3,5)} S_{23} = c_6, D_{35} \uparrow^{(3,5)} S_{23} = c_7, D_{37} \uparrow^{(3,5)} S_{23} = c_8,\ D_{43} \uparrow^{(3,5)} S_{23} = c_9.$

Now, on $(7, \alpha)$-regular classes we have:

1) $\langle 14,5,3,1\rangle = \langle 14,5,3,1\rangle'$.
2) $\langle 12,7,3,1\rangle = \langle 12,7,3,1\rangle'$.
3) $\langle 10,7,5,1\rangle = \langle 10,7,5,1\rangle'$.
4) $\langle 8,7,5,3\rangle = \langle 8,7,5,3\rangle'$.
5) $\langle 10,5,4,3,1\rangle^{*} = \langle 10,7,5,1\rangle + \langle 17,5,1\rangle^{*} - \langle 10,8,5\rangle^{*} - \langle 12,10,1\rangle^{*}$.
6) $\langle 8,6,5,31\rangle^{*} = \langle 8,7,5,3\rangle + \langle 10,8,5\rangle - \langle 12,8,3\rangle^{*} + \langle 15,5,3\rangle + \langle 19,3.1\rangle$.
7) $\langle 14,5,3,1\rangle = \langle 12,7,3,1\rangle - \langle 10,7,5,1\rangle + \langle 8,7,5,3\rangle$.
8) $\langle 8,7,5,3\rangle = \langle 10,7,5,1\rangle - \langle 12,7,3,1\rangle + \langle 14,5,3,1\rangle$.

Approximation matrix contains at most 9 columns since there are 8 equations corresponding to the spin characters of S_{23} in B_5 [A.K.Yaseen 1987], and Since $c_1, \dots, c_9$ are linearly independent, $c_i - c_j$ is not p.s to S_{23} for all $1 \le i < j \le 9$ then we get the decomposition matrix for B_5 ■.

***Lemma*(3. 2)**:

The decomposition matrix for the block B_4 is $D_{23,7}{}^{4}$ (as in appendix 2).

Proof:

By using $(r, \bar{r})$-inducing of p.i.s. for S_{22} to S_{23} we get:

$D_{45}\uparrow^{(4,4)} S_{23}=k_1,\quad D_{46}\uparrow^{(4,4)} S_{23}=k_2,\quad D_{84}\uparrow^{(1,0)} S_{23}=c_5,\quad D_{85}\uparrow^{(1,0)} S_{23}=c_6,$
$D_{48}\uparrow^{(4,4)} S_{23}=k_3,\quad D_{49}\uparrow^{(4,4)} S_{23}=k_4,\quad D_{86}\uparrow^{(1,0)} S_{23}=c_{11},\quad D_{87}\uparrow^{(1,0)} S_{23}=c_{12},$
$D_{51}\uparrow^{(4,4)} S_{23}=k_5,\quad D_{88}\uparrow^{(1,0)} S_{23}=c_{15},\quad D_{89}\uparrow^{(1,0)} S_{23}=c_{16},\quad D_{53}\uparrow^{(4,4)} S_{23}=k_6.$

Table(2)

	Ψ_1	Ψ_2	φ_5	φ_6	Ψ_3	Ψ_4	φ_{11}	φ_{12}	Ψ_5	φ_{15}	φ_{16}	Ψ_6	φ_1	φ_2
$\langle 19,4\rangle$	1												a	
$\langle 19,4\rangle'$	1													a
$\langle 18,5\rangle$	1	1											b	
$\langle 18,5\rangle'$	1	1												b
$\langle 14,5,4\rangle^*$		2	1	1									c	c
$\langle 13,5,4,1\rangle$			1		1								d	
$\langle 13,5,4,1\rangle'$				1	1									d
$\langle 12,11\rangle$		1				1								
$\langle 12,11\rangle'$		1				1								
$\langle 12,7,4\rangle^*$	2	2	1	1		2	1	1					f	f
$\langle 12,6,4,1\rangle$			1		1		1		1				g	
$\langle 12,6,4,1\rangle'$				1	1			1	1					g
$\langle 12,5,4,2\rangle$					1				1				h	
$\langle 12,5,4,2\rangle'$					1				1					h
$\langle 11,7,5\rangle^*$	2					2	1	1		1	1		i	i
$\langle 11,6,5,1\rangle$						2	1		1	1		1	j	
$\langle 11,6,5,1\rangle'$						2		1	1		1	1		j
$\langle 11,5,4,3\rangle$									1			1	m	
$\langle 11,5,4,3\rangle'$									1			1		m
$\langle 8,6,5,4\rangle$						2				1	1	1	n	
$\langle 8,6,5,4\rangle'$						2				1	1	1		n
$\langle 7,6,5,4,1\rangle^*$										1	1		z	z
	k_1	k_2	c_5	c_6	k_3	k_4	c_{11}	c_{12}	k_5	c_{15}	c_{16}	k_6	Y_1	Y_2

On $(7,\alpha)$-regular classes we have:

1) $\langle 12,7,4\rangle^*=\langle 19,4\rangle+\langle 19,4\rangle'+\langle 12,6,4,1\rangle+\langle 12,6,4,1\rangle'+\langle 12,11\rangle+\langle 12,11\rangle'-\langle 12,5,4,2\rangle-\langle 12,5,4,2\rangle'$.
2) $\langle 11,7,5\rangle^*=\langle 18,5\rangle+\langle 18,5\rangle'+\langle 11,6,5,1\rangle+\langle 11,6,5,1\rangle'-\langle 11,5,4,3\rangle-\langle 11,5,4,3\rangle'-\langle 12,11\rangle-\langle 12,11\rangle'$.
3) $\langle 14,5,4\rangle^*=\langle 12,7,4\rangle^*+\langle 7,6,5,4,1\rangle^*-\langle 11,7,5\rangle^*$.
4) $\langle 7,6,5,4,1\rangle^*=\langle 11,6,5,1\rangle+\langle 11,6,5,1\rangle'+\langle 18,5\rangle+\langle 18,5\rangle'+\langle 14,5,4\rangle^*-\langle 12,7,4\rangle^*-\langle 11,5,4,3\rangle-\langle 11,5,4,3\rangle'-\langle 12,11\rangle-\langle 12,11\rangle'$.

So, there are 18 columns to the spin characters of S_{23} in B_4.

Since $\langle 19,4\rangle\neq\langle 19,4\rangle'$ on $(7,\alpha)$-regular classes then k_1 is split or there are two columns. Suppose there are two columns such as Y_1 and Y_2(table (2)).To describe columns Y_1 and Y_2 :

1. $\langle 19,4\rangle\downarrow S_{22}=(\langle 18,4\rangle^*)^1+(\langle 19,3\rangle^*)^1$ has 2 of i.m.s.(see appendix 1) so we have $a\in\{0,1\}$.

2. $\langle 18,5\rangle \downarrow S_{22} = (\langle 17,5\rangle^*)^2 + (\langle 18,4\rangle^*)^1$ has 3 of i.m.s. so we have $b \in \{0,1\}$.
3. $\langle 14,5,4\rangle^* \downarrow S_{22} = (\langle 13,5,4\rangle)^1 + (\langle 13,5,4\rangle')^1 + (\langle 14,5,3\rangle)^2 + (\langle 14,5,3\rangle')^2$ has 6 of i.m.s. we have $c \in \{0,1\}$, if $c = 2$ so we have a contradiction.
4. $\langle 13,5,4,1\rangle \downarrow S_{22} = (\langle 12,5,4,1\rangle^*)^1 + (\langle 13,5,3,1\rangle^*)^2 + (\langle 13,5,4\rangle)^1$ has 4 of i.m.s. so we have $d \in \{0,1,2\}$.
5. $\langle 12,7,4\rangle^* \downarrow S_{22} = (\langle 11,7,4\rangle)^2 + (\langle 11,7,4\rangle')^2 + (\langle 12,6,4\rangle)^2 + (\langle 12,6,4\rangle')^2 + (\langle 12,7,3\rangle)^5 + (\langle 12,7,3\rangle')^5$ has 18 of i.m.s. so we have $f \in \{0,1,\ldots,4\}$.
6. $\langle 12,6,4,1\rangle \downarrow S_{22} = (\langle 11,6,4,1\rangle^*)^2 + (\langle 12,5,4,1\rangle^*)^1 + (\langle 12,6,3,1\rangle^*)^4 + (\langle 12,6,4\rangle)^2$ has 9 of i.m.s. so we have $g \in \{0,1,\ldots,5\}$.
7. $\langle 12,5,4,2\rangle \downarrow S_{22} = (\langle 11,5,4,2\rangle^*)^1 + (\langle 12,5,3,2\rangle^*)^2 + (\langle 12,5,4,1\rangle^*)^1$ has 4 of i.m.s. so we have $h \in \{0,1,2\}$.
8. $\langle 11,7,5\rangle^* \downarrow S_{22} = (\langle 10,7,5\rangle)^4 + (\langle 10,7,5\rangle')^4 + (\langle 11,6,5\rangle)^2 + (\langle 11,6,5\rangle')^2 + (\langle 11,7,4\rangle)^2 + (\langle 11,7,4\rangle')^2$ has 16 of i.m.s. so we have $i \in \{0,1,\ldots,4\}$.
9. $\langle 11,6,5,1\rangle \downarrow S_{22} = (\langle 10,6,5,1\rangle^*)^6 + (\langle 11,6,4,1\rangle^*)^2 + (\langle 11,6,5\rangle)^2$ has 10 of i.m.s. so we have $j \in \{0,1,\ldots,4\}$.
10. $\langle 11,5,4,3\rangle \downarrow S_{22} = (\langle 10,5,4,3\rangle^*)^2 + (\langle 11,5,4,2\rangle^*)^1$ has 3 of i.m.s. so we have $m \in \{0,1\}$.
11. $\langle 8,6,5,4\rangle \downarrow S_{22} = (\langle 7,6,5,4\rangle^*)^2 + (\langle 8,6,5,3\rangle^*)^5$ has 7 of i.m.s. so we have $n \in \{0,1,2\}$.
12. $\langle 7,6,5,4,1\rangle^* \downarrow S_{22} = (\langle 7,6,5,3,1\rangle)^1 + (\langle 7,6,5,3,1\rangle')^1 + (\langle 7,6,5,4\rangle^*)^2$ has 4 of i.m.s. so we have $z \in \{0,1\}$.

Take $a = 1$, since the restriction of the following intersections:

$\langle 19,4\rangle \downarrow S_{22} \cap \langle 14,5,4\rangle^* \downarrow S_{22}$, $\langle 19,4\rangle \downarrow S_{22} \cap \langle 13,5,4,1\rangle \downarrow S_{22}$, $\langle 19,4\rangle \downarrow S_{22} \cap \langle 12,6,4,1\rangle \downarrow S_{22}$, $\langle 19,4\rangle \downarrow S_{22} \cap \langle 12,5,4,2\rangle \downarrow S_{22}$, $\langle 19,4\rangle \downarrow S_{22} \cap \langle 11,6,5,1\rangle \downarrow S_{22}$, $\langle 19,4\rangle \downarrow S_{22} \cap \langle 11,5,4,3\rangle \downarrow S_{22}$,

$\langle 19,4\rangle \downarrow S_{22} \cap \langle 8,6,5,4\rangle \downarrow S_{22}$ and $\langle 19,4\rangle \downarrow S_{22} \cap \langle 7,6,5,4,1\rangle \downarrow S_{22}$, has no i.m.s in the intersections, so we have $c = d = g = h = j = m = n = z = 0$.

- $\langle 19,4\rangle \downarrow S_{22} \cap \langle 18,5\rangle \downarrow S_{22}$ has 2 of i.m.s for S_{22}
  $\therefore \langle 19,4\rangle \cap \langle 18,5\rangle = \Psi_1 + \varphi_1$ if $b = 1$,
  $= \Psi_1$ if $b = 0$.
- $\langle 19,4\rangle \downarrow S_{22} \cap \langle 12,7,4\rangle^* \downarrow S_{22}$ has 2 of i.m.s for S_{22}
  $\therefore \langle 19,4\rangle \cap \langle 12,7,4\rangle^* = \Psi_1 + \varphi_1$ if $f \in \{1,2,3,4\}$,
  $= \Psi_1$ if $f = 0$.
- $\langle 19,4\rangle \downarrow S_{22} \cap \langle 11,7,5\rangle^* \downarrow S_{22}$ has 2 of i.m.s for S_{22}
  $\therefore \langle 19,4\rangle \cap \langle 11,7,5\rangle^* = \Psi_1 + \varphi_1$ if $i \in \{1,2,3,4\}$,
  $= \Psi_1$ if $i = 0$.

Then, we have:

$Y_1 = \langle 19,4\rangle + b\,\langle 18,5\rangle + f\langle 12,7,4\rangle^* + i < 11,7,5 >^*$;$Y_2 = \langle 19,4\rangle' + b\langle 18,5\rangle' + f < 12,7,4 >^* + i < 11,7,5 >^*$; such that $b \in \{0,1\}, f \in \{0,1,2,3,4\}, i \in \{0,1,2,3,4\}$.

Since inducing m.s. is m.s. [J. F. Humphreys 1977], so we have

- $(\langle 17,5\rangle^* - \langle 19,3\rangle^*) \uparrow^{(4,4)} S_{23} = \langle 18,5\rangle + \langle 18,5\rangle' - \langle 19,4\rangle - \langle 19,4\rangle'$,
  $\therefore b \geq a \;\Rightarrow b = a = 1$ **…………………(3.1).**
- $(\langle 12,7,3\rangle - \langle 19,3\rangle^*) \uparrow^{(4,4)} S_{23} = \langle 12,7\rangle - \langle 19,4\rangle - \langle 19,4\rangle'$,
  $\therefore f \geq a$ **……………………………(3.2).**
- $(\langle 19,3\rangle^* - \langle 12,7,3\rangle + \langle 12,10\rangle^* + \langle 12,6,3,1\rangle^*) \uparrow^{(4,4)} S_{23} = \langle 19,4\rangle + \langle 19,4\rangle' - \langle 12,7,4\rangle^* + \langle 12,11\rangle + \langle 12,11\rangle' + \langle 12,6,4\rangle + \langle 12,6,4\rangle'$,
  $\therefore a \geq f$ **………………………………(3.3).**
  $\Rightarrow f = a = 1$ **………………… (3.4)** (from**(3.2)&(3.3)**).
- $(\langle 19,3\rangle^* - \langle 10,7,5\rangle + \langle 10,6,5,1\rangle^*) \uparrow^{(4,4)} S_{23} = \langle 19,4\rangle + \langle 19,4\rangle' - \langle 11,7,5\rangle^* + \langle 11,6,5,1\rangle + \langle 11,6,5,1\rangle'$,
  $\therefore a \geq i \Rightarrow i = a = 1$ **……………………(3.5),**

From **(3.1)**, **(3.4),** and **(3.5)** we get on $a = b = f = i = 1$ so k_1 splits.

Since $\langle 18,5\rangle \neq \langle 18,5\rangle'$ on $(7,\alpha)$- regular classes then either k_2 is split or there are two columns, we take $b = 1$ $(a = 0)$ and since:

$\langle 18,5\rangle \downarrow S_{22} \cap \langle 13,5,4,1\rangle \downarrow S_{22}$, $\langle 18,5\rangle \downarrow S_{22} \cap \langle 12,6,4,1\rangle \downarrow S_{22}$, $\langle 18,5\rangle \downarrow S_{22} \cap \langle 12,5,4,2\rangle \downarrow S_{22}$, $\langle 18,5\rangle \downarrow S_{22} \cap \langle 11,6,5,1\rangle \downarrow S_{22}$, $\langle 18,5\rangle \downarrow S_{22} \cap \langle 11,5,4,3\rangle \downarrow S_{22}$, $\langle 18,5\rangle \downarrow S_{22} \cap \langle 8,6,5,4\rangle \downarrow S_{22}$

and $\langle 18,5\rangle \downarrow S_{22} \cap \langle 7,6,5,4,1\rangle^* \downarrow S_{22}$, has no i.m.s in the intersections, so we have $d = g = h = j = m = n = z = 0$, then we have:

$Y_1 = \langle 18,5\rangle + c\langle 14,5,4\rangle^* + f < 12,7,4 >^* + i < 11,7,5 >^*; Y_2 = \langle 18,4\rangle' + c\langle 14,5,4\rangle^* + f < 12,7,4 >^* + i < 11,7,5 >^*$; such that $c \in \{0,1\}, f \in \{0,1,2,3,4\}$, and $i \in \{0,1,2,3,4\}$,

And same discussion, we have on $b = c = f = 1$, so we have:

$Y_1 = \langle 18,4\rangle + \langle 14,5,4\rangle^* + \langle 12,7,4\rangle^* + i\langle 11,7,5\rangle^*, Y_2 = \langle 18,4\rangle' + \langle 14,5,4\rangle^* + \langle 12,7,4\rangle^* i\langle 11,7,5\rangle^*$ which is not p.s. since $\deg Y_1 \not\equiv 0 \bmod 7^3$ and $\deg Y_2 \not\equiv 0 \bmod 7^3$, $\forall i \in \{0,1,\ldots,4\}$, so $\boldsymbol{k_2}$ splits.

Since $\langle 12,11\rangle \neq \langle 12,11\rangle'$ on $(7,\alpha)$- regular classes and since

$\langle 12,11\rangle \downarrow S_{22} = (\langle 12,10\rangle^*)^2$ and from table(2) then k_4 must splits .

Since $\langle 12,5,4,2\rangle \neq \langle 12,5,4,2\rangle'$ on $(7,\alpha)$- regular classes then k_3 or k_5 splits or there are another two columns.

Suppose there are other two columns Y_1, Y_2 see table (1).

Let $h \in \{1,2\}$ and same discussion we have on $h = d = g$, then k_3 is splits.

the second probability leads to the first probability then either k_3 or k_5 splits suppose k_5 splits, since $\langle 11,5,4,3\rangle \neq \langle 11,5,4,3\rangle'$ on $(7,\alpha)$- regular classes then k_6 splits or there are other two columns but $\langle 8,6,5,4\rangle \neq \langle 8,6,5,4\rangle'$ on $(7,\alpha)$- regular classes then either k_6 splits or there are other two columns these in two cases we get contradiction.

If k_6 splits and $\langle 8,6,5,4\rangle \neq \langle 8,6,5,4\rangle'$ on $(7,\alpha)$- regular classes then we must find another two columns so we have contradiction, then k_3 must is splits.

Since $\langle 11,5,4,3\rangle \neq \langle 11,5,4,3\rangle'$ on $(7,\alpha)$- regular classes then k_5 or k_6 splits or there are other two columns.

Suppose there are two columns. Let m=1 and same discussion we have on $n=j=m$. then k_6 splits.

Since $\langle 8,6,5,4\rangle \neq \langle 8,6,5,4\rangle'$ on $(7,\alpha)$- regular classes then so k_5 must splits so we get the decomposition matrix for B_4 ■.

Lemma(3.3):

Decomposition matrix for the block B_3 is $D_{23,7}{}^3$ (as in appendix 2).

Proof:

By using $(r,\bar{r})$-inducing of p.i.s. for S_{22} to S_{23} we get:

$D_{45}\uparrow^{(6,2)} S_{23}=k_1$, $D_{46}\uparrow^{(6,2)} S_{23}=k_2$, $D_{90}\uparrow^{(1,0)} S_{23}=c_5$, $D_{91}\uparrow^{(1,0)} S_{23}=c_6$, $D_{49}\uparrow^{(6,2)} S_{23}=k_3$, $D_{50}\uparrow^{(6,2)} S_{23}=k_4$, $D_{59}\uparrow^{(5,3)} S_{23}=k_5$, $D_{60}\uparrow^{(5,3)} S_{23}=k_6$, $D_{52}\uparrow^{(6,2)} S_{23}=k_7$, $D_{53}\uparrow^{(6,2)} S_{23}=k_8$.

Table(3)

	Ψ_1	Ψ_2	φ_5	φ_6	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	φ_1	φ_2
$\langle 20,3\rangle$	1										a	
$\langle 20,3\rangle'$	1											a
$\langle 17,6\rangle$	1	1									b	
$\langle 17,6\rangle'$	1	1										b
$\langle 14,6,3\rangle^*$		2	1	1							c	c
$\langle 13,10\rangle$		1			1						d	
$\langle 13,10\rangle'$		1			1							d
$\langle 13,7,3\rangle^*$	2	2	1	1	2	2					e	e
$\langle 13,6,3,1\rangle$			1			1	1				f	
$\langle 13,6,3,1\rangle'$				1		1	1					f
$\langle 13,5,3,2\rangle$							1				g	
$\langle 13,5,3,2\rangle'$							1					g
$\langle 12,6,3,2\rangle$						1	1	1			h	
$\langle 12,6,3,2\rangle'$						1	1	1				h
$\langle 10,7,6\rangle^*$	2				2	2			2		i	i
$\langle 10,6,5,2\rangle$					2	1		1	1	1	j	
$\langle 10,6,5,2\rangle'$					2	1		1	1	1		j
$\langle 10,6,4,3\rangle$								1		1	m	
$\langle 10,6,4,3\rangle'$								1		1		m
$\langle 9,6,5,3\rangle$					2				2	1	n	
$\langle 9,6,5,3\rangle'$					2				2	1		n
$\langle 7,6,5,3,2\rangle^*$									2		z	z
	k_1	k_2	c_5	c_6	k_3	k_4	k_5	k_6	k_7	k_8	Y_1	Y_2

Now, on $(7,\alpha)$-regular classes we have:

1) $\langle 7,6,5,3,2\rangle^* = \langle 10,7,6\rangle^* - \langle 13,7,3\rangle^* + \langle 14,6,3\rangle^*$;
2) $\langle 10,7,6\rangle^* = \langle 10,6,5,2\rangle + \langle 10,6,5,2\rangle' - \langle 10,6,4,3\rangle - \langle 10,6,4,3\rangle' - \langle 13,10\rangle - \langle 13,10\rangle' + \langle 17,6\rangle + \langle 17,6\rangle'$;
3) $\langle 13,7,3\rangle^* = \langle 13,6,3\rangle + \langle 13,6,3\rangle' + \langle 13,10\rangle + \langle 13,10\rangle' + \langle 20,3\rangle + \langle 20,3\rangle' - \langle 13,5,3,2\rangle - \langle 13,5,3,2\rangle'$ and,
4) $\langle 14,6,3\rangle^* = \langle 13,10\rangle + \langle 13,10\rangle' + \langle 13,6,3,1\rangle + \langle 13,6,3,1\rangle' - \langle 13,5,3,2\rangle - \langle 13,5,3,2\rangle' - \langle 10,7,4\rangle^* + \langle 20,3\rangle + \langle 20,3\rangle'$.

So there are 18 columns to the spin characters of S_{23} in B_3.

Since $\langle 20,3\rangle \neq \langle 20,3\rangle'$ on $(7,\alpha)$-regular classes then k_1 is split or there are two columns. Suppose there are two columns such as Y_1 and Y_2(Table (3)). To describe columns Y_1 and Y_2 :

1. $\langle 20,3\rangle \downarrow S_{22} = (\langle 19,3\rangle^*)^1 + (\langle 20,2\rangle^*)^1$ has 2 of i.m.s.(see appendix 1) so we have $a \in \{0,1\}$.
2. $\langle 17,6\rangle \downarrow S_{22} = (\langle 16,6\rangle^*)^2 + (\langle 17,5\rangle^*)^2$ has 4 of i.m.s. so we have $b \in \{0,1,2\}$.
3. $\langle 14,6,3\rangle^* \downarrow S_{22} = (\langle 13,6,3\rangle)^1 + (\langle 13,6,3\rangle')^1 + (\langle 14,5,3\rangle)^2 + (\langle 14,5,3\rangle')^2 + (\langle 14,6,2\rangle)^2 + (\langle 14,6,2\rangle')^2$ has 10 of i.m.s. we have $c \in \{0,1,2,3\}$.
4. $\langle 13,10\rangle \downarrow S_{22} = (\langle 12,10\rangle^*)^2 + (\langle 13,9\rangle^*)^2$ has 4 of i.m.s. so we have $d \in \{0,1,2\}$.
5. $\langle 13,7,3\rangle^* \downarrow S_{22} = (\langle 12,7,3\rangle)^5 + (\langle 12,7,3\rangle')^5 + (\langle 13,6,3\rangle)^1 + (\langle 13,6,3\rangle')^1 + (\langle 13,7,2\rangle)^5 + (\langle 13,7,2\rangle')^5$ has 22 of i.m.s. so we have $e \in \{0,1,\ldots,6\}$.
6. $\langle 13,6,3,1\rangle \downarrow S_{22} = (\langle 12,6,3,1\rangle^*)^4 + (\langle 13,5,3,1\rangle^*)^2 + (\langle 13,6,2,1\rangle^*)^3 + (\langle 13,6,3\rangle)^1$ has 10 of i.m.s. so we have $f \in \{0,1,\ldots,7\}$.
7. $\langle 13,5,3,2\rangle \downarrow S_{22} = (\langle 12,5,3,2\rangle^*)^2 + (\langle 13,4,3,2\rangle^*)^1 + (\langle 13,5,3,1\rangle^*)^2$ has 5 of i.m.s. so we have $g \in \{0,1,2,3,4\}$.
8. $\langle 12,6,3,2\rangle \downarrow S_{22} = (\langle 11,6,3,2\rangle^*)^3 + (\langle 12,5,3,2\rangle^*)^2 + (\langle 12,6,3,1\rangle)^4$ has 9 of i.m.s. so we have $h \in \{0,1,\ldots,6\}$.
9. $\langle 10,7,6\rangle^* \downarrow S_{22} = (\langle 9,7,6\rangle)^4 + (\langle 9,7,6\rangle')^4 + (\langle 10,7,5\rangle)^4 + (\langle 10,7,5\rangle')^4$ has 16 of i.m.s. so we have $i \in \{0,1,2,\ldots,4\}$.
10. $\langle 10,6,5,2\rangle \downarrow S_{22} = (\langle 9,6,5,2\rangle^*)^6 + (\langle 10,6,4,2\rangle^*)^3 + (\langle 10,6,5,1\rangle^*)^6$ has 15 of i.m.s. so we have $j \in \{0,1,2,\ldots,9\}$.
11. $\langle 10,6,4,3\rangle \downarrow S_{22} = (\langle 9,6,4,3\rangle^*)^2 + (\langle 10,5,4,3\rangle^*)^2 + (\langle 10,6,4,2\rangle^*)^3$ has 7 of i.m.s. so we have $m \in \{0,1,2,\ldots,5\}$.
12. $\langle 9,6,5,3\rangle \downarrow S_{22} = (\langle 8,6,5,3\rangle^*)^5 + (\langle 9,6,4,3\rangle^*)^2 + (\langle 9,6,5,2\rangle^*)^6$ has 13 of i.m.s. so we have $n \in \{0,1,2,\ldots,8\}$.
13. $\langle 7,6,5,3,2\rangle^* \downarrow S_{22} = (\langle 7,6,4,3,2\rangle)^1 + (\langle 7,6,4,3,2\rangle')^1 + (\langle 7,6,5,3,1\rangle)^1 + (\langle 7,6,5,3,1\rangle')^1$ has 4 of i.m.s. so we have $z \in \{0,1\}$.

Take $a = 1$, and since :

$\langle 20,3\rangle \downarrow S_{22} \cap \langle 14,6,3\rangle^* \downarrow S_{22}$, $\langle 20,3\rangle \downarrow S_{22} \cap \langle 13,10\rangle \downarrow S_{22}$, $\langle 20,3\rangle \downarrow S_{22} \cap \langle 13,6,3,1\rangle \downarrow S_{22}$, $\langle 20,3\rangle \downarrow S_{22} \cap \langle 13,5,3,2\rangle \downarrow S_{22}$, $\langle 20,3\rangle \downarrow S_{22} \cap \langle 12,6,3,2\rangle \downarrow S_{22}$,$\langle 20,3\rangle \downarrow S_{22} \cap \langle 10,6,5,2\rangle \downarrow S_{22}$,

$\langle 20,3\rangle \downarrow S_{22} \cap \langle 10,6,4,3\rangle \downarrow S_{22}$, $\langle 20,3\rangle \downarrow S_{22} \cap \langle 9,6,5,3\rangle \downarrow S_{22}$ and, $\langle 20,3\rangle \downarrow S_{22} \cap \langle 7,6,5,3,2\rangle^* \downarrow S_{22}$, has no i.m.s in the intersections, so we have $c = d = f = g = h = j = m = n = z = 0$, then have:

$Y_1 = \langle 20,3\rangle + b\,\langle 17,6\rangle + e\langle 13,7,3\rangle^* + i < 10,7,6 >^*$;

$Y_2 = \langle 20,3\rangle' + b\langle 17,6\rangle' + e < 13,7,3 >^* + i < 10,7,6 >^*$;

such that $b \in \{0,1,2\}, e \in \{0,1,2,\dots,6\}, i \in \{0,1,2,3,4\}$,

and same discussion we have on $a = b = e = i = 1$, so k_1 splits .

Since $\langle 17,6\rangle \neq \langle 17,6\rangle'$ on $(7,\alpha)$- regular classes then either k_2 is split or there are two columns, we take $b \in \{1,2\}$ and Since:

$\langle 17,6\rangle \downarrow S_{22} \cap \langle 13,6,3,1\rangle \downarrow S_{22}$, $\langle 17,6\rangle \downarrow S_{22} \cap \langle 13,5,3,2\rangle \downarrow S_{22}$, $\langle 17,6\rangle \downarrow S_{22} \cap \langle 12,6,3,2\rangle \downarrow S_{22}$, $\langle 17,6\rangle \downarrow S_{22} \cap \langle 10,6,5,2\rangle \downarrow S_{22}$, $\langle 17,6\rangle \downarrow S_{22} \cap \langle 10,6,4,3\rangle \downarrow S_{22}$, $\langle 17,6\rangle \downarrow S_{22} \cap \langle 9,6,5,3\rangle \downarrow S_{22}$

and $\langle 17,6\rangle \downarrow S_{22} \cap \langle 7,6,5,3,2\rangle^* \downarrow S_{22}$, has no i.m.s in the intersections, so we have $f = g = h = j = m = n = z = 0$, so we have:

$Y_1 = b\langle 17,6\rangle + c\langle 14,6,3\rangle^* + d\langle 13,10\rangle + e\langle 13,7,3\rangle^* + i < 10,7,6 >^*$;

$Y_2 = b\langle 17,6\rangle' + c\langle 14,6,3\rangle^* + d\langle 13,10\rangle' + e\langle 13,7,3\rangle^* + i < 10,7,6 >^*$;

such that $b \in \{1,2\}, c \in \{0,1,2,3\}, d \in \{0,1,2\}, e \in \{0,1,\dots,6\}$ *and* $i \in \{0,1,\dots,4\}$,

and same discussion we have on $b = c = d = e$ and $i \in \{0,1,2\}$.

But $\deg Y_1 \equiv 0 \bmod 7^3$ and $\deg Y_2 \equiv 0 \bmod 7^3$ only when $c = d = e = b$ and $i = 0$, so k_2 splits.

Since $\langle 13,10\rangle \neq \langle 13,10\rangle'$ on $(7,\alpha)$- regular classes then either k_3 is split or there are two columns, we take $d \in \{1,2\}$ and since:

$\langle 13,10\rangle \downarrow S_{22} \cap \langle 13,6,3,1\rangle \downarrow S_{22}$, $\langle 13,10\rangle \downarrow S_{22} \cap \langle 13,5,3,2\rangle \downarrow S_{22}$, $\langle 13,10\rangle \downarrow S_{22} \cap \langle 12,6,3,2\rangle \downarrow S_{22}$, $\langle 13,10\rangle \downarrow S_{22} \cap \langle 10,6,4,3\rangle \downarrow S_{22}$, and $\langle 13,10\rangle \downarrow S_{22} \cap \langle 7,6,5,3,2\rangle^* \downarrow S_{22}$,

Has no i.m.s in the intersections so we have $f = g = h = m = z = 0$.

So we have:

$Y_1 = c\langle 14,6,3\rangle^* + d\langle 13,10\rangle + e\langle 13,7,3\rangle^* + i < 10,7,6 >^* + j\langle 10,6,5,2\rangle + n\langle 9,6,5,3\rangle$;

$Y_2 = c\langle 14,6,3\rangle^* + d\langle 13,10\rangle' + e\langle 13,7,3\rangle^* + i < 10,7,6 >^* + j\langle 10,6,5,2\rangle' + n\langle 9,6,5,3\rangle'$;

such that $c \in \{0,1,2,3\}, d \in \{1,2\}, e \in \{0,1,\dots,6\}, i \in \{0,1,\dots,4\}, j \in \{0,1,\dots,9\}$ *and* $i \in \{0,1,\dots,4\}$.

and same discussion we have on $d = e = i$, $c \in \{0,1,2\}$, $j \in \{2,4\}$, $n = j$.

But the $\deg Y_1 \equiv 0 \bmod 7^3$ and $\deg Y_2 \equiv 0 \bmod 7^3$ only when $c = 0$, $d = e = i = 1$ and $i = n = 2$, or $c = 0$, $d = e = i = 2$ and $i = n = 4$ so k_3 splits.

Since $\langle 13,5,3,2\rangle \neq \langle 13,5,3,2\rangle'$ on $(7,\alpha)$- regular classes then k_5 splits or there are other two columns.

Suppose there are two columns, we take $g \in \{1,2,3,4\}$ and same discussion we have on $g = h = f$ then k_5 splits.

Since $\langle 13,6,3,1\rangle \neq \langle 13,6,3,1\rangle'$ on $(7,\alpha)$- regular classes then k_4 splits or there are other two columns.

Suppose there are two columns, and we take $f \in \{1,2,\ldots,7\}$ and same discussion we have on

$e = h = i = j = f$, then k_4 splits.

Since $\langle 12,6,3,2\rangle \neq \langle 12,6,3,2\rangle'$ on $(7,\alpha)$- regular classes then k_6 splits or there are other two columns.

Suppose there are two columns, and we take $h \in \{1,2,\ldots,6\}$, and we get deg $Y_1 \not\equiv 0 \bmod 7^3$ and deg $Y_2 \not\equiv 0 \bmod 7^3$ when $i \neq 0$ and same discussion we have on $m = j = h$, then k_6 splits.

Since $\langle 10,6,4,3\rangle \neq \langle 10,6,4,3\rangle'$ on $(7,\alpha)$- regular classes then k_8 splits or there are other two columns.

Suppose there are two columns,

$Y_1 = j\langle 10,6,5,2\rangle + +m\langle 10,6,4,3\rangle + n\langle 9,6,5,3\rangle;$

$Y_2 = j\langle 10,6,5,2\rangle' + m\langle 10,6,4,3\rangle' + n\langle 9,6,5,3\rangle';$

such that $j \in \{0,1,\ldots,9\}, m \in \{1,2,\ldots,5\}$, and $n \in \{0,1,\ldots,8\}$.

Let $m \in \{1,2,\ldots,5\}$, and since:

- $(\langle 9,6,5,2\rangle^* - \langle 9,6,4,3\rangle^* + \langle 7,6,4,3,2\rangle)\uparrow^{(5,3)} S_{23} = \langle 10,6,5,2\rangle + \langle 10,6,5,2\rangle' + \langle 9,6,5,3\rangle + \langle 9,6,5,3\rangle' - \langle 10,6,4,3\rangle - \langle 10,6,4,3\rangle' - \langle 9,6,5,3\rangle - \langle 9,6,5,3\rangle' + \langle 7,6,5,3,2\rangle^*$,
  $\therefore j \geq m$ **……………………………(3.6).**
- $(\langle 9,6,4,3\rangle^* + \langle 7,6,4,3,2\rangle - \langle 9,6,5,2\rangle^* + \langle 13,7,2\rangle + \langle 13,9\rangle^*)\uparrow^{(5,3)} S_{23} = \langle 10,6,4,3\rangle + \langle 10,6,4,3\rangle' + \langle 9,6,5,3\rangle + \langle 9,6,5,3\rangle' + \langle 7,6,5,3,2\rangle' - \langle 10,6,5,2\rangle - \langle 10,6,5,2\rangle' - \langle 9,6,5,3\rangle - \langle 9,6,5,3\rangle' + \langle 13,7,3\rangle^* + \langle 13,10\rangle + \langle 13,10\rangle'$,
  $\therefore m \geq j$ **……………………………..(3.7).**
  $\Rightarrow m = j$ **…………………… (3.8)** (from**(3.6)&(3.7)**).

Then we have deg $Y_1 \equiv 0 \bmod 7^3$ and deg $Y_2 \equiv 0 \bmod 7^3$ only when $j = m = n$ so k_8 splits.

Since $\langle 9,6,5,3\rangle \neq \langle 9,6,5,3\rangle'$ on $(7,\alpha)$- regular classes then so k_7 must splits so we get the decomposition matrix for B_3 ■

Lemma (3.4):

Decomposition matrix for the block B_2 is ${D_{23,7}}^2$ (see appendix 2).

Proof:

By using (1,0)-inducing of p.i.s. for S_{22} to S_{23} we get:

$D_3 \uparrow^{(1,0)} S_{23} = k_1$, $D_{13} \uparrow^{(1,0)} S_{23} = k_2$, $D_{11} \uparrow^{(1,0)} S_{23} = c_5$, $D_{12} \uparrow^{(1,0)} S_{23} = c_6$,
$D_9 \uparrow^{(1,0)} S_{23} = c_7$, $D_{10} \uparrow^{(1,0)} S_{23} = c_8$, $D_{25} \uparrow^{(1,0)} S_{23} = c_9$, $D_{26} \uparrow^{(1,0)} S_{23} = c_{10}$,
$D_5 \uparrow^{(1,0)} S_{23} = c_{11}$, $D_6 \uparrow^{(1,0)} S_{23} = c_{12}$, $D_{27} \uparrow^{(1,0)} S_{23} = k_3$, $D_{28} \uparrow^{(1,0)} S_{23} = k_4$,
$D_7 \uparrow^{(1,0)} S_{23} = c_{13}$, $D_8 \uparrow^{(1,0)} S_{23} = c_{14}$, $D_{39} \uparrow^{(1,0)} S_{23} = k_5$, $D_{43} \uparrow^{(1,0)} S_{23} = c_{17}$,
$D_3 \uparrow^{(1,0)} S_{44} = c_{18}$,

Now we have $k_5 = k_3 + k_4 - c_{11} - c_{12}$ either $k_3 - c_{11}$, $k_4 - c_{12}$ are principal.

Let $c_{15} = k_4 - c_{12}$ and $c_{16} = k_3 - c_{11}$.

Table(4)

$\langle 22,1\rangle$	1															
$\langle 22,1\rangle'$	1															
$\langle 15,8\rangle$	1	2														
$\langle 15,8\rangle'$	1	2														
$\langle 15,7,1\rangle^*$	4	4	1	1												
$\langle 15,5,2,1\rangle$			1		1											
$\langle 15,5,2,1\rangle'$				1		1										
$\langle 15,4,3,1\rangle$					1											
$\langle 15,4,3,1\rangle'$						1										
$\langle 14,8,1\rangle^*$	4	4	1	1			1	1								
$\langle 12,8,2,1\rangle$	2		1		1		1		1							
$\langle 12,8,2,1\rangle'$	2			1		1		1		1						
$\langle 11,8,3,1\rangle$					1				1		1					
$\langle 11,8,3,1\rangle'$						1				1		1				
$\langle 10,8,4,1\rangle$									1		1		1			
$\langle 10,8,4,1\rangle'$										1		1		1		
$\langle 9,8,5,1\rangle$	2						1	1	1				1		1	1
$\langle 9,8,5,1\rangle'$	2						1	1		1				1	1	1
$\langle 8,7,5,2,1\rangle^*$							1	1					1	1	1	1
$\langle 8,7,4,3,2,1\rangle^*$													1	1	1	1
$\langle 8,5,4,3,2,1\rangle$															1	
$\langle 8,5,4,3,2,1\rangle$																1
	k_1	k_2	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}

Now $k_1$is split to $c_1$and c_2 [A.O.Morris and A.K.Yassen 1988].

Since $\langle 15,8,1\rangle$ and $\langle 15,8,1\rangle'$ are projective indecomposable spin characters of S_{24}(of defect 0 in S_{24} ,$p = 7$) and :

$\langle 15,8,1\rangle \downarrow S_3 = \langle 15,8\rangle + \langle 15,7,1\rangle^* + \langle 14,8,1\rangle^*$,

$\langle 15,8,1\rangle' \downarrow S_3 = \langle 15,8\rangle' + \langle 15,7,1\rangle^* + \langle 14,8,1\rangle^*$.

and since $(\frac{1}{2})k_2$ is a principal character of S_{23} [G.D.James and A.Kerber 1981] then $(\frac{1}{2})k_2$ must split to c_3 and c_4 .

Case : $c_3 \not\subset c_1$:

Suppose c_3 is subtracted from c_1, then;

$(c_1 - c_3) \downarrow_{(1,0)} S_{22} = D_1 + D_{14} - D_{13}$,

is not p.s. for S_{22}(see appendix1) hence: c_3 is not subtracted from c_1. Since c_1, c_2 are associated columns and c_3, c_4 are associated columns, then c_4 is not subtracted from c_2, so we get the decomposition matrix for B_2 ■

Section (4) block of defect three

All i.m.s. of the decomposition matrix for the block B_1 are double we have $\langle\beta\rangle = \langle\beta\rangle'$ on $(7,\alpha)$ −regular classes.

***Theorem* (4 . 1)**:

Decomposition matrix for S_{23} is [appendix 2].

Proof:

We determine all except the block B_1. Now we find the decomposition matrix for the block B_1 By using $(r,\bar{r})$-inducing of p.i.s. of S_{22} to S_{23} we get on:

$D_1 \uparrow^{(2,6)} S_{23} = c_1$, $D_{54} \uparrow^{(1,0)} S_{23} = c_2$, $D_5 \uparrow^{(2,6)} S_{23} = c_3$, $D_7 \uparrow^{(2,6)} S_{23} = c_4$,
$D_{72} \uparrow^{(5,3)} S_{23} = c_5$, $D_{13} \uparrow^{(2,6)} S_{23} = c_6$, $D_{11} \uparrow^{(2,6)} S_{23} = 2c_7$, $D_{17} \uparrow^{(2,6)} S_{23} = c_8$,
$D_{56} \uparrow^{(1,0)} S_{23} = 2c_9$, $D_{57} \uparrow^{(1,0)} S_{23} = c_{10}$, $D_{58} \uparrow^{(1,0)} S_{23} = c_{11}$, $D_{23} \uparrow^{(2,6)} S_{23} = c_{12}$,
$D_{25} \uparrow^{(2,6)} S_{23} = 2c_{13}$, $D_{29} \uparrow^{(2,6)} S_{23} = c_{14}$, $D_{31} \uparrow^{(2,6)} S_{23} = c_{15}$, $D_{74} \uparrow^{(5,3)} S_{23} = c_{16}$,
$D_{33} \uparrow^{(2,6)} S_{23} = c_{17}$, $D_{35} \uparrow^{(2,6)} S_{23} = c_{18}$, $D_{37} \uparrow^{(2,6)} S_{23} = c_{19}$, $D_{61} \uparrow^{(1,0)} S_{23} = c_{20}$,
$D_{62} \uparrow^{(1,0)} S_{23} = c_{21}$, $D_{43} \uparrow^{(2,6)} S_{23} = 2c_{22}$, $D_3 \uparrow^{(2,6)} S_{23} = k_1$, $D_9 \uparrow^{(2,6)} S_{23} = k_2$,
$D_{15} \uparrow^{(2,6)} S_{23} = k_3$, $D_{19} \uparrow^{(2,6)} S_{23} = k_4$, $D_{21} \uparrow^{(2,6)} S_{23} = k_5$, $D_{27} \uparrow^{(2,6)} S_{23} = k_6$,
$D_{39} \uparrow^{(2,6)} S_{23} = k_7$, $D_{41} \uparrow^{(2,6)} S_{23} = k_8$. $D_{55} \uparrow^{(1,0)} S_{23} = k_9$

Now we have

$k_1 = c_2 + c_7 + c_{13}$, $k_2 = c_5 + c_7$, $k_3 = c_7 + c_9$, $k_4 = c_{10} + c_{13}$,$k_5 = c_{11} + 2c_{13}$, $k_6 = c_{13} + c_{16}$, $k_7 = c_{20} + c_{22}$, $k_8 = c_{21} + c_{22}$ and $k_9 = c_6 + c_9$.

Since $(c_3 - c_4) \downarrow_{(2,6)} S_{22}$, $(c_1 - c_6) \downarrow_{(1,0)} S_{22}$, $(c_{11} - c_{18}) \downarrow_{(1,0)} S_{22}$, $(c_{14} - c_{17}) \downarrow_{(1,0)} S_{22}$ and $(c_{15} - c_{18}) \downarrow_{(1,0)} S_{22}$, are not p.s. , so $c_4 \not\subset c_3$, $c_6 \not\subset c_1$, $c_{18} \not\subset c_{11}$, $c_{17} \not\subset c_{14}$and $c_{18} \not\subset c_{15}$, so we get the approximation matrix.

< 23 >*	**1**																					
< 21, 2 >	**1**	**1**																				
< 21, 2 >′	**1**	**1**																				
< 20, 2, 1 >*		**1**	**1**																			
< 18, 3, 2 >*			**1**	**1**																		
< 17, 4, 2 >*			**1**	**1**	**1**																	
< 16, 7 >	**1**	**1**				**1**																
< 16, 7 >′	**1**	**1**				**1**																

$<16,6,1>^*$	2	1	1			1	1															
$<16,5,2>^*$			1		1		1	1														
$<16,4,3>^*$					1			1														
$<15,6,2>^*$	2					1	1	1	1													
$<14,9>$	1					1				1												
$<14,9>'$	1					1				1												
$<14,7,2>^*$	4	2				2			2	2	2											
$<14,6,2,1>$								1	1		1	1										
$<14,6,2,1>'$								1	1		1	1										
$<14,4,3,2>$								1				1										
$<14,4,3,2>'$								1				1										
$<13,9,1>^*$	2					1	1			1			1									
$<13,8,2>^*$	4	2	1			1	1	1	1	2	2		1	1								
$<13,7,2,1>$	2	1						1	1	1	2	1		1	1							
$<13,7,2,1>'$	2	1						1	1	1	2	1		1	1							
$<13,4,3,2,1>^*$												1			1							
$<12,9,2>^*$	2		1		1		1	1					1	1		1						
$<11,10,2>^*$					1											1						
$<11,9,3>^*$			1	1	1			1						1		1	1					
$<11,7,3,2>$								1			1	1		1	1		1	1				
$<11,7,3,2>'$								1			1	1		1	1		1	1				
$<11,6,3,2,1>^*$											1	1			1			1				
$<10,9,4>^*$			1	1										1			1		1			
$<10,7,4,2>$											1			1	1	1	1	1	1	1		
$<10,7,4,2>'$											1			1	1	1	1	1	1	1		
$<10,6,4,2,1>^*$											1				1			1		1		
$<9,8,6>^*$	2	2	1							2	2			1					1		2	
$<9,7,6,1>$	2	1								1	1		1	1	1				1		1	1
$<9,7,6,1>'$	2	1								1	1		1	1	1				1		1	1
$<9,7,5,2>$	2									2	1		1	1	1	1			2	1	2	1
$<9,7,5,2>'$	2									2	1		1	1		1			2	1	2	1
$<9,7,4,3>$																1			1	1	1	
$<9,7,4,3>'$																1			1	1	1	
$<9,6,5,2,1>^*$										2	1								2	1	2	
$<9,6,4,3,1>^*$															1				2	1	1	1
$<9,5,4,3,2>^*$															1							1
$<8,7,6,2>$													1						1			1
$<8,7,6,2>'$													1						1			1
$<8,6,4,3,2>^*$																			2		2	1
$<7,6,4,3,2,1>$																					1	
$<7,6,4,3,2,1>'$																					1	
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}	c_{21}	c_{22}

From lemmas and theorems above we can find the 7-Modular Projective (spin) Characters of The symmetric group S_{23} .

Appendix 1

The decomposition matrix for the spin characters of $S_{22}, p = 7$

⟨22⟩	1																																											
⟨22⟩′		1																																										
⟨21,1⟩*	1	1	1	1																																								
⟨19,2,1⟩			1		1																																							
⟨19,2,1⟩′				1		1																																						
⟨18,3,1⟩					1		1																																					
⟨18,3,1⟩′						1		1																																				
⟨17,4,1⟩					1		1		1																																			

⟨17,4,1⟩′						1		1		1												
⟨16,5,1⟩			1		1				1		1											
⟨16,5,1⟩′				1		1				1		1										
⟨15,7⟩*	1	1	1	1									1	1								
⟨15,6,1⟩	2		1								1		1		1							
⟨15,6,1⟩′		2		1								1		1		1						
⟨15,5,2⟩									1		1				1		1					
⟨15,5,2⟩′										1		1				1		1				
⟨15,4,3⟩									1								1					
⟨15,4,3⟩′										1								1				
⟨14,8⟩*	1	1											1	1					1	1		
⟨14,7,1⟩	4		1	1									2		2				2		1	1
⟨14,7,1⟩′		4	1	1										2		2				2	1	1
⟨14,5,2,1⟩*															1	1	1	1			1	1
⟨14,4,3,1⟩*																	1	1				
⟨13,8,1⟩	2		1	1							1		1		1				2		1	1
⟨13,8,1⟩′		2	1	1								1		1		1				2	1	1
⟨12,9,1⟩									1		1											
⟨12,9,1⟩′										1		1										
⟨12,8,2⟩	2		1	1	1				1		1				1		1		1		1	1
⟨12,8,2⟩′		2	1	1		1				1		1				1		1		1	1	1
⟨12,7,2,1⟩*	2	2	1	1											1	1	1	1	1	1	2	2
⟨12,4,3,2,1																						
⟨12,4,3,2,1																						
⟨11,10,1⟩									1													
⟨11,10,1⟩′										1												
⟨11,8,3⟩					1		1		1								1					
⟨11,8,3⟩′						1		1		1								1				
⟨11,7,3,1⟩*																	1	1			1	1
⟨11,5,3,2,1																					1	
⟨11,5,3,2,1																						1
⟨10,8,4⟩					1		1															
⟨10,8,4⟩′						1		1														
⟨10,7,4,1⟩*																					1	1
⟨10,5,4,2,1																					1	
⟨10,5,4,2,1																						1
⟨9,8,5⟩	2		1	1	1														2		1	1
⟨9,8,5⟩′		2	1	1		1														2	1	1
⟨9,7,5,1⟩*	2	2	1	1															2	2	2	2
⟨9,5,4,3,1⟩																						
⟨9,5,4,3,1⟩																						
⟨8,7,6,1⟩*			1	1																	1	1
⟨8,7,5,2⟩*																			1	1	1	1
⟨8,7,4,3⟩*																						
⟨8,6,5,2,1⟩																			2		1	
⟨8,6,5,2,1⟩																				2		1
⟨8,6,4,3,1⟩																						
⟨8,6,4,3,1⟩																						
⟨8,5,4,3,2⟩																						
⟨8,5,4,3,2⟩																						
⟨7,5,4,3,2,																						
	D_{1}	D_{2}	D_{3}	D_{4}	D_{5}	D_{6}	D_{7}	D_{8}	D_{9}	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}

⟨17,4,1⟩′																						
⟨16,5,1⟩																						
⟨16,5,1⟩′																						
⟨15,7⟩*																						
⟨15,6,1⟩																						
⟨15,6,1⟩′																						
⟨15,5,2⟩																						
⟨15,5,2⟩′																						
⟨15,4,3⟩																						
⟨15,4,3⟩′																						
⟨14,8⟩*																						
⟨14,7,1⟩																						
⟨14,7,1⟩′																						
⟨14,5,2,1⟩*	1	1																				
⟨14,4,3,1⟩*	1	1																				
⟨13,8,1⟩			1																			
⟨13,8,1⟩′				1																		
⟨12,9,1⟩			1		1																	
⟨12,9,1⟩′				1		1																
⟨12,8,2⟩			1		1		1															
⟨12,8,2⟩′				1		1		1														
⟨12,7,2,1⟩*	1	1					1	1	1	1												
⟨12,4,3,2,1	1								1													
⟨12,4,3,2,1		1								1												
⟨11,10,1⟩					1																	
⟨11,10,1⟩′						1																
⟨11,8,3⟩					1		1				1											
⟨11,8,3⟩′						1		1				1										
⟨11,7,3,1⟩*	1	1					1	1	1	1	1	1	1	1								
⟨11,5,3,2,1	1								1				1									
⟨11,5,3,2,1		1								1				1								
⟨10,8,4⟩							1				1				1							
⟨10,8,4⟩′								1				1				1						
⟨10,7,4,1⟩*					1	1	1	1	1	1	1	1	1	1	1	1	1	1				
⟨10,5,4,2,1									1				1				1					
⟨10,5,4,2,1										1				1				1				
⟨9,8,5⟩							1								1				2			
⟨9,8,5⟩′								1								1				2		
⟨9,7,5,1⟩*			1	1	1	1	1	1	1	1					1	1	1	1	2	2	1	1
⟨9,5,4,3,1⟩									1								1				1	
⟨9,5,4,3,1⟩										1								1				1
⟨8,7,6,1⟩*			1	1																	1	1
⟨8,7,5,2⟩*			1	1	1	1									1	1	1	1	1	1	1	1
⟨8,7,4,3⟩*					1	1									1	1	1	1	1	1		
⟨8,6,5,2,1⟩															2		1		2			
⟨8,6,5,2,1⟩																2		1		2		
⟨8,6,4,3,1⟩															2		1		2		1	
⟨8,6,4,3,1⟩																2		1		2		1
⟨8,5,4,3,2⟩																			1		1	
⟨8,5,4,3,2⟩																				1		1
⟨7,5,4,3,2,																			1	1		
	D_{23}	D_{24}	D_{25}	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}	D_{37}	D_{38}	D_{39}	D_{40}	D_{41}	D_{42}	D_{43}	D_{44}

The spin characters	The decomposition matrix for the block B_2								
$\langle 19,3\rangle^*$	1								
$\langle 17,5\rangle^*$	1	1							
$\langle 14,5,3\rangle$		1	1						
$\langle 14,5,3\rangle'$		1	1						
$\langle 13,5,3,1\rangle^*$			1	1					

$\langle 12,10\rangle^*$		1			1				
$\langle 12,7,3\rangle$	1	1	1		1	1			
$\langle 12,7,3\rangle'$	1	1	1		1	1			
$\langle 12,6,3,1\rangle^*$			1	1		1	1		
$\langle 12,5,3,2\rangle^*$				1			1		
$\langle 10,7,5\rangle$	1				1	1		1	
$\langle 10,7,5\rangle'$	1				1	1		1	
$\langle 10,6,5,1\rangle^*$					2	1	1	1	1
$\langle 10,5,4,3\rangle^*$							1		1
$\langle 8,6,5,3\rangle^*$					2			2	1
$\langle 7,6,5,3,1\rangle$								1	
$\langle 7,6,5,3,1\rangle'$								1	
	D_{45}	D_{46}	D_{47}	D_{48}	D_{49}	D_{50}	D_{51}	D_{52}	D_{53}

The spin characters	The decomposition matrix for the block B_3								
$\langle 20,2\rangle^*$	1								
$\langle 16,6\rangle^*$	1	1							
$\langle 14,6,2\rangle$		1	1						
$\langle 14,6,2\rangle'$		1	1						
$\langle 13,9\rangle^*$		1		1					
$\langle 13,7,2\rangle$	1	1	1	1	1				
$\langle 13,7,2\rangle'$	1	1	1	1	1				
$\langle 13,6,2,1\rangle^*$			1		1	1			
$\langle 13,4,3,2\rangle^*$						1			
$\langle 11,6,3,2\rangle^*$					1	1	1		
$\langle 10,6,4,2\rangle^*$					1		1	1	
$\langle 9,7,6\rangle$	1			1	1				1
$\langle 9,7,6\rangle'$	1			1	1				1
$\langle 9,6,5,2\rangle^*$				2	1			1	2
$\langle 9,6,4,3\rangle^*$								1	1
$\langle 7,6,4,3,2\rangle$									1
$\langle 7,6,4,3,2\rangle'$									1
	D_{54}	D_{55}	D_{56}	D_{57}	D_{58}	D_{59}	D_{60}	D_{61}	D_{62}

The spin characters	Decomposition matrix for the block B_4		
$\langle 18,4\rangle^*$	1		
$\langle 11,7,4\rangle$	1	1	
$\langle 11,7,4\rangle'$	1	1	
$\langle 11,6,4,1\rangle^*$		1	1
$\langle 11,5,4,2\rangle^*$			1
	D_{63}	D_{64}	D_{65}

The spin characters	Decomposition matrix for the block B_5					
$\langle 17,3,2\rangle$	1					
$\langle 17,3,2\rangle'$		1				
$\langle 10,9,3\rangle$	1		1			
$\langle 10,9,3\rangle'$		1		1		
$\langle 10,7,3,2\rangle^*$			1	1	1	1
$\langle 10,6,3,2,1\rangle$					1	
$\langle 10,6,3,2,1\rangle'$						1
	D_{66}	D_{67}	D_{68}	D_{69}	D_{70}	D_{71}

The spin characters	Decomposition matrix for the block B_6					
$\langle 16,4,2\rangle$	1					
$\langle 16,4,2\rangle'$		1				
$\langle 11,9,2\rangle$	1		1			

The spin characters	Decomposition matrix for the block B_7		
$\langle 16,3,2,1\rangle^*$	1		
$\langle 10,9,2,1\rangle^*$	1	1	
$\langle 9,8,3,2\rangle^*$		1	1

$\langle 11,9,2\rangle'$		1		1		
$\langle 9,7,4,2\rangle^*$			1	1	1	1
$\langle 9,6,4,2,1\rangle$					1	
$\langle 9,6,4,2,1\rangle'$						1
	D_{72}	D_{73}	D_{74}	D_{75}	D_{76}	D_{77}

$\langle 9,7,3,2,1\rangle$			1
$\langle 9,7,3,2,1\rangle'$			1
	D_{78}	D_{79}	D_{80}

The spin characters	Decomposition matrix for the block B_8		
$\langle 15,4,2,1\rangle^*$	1		
$\langle 11,8,2,1\rangle^*$	1	1	
$\langle 9,8,4,1\rangle^*$		1	1
$\langle 8,7,4,2,1\rangle$			1
$\langle 8,7,4,2,1\rangle$			1
	D_{81}	D_{82}	D_{83}

The spin characters	The decomposition matrix for the block B_9					
$\langle 13,5,4\rangle$	1					
$\langle 13,5,4\rangle'$		1				
$\langle 12,6,4\rangle$	1		1			
$\langle 12,6,4\rangle'$		1		1		
$\langle 11,6,5\rangle$			1		1	
$\langle 11,6,5\rangle'$				1		1
$\langle 7,6,5,4\rangle^*$					1	1
	D_{84}	D_{85}	D_{86}	D_{87}	D_{88}	D_{89}

The blocks of defect 0 are:

$$\langle 13,6,3\rangle = D_{90},\ \ \langle 13,6,3\rangle' = D_{91},\ \langle 12,5,4,1\rangle^* = D_{92}, \langle 10,8,3,1\rangle^* = D_{93}$$

Appendix 2

The decomposition matrix for the spin characters of $S_{23}, p = 7$

$< 23 >^*$	1																					
$< 21,2 >$	1	1																				
$< 21,2 >'$	1	1																				
$< 20,2,1 >^*$		1	1																			
$< 18,3,2 >^*$			1	1																		
$< 17,4,2 >^*$			1	1	1																	
$< 16,7 >$	1	1				1																
$< 16,7 >'$	1	1				1																
$< 16,6,1 >^*$	2	1	1			1	1															
$< 16,5,2 >^*$			1		1		1	1														
$< 16,4,3 >^*$					1			1														
$< 15,6,2 >^*$	2					1	1	1	1													
$< 14,9 >$	1					1				1												
$< 14,9 >'$	1					1				1												
$< 14,7,2 >^*$	4	2				2			2	2	2											
$< 14,6,2,1 >$								1	1		1	1										
$< 14,6,2,1 >'$								1	1		1	1										
$< 14,4,3,2 >$								1				1										
$< 14,4,3,2 >'$								1				1										
$< 13,9,1 >^*$	2					1	1			1			1									
$< 13,8,2 >^*$	4	2	1			1	1	1	1	2	2		1	1								
$< 13,7,2,1 >$	2	1						1	1	1	2	1		1	1							
$< 13,7,2,1 >'$	2	1						1	1	1	2	1		1	1							
$< 13,4,3,2,1 >^*$												1			1							
$< 12,9,2 >^*$	2		1		1		1	1					1	1		1						
$< 11,10,2 >^*$					1											1						
$< 11,9,3 >^*$			1	1	1			1						1		1	1					
$< 11,7,3,2 >$								1			1	1		1	1		1	1				
$< 11,7,3,2 >'$								1			1	1		1	1		1	1				
$< 11,6,3,2,1 >^*$											1	1			1			1				

$<10,9,4>^*$			1	1										1			1		1			
$<10,7,4,2>$											1			1	1	1	1	1	1	1		
$<10,7,4,2>'$											1			1	1	1	1	1	1	1		
$<10,6,4,2,1>^*$											1				1			1		1		
$<9,8,6>^*$	2	2	1							2	2			1					1		2	
$<9,7,6,1>$	2	1								1	1		1	1	1				1		1	1
$<9,7,6,1>'$	2	1								1	1		1	1	1				1		1	1
$<9,7,5,2>$	2									2	1		1	1	1	1			2	1	2	1
$<9,7,5,2>'$	2									2	1		1	1		1			2	1	2	1
$<9,7,4,3>$																1			1	1	1	
$<9,7,4,3>'$																1			1	1	1	
$<9,6,5,2,1>^*$										2	1								2	1	2	
$<9,6,4,3,1>^*$															1				2	1	1	1
$<9,5,4,3,2>^*$															1							1
$<8,7,6,2>$													1						1			1
$<8,7,6,2>'$													1						1			1
$<8,6,4,3,2>^*$																			2		2	1
$<7,6,4,3,2,1>$																					1	
$<7,6,4,3,2,1>'$																					1	
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}

The spin characters	The decomposition matrix for the block B_2																	
$\langle 22,1\rangle$	4																	
$\langle 22,1\rangle'$		4																
$\langle 15,8\rangle$	1		2															
$\langle 15,8\rangle'$		1		2														
$\langle 15,7,1\rangle^*$	5	5	2	2	1	1												
$\langle 15,5,2,1\rangle$					1		1											
$\langle 15,5,2,1\rangle'$						1		1										
$\langle 15,4,3,1\rangle$							1											
$\langle 15,4,3,1\rangle'$								1										
$\langle 14,8,1\rangle^*$	5	5	2	2	1	1			1	1								
$\langle 12,8,2,1\rangle$	8				1		1		1		1							
$\langle 12,8,2,1\rangle'$		8				1		1		1		1						
$\langle 11,8,3,1\rangle$							1				1		1					
$\langle 11,8,3,1\rangle'$								1				1		1				
$\langle 10,8,4,1\rangle$											1		1		1			
$\langle 10,8,4,1\rangle'$												1		1		1		
$\langle 9,8,5,1\rangle$	8								1	1	1				1		1	1
$\langle 9,8,5,1\rangle'$		8							1	1		1				1	1	1
$\langle 8,7,5,2,1\rangle^*$									1	1					1	1	1	1
$\langle 8,7,4,3,1\rangle^*$															1	1	1	1
$\langle 8,5,4,3,2,1\rangle$																	1	
$\langle 8,5,4,3,2,1\rangle'$																		1
	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

The spin characters	The decomposition matrix for the block B_3																	
$\langle 20,3\rangle$	1																	
$\langle 20,3\rangle'$		1																
$\langle 17,6\rangle$	1		1															

$\langle 17,6\rangle'$		1		1														
$\langle 14,6,3\rangle^*$			1	1	1	1												
$\langle 13,10\rangle$			1				1											
$\langle 13,10\rangle'$				1				1										
$\langle 13,7,3\rangle^*$	1	1	1	1	1	1	1	1	1	1								
$\langle 13,6,3,1\rangle$					1				1		1							
$\langle 13,6,3,1\rangle'$						1				1		1						
$\langle 13,5,3,2\rangle$											1							
$\langle 13,5,3,2\rangle'$												1						
$\langle 12,6,3,2\rangle$									1		1		1					
$\langle 12,6,3,2\rangle'$										1		1		1				
$\langle 10,7,6\rangle^*$	1	1					1	1	1	1					1	1		
$\langle 10,6,5,2\rangle$							2		1				1		1		1	
$\langle 10,6,5,2\rangle'$								2		1				1		1		1
$\langle 10,6,4,3\rangle$													1				1	
$\langle 10,6,4,3\rangle'$														1				1
$\langle 9,6,5,3\rangle$							2								2		1	
$\langle 9,6,5,3\rangle'$								2								2		1
$\langle 7,6,5,3,2\rangle^*$															1	1		
	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}	d_{57}	d_{58}

The spin characters	The decomposition matrix for the block B_4																	
$\langle 19,4\rangle$	1																	
$\langle 19,4\rangle'$		1																
$\langle 18,5\rangle$	1		1															
$\langle 18,5\rangle'$		1		1														
$\langle 14,5,4\rangle^*$			1	1	1	1												
$\langle 13,5,4,1\rangle$					1		1											
$\langle 13,5,4,1\rangle'$						1		1										
$\langle 12,11\rangle$			1						1									
$\langle 12,11\rangle'$				1						1								
$\langle 12,7,4\rangle^*$	1	1	1	1	1	1			1	1	1	1						
$\langle 12,6,4,1\rangle$					1		1				1		1					
$\langle 12,6,4,1\rangle'$						1		1				1		1				
$\langle 12,5,4,2\rangle$							1						1					
$\langle 12,5,4,2\rangle'$								1						1				
$\langle 11,7,5\rangle^*$	1	1							1	1	1	1			1	1		
$\langle 11,6,5,1\rangle$									2		1		1		1		1	
$\langle 11,6,5,1\rangle'$										2		1		1		1		1
$\langle 11,5,4,3\rangle$													1				1	
$\langle 11,5,4,3\rangle'$														1				1
$\langle 8,6,5,4\rangle$									2						1	1	1	
$\langle 8,6,5,4\rangle'$										2					1	1		1
$\langle 7,6,5,4,1\rangle$															1	1		
	d_{59}	d_{60}	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	d_{66}	d_{67}	d_{68}	d_{69}	d_{70}	d_{71}	d_{72}	d_{73}	d_{74}	d_{75}	d_{76}

The spin characters	The decomposition matrix for the block B_5								
$\langle 19,3,1\rangle^*$	1								
$\langle 17,5,1\rangle^*$	1	1							

$\langle 15,5,3\rangle^*$		1	1						
$\langle 14,5,3,1\rangle$			1	1					
$\langle 14,5,3,1\rangle'$			1	1					
$\langle 12,10,1\rangle^*$		1			1				
$\langle 12,8,3\rangle^*$	1	1	1		1	1			
$\langle 12,7,3,1\rangle$			1	1		1	1		
$\langle 12,7,3,1\rangle'$			1	1		1	1		
$\langle 12,5,3,2,1\rangle^*$				1			1		
$\langle 10,8,5\rangle^*$	1					1		1	
$\langle 10,7,5,1\rangle$					1	1	1	1	1
$\langle 10,7,5,1\rangle'$					1	1	1	1	1
$\langle 10,5,4,3,1\rangle^*$							1		1
$\langle 8,7,5,3\rangle$					1			1	1
$\langle 8,7,5,3\rangle'$					1			1	1
$\langle 8,6,5,3,1\rangle^*$								2	1
	d_{77}	d_{78}	d_{79}	d_{80}	d_{81}	d_{82}	d_{83}	d_{84}	d_{85}

The spin characters	Decomposition matrix for the block B_6		
$\langle 18,4,1\rangle^*$	1		
$\langle 11,8,4\rangle^*$	1	1	
$\langle 11,7,4,1\rangle$		1	1
$\langle 11,7,4,1\rangle'$		1	1
$\langle 11,5,4,2,1$			1
	d_{86}	d_{87}	d_{88}

The spin characters	The decomposition matrix for the block B_7					
$\langle 17,3,2,1\rangle$	1					
$\langle 17,3,2,1\rangle'$		1				
$\langle 10,9,3,1\rangle$	1		1			
$\langle 10,9,3,1\rangle'$		1		1		
$\langle 10,8,3,2\rangle$			1		1	
$\langle 10,8,3,2\rangle'$				1		1
$\langle 10,7,3,2,1\rangle^*$					1	1
	d_{89}	d_{90}	d_{91}	d_{92}	d_{93}	d_{94}

The spin characters	The decomposition matrix for the block B_8					
$\langle 16,4,2,1\rangle$	1					
$\langle 16,4,2,1\rangle'$		1				
$\langle 11,9,2,1\rangle$	1		1			
$\langle 11,9,2,1\rangle'$		1		1		
$\langle 9,8,4,2\rangle$			1		1	
$\langle 9,8,4,2\rangle'$				1		1
$\langle 9,7,4,2,1\rangle^*$					1	1
	d_{95}	d_{96}	d_{97}	d_{98}	d_{99}	d_{100}

The blocks of defect 0 are:

$\langle 13,6,4\rangle^* = \mathrm{d}_{101}$, $\langle 12,6,5\rangle^* = d_{102}$,

$\langle 11,6,4,2\rangle = \mathrm{d}_{103}$, $\langle 11,6,4,2\rangle' = \mathrm{d}_{104}$ and $\langle 9,8,3,2,1\rangle^* = \mathrm{d}_{105}$

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المشخصات المعيارية قياس $p = 7$ لزمرة التمثيل $\bar{S}_{23}$

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الملخص:

في هذا البحث تم ايجاد المشخصات المعيارية للزمر التمثيلية (زمرة الغطاء) $\bar{S}_{23}$ قياس $p = 7$ والتي نحصل منها على المشخصات المعيارية الأسقاطية غير القابلة للتحليل للزمرة S_{23} قياس $p = 7$, كذلك اعطينا مصفوفة التجزئة قياس $p = 7$ للزمرة $\bar{S}_{23}$.