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On New Coincidence and Fixed Point Results for Single-Valued Maps in Partial Metric Spaces

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Abstract

In the present paper, we are interested in the study of the concept (T, f) generalized condition (B) and combined with the concept of partial metric space and prove some common fixed point results in such a space. Our results generalize, extend and unify many results. Example is given showing the validly of our results.

Keywords: fixed point, weak*compatible maps , partial metric space.

1- Introduction

The concept of metric space, introduced by Frechet in 1906, [6] is one of the cornerstones of Mathematics and other Sciences due to its importance and application. This notion has been extended, in proved and generalized in many different ways for example, partial metric space, Cone metric space, G -metric space, Fuzzy metric space and so on.

In this paper, we pay attention to the concept of partial metric space. The notion of partial metric space was introduced by Matthews [10] as a part of the study of denotational semantics of data flow networks. In partial metric spaces, the distance of a point in the self may not be zero.

Recently, several papers have been published on fixed point theorems in partial metric space (see, for instance [3], [7], [10] [12], [14] and [51]).

The aim of this paper is to generalize various known results and to give an example to illustrate our main results.

2-Preliminaries: Compatible with Matthews [10,11] and Hashim [7], the following definition and results will be needed in the sequel.

Definition2.1 [10,11] A partial metric on a non-empty set X is a function $p: X \times X \to \Re^+$ (where $\Re^+$ denotes the set of all non-negative real numbers), satisfying the following conditions:

(P1) $x = y \Leftrightarrow p(x,x) = p(y,y) = p(x,y)$

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(P2) $p(x,x) \le p(x,y)$
(P3) $p(x,y) = p(y,x)$
(P4) $p(x,z) \le p(x,y) + p(y,z) - p(y,y)$
for all $x,y,z \in X$. Then (X,p) is called partial metric space.
For a partial metric p on X, the function

$d_p : X \times X \to \Re^+$ defined by
$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ is a (usual) metric on X.

Each partial metric p on X generates a T_0-Topology $\tau(p)$ on X with the space of family of open p-balls
$\{B_p(x;\varepsilon); x \in X, \varepsilon > 0\}$, where
$B_p(x;\varepsilon) = \{y \in X; p(x,y) < p(x,x) + \varepsilon\}$
for all $x \in X$ and $\varepsilon > 0$.

Example 2.2[10,11] The pair (R^+, p) where $p(x,y) = \max\{x,y\}$ for all $x,y \in R^+$ is a partial metric space.

Definition 2.3 [10,11]

(1) A sequence $\{x_n\}$ in a partial metric space (X,p) is converged to x if and only if $\lim_{n\to\infty} p(x,x_n) = p(x,x)$

(2) A sequence $\{x_n\}$ in a partial metric space (X,p) is called Cauchy if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists (and is finite).

(3) A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to $\tau(p)$, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$

Lemma 2.4 [10,11] Let (X,p) be a partial metric space. Then

(1) A sequence $\{x_n\}$ is a Cauchy in a partial metric space (X,p) if and only if $\{x_n\}$ is a Cauchy in a metric space (X,d_p),

(2) A partial metric space (X,p) is complete if and only if a metric space (X,d_p) is complete.

Moreover, $\lim_{n\to\infty} d_p(x_n,x) = 0$
$\Leftrightarrow p(x,x) = \lim_{n\to\infty} p(x,x_n) = \lim_{n,m\to\infty} p(x_n,x_m)$

Definition 2.5 [1,5] Two self maps, f and T of a non-empty set X are weakly compatible if they commute at their coincidence points, that is, $fx = Tx$ ($x \in X$) implies $fTx = Tfx$.

Definition 2.6 [8] Two self maps f and T of a non-empty set X are weak*compatible if they commute at one of their coincidence points, that is, if there exists a point $x \in X$ such that $fx = Tx$ then $fTx = Tfx$.

This concept is available as early as 1987 in [13]. However, we are giving a formal name, viz., " weak* compatible " in this paper. The following example shows that weak* compatible maps are indeed more general than weak compatible maps [9].

Example 2.7 [8]Let $fx = x^2/4$ and $Tx = x^3$ for $x \in [0, 1/4]$. Then f and T have two coincidence points viz., 0 and $1/4$. Evidently, they commute at 0 but not at $1/4$.

Definition 2.9 A self maps T on a partial metric space X is said to satisfy generalization almost contradiction type associated with self map f of X if there exits $\alpha \in [0,1)$ and $L \ge 0$ such that,
$p(Tx,Ty) \le \alpha M_p(x,y) + LN_p(x,y)$ for all

$x, y \in X$, where

$$M_p(x,y) = \max\{p(fx, fy), p(fx,Tx), p(fy,Ty), \tfrac{1}{2}[p(fx,Ty)+p(fy,Tx)]\}$$

$$N_p(x,y) = \min\{p(fx,Tx), p(fy,Ty), p(fx,Ty), p(fy,Tx)\}$$

3-Main Results

Theorem 3.1 Let (X, p) be a partial metric space and $f, T : X \to X$ be such that,

$$p(Tx,Ty) \le \alpha M_p(x,y) + LN_p(x,y) \quad (3.1)$$

for all $x, y \in X$, where $\alpha \in [0,1)$, $L \ge 0$, $\alpha + 2L < 1$, and

$$M_p(x,y) = \max\{p(fx, fy), p(fx,Tx), p(fy,Ty), \tfrac{1}{2}[p(fx,Ty)+p(fy,Tx)]\}$$

$$N_p(x,y) = \min\{p(fx,Tx), p(fy,Ty), p(fx,Ty), p(fy,Tx)\}$$

$$TX \subseteq fX \quad (3.2),$$

and if one of fX or TX is a complete subspace of X . Then f and T have a coincidence point. Moreover, if (f, T) is weak* compatible. Then f and T have a unique fixed point in X .

Proof: Let x_0 be an arbitrary point in X , since $TX \subseteq fX$. choose a point $x_1 \in X$ such that $fx_1 = Tx_0$. Continuing this process we have $fx_n = Tx_{n-1}$, $n = 1, 2, ...$

Now,

if $fx_n = fx_{n+1}$ for some n , then, f and T have a coincidence point.

Suppose, further, that $fx_n \neq fx_{n+1}$ for all $n \in N$, (N the natural numbers).

Now,

$$M_p(x_{n-1}, x_n) = \max\{p(fx_{n-1}, fx_n), p(fx_{n-1},Tx_{n-1}), p(fx_n,Tx_n), \frac{1}{2}[p(fx_{n-1},Tx_n)+p(fx_n,Tx_{n-1})]\}$$

$$\le \max\{p(fx_{n-1},fx_n), p(fx_{n-1},fx_n), p(fx_n,fx_{n+1}), p(fx_n,fx_{n+1}), \frac{1}{2}[p(fx_{n-1},fx_{n+1})+p(fx_n,fx_n)]\}$$

$$\le \max\{ p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1}), \tfrac{1}{2}[p(fx_{n-1}, fx_n)+p(fx_n, fx_{n+1})]\},$$

$$\le \max\{ p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1})\},$$

and

$$N_p(x_{n-1}, x_n) = \min\{p(fx_{n-1},Tx_{n-1}), p(fx_n,Tx_n), p(fx_{n-1},Tx_n), p(fx_n,Tx_{n-1})\}$$

$$\le \min\{p(fx_{n-1},fx_n), p(fx_n,fx_{n+1}), p(fx_{n-1},fx_{n+1}), p(fx_n,fx_n)\}$$

$$= \min\{p(fx_{n-1}, fx_{n+1}), p(fx_n, fx_n)\} ,$$

we obtain that,

$$p(fx_{n+1},fx_n) = p(Tx_n,Tx_{n-1})$$

$$\le \alpha \max\{p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1})\} + L\min\{p(fx_{n-1}, fx_{n+1}), p(fx_n, fx_n)\}$$

Now, we have four cases:

(1) If

$$max\{p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1})\} = p(fx_{n-1}, fx_n)$$

and

$$min\{p(fx_{n-1},fx_{n+1}), p(fx_n,fx_n)\} = p(fx_{n-1},fx_{n+1})$$

$$p(fx_{n+1}, fx_n) \le \alpha p(fx_{n-1}, fx_n) + Lp(fx_{n-1}, fx_{n+1})$$

$$\le \alpha p(fx_{n-1}, fx_n) + L[p(fx_{n-1}, fx_n)+p(fx_n, fx_{n+1})-p(fx_n, fx_n)]$$

$$\le \alpha p(fx_{n-1}, fx_n) + L\, p(fx_{n-1}, fx_n) + Lp(fx_n, fx_{n+1})$$

it follow that

$$p(fx_n, fx_{n+1}) \le \frac{\alpha + L}{1-L} p(fx_{n-1}, fx_n)$$

Since, $\beta_1 = \dfrac{\alpha+L}{1-L} < 1$. Therefore

$$p(fx_n, fx_{n+1}) \le \beta_1 p(fx_{n-1}, fx_n)$$

(2) If

$\max\{p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1})\} = p(fx_{n-1}, fx_n)$ and

$\min\{p(fx_{n-1}, fx_{n+1}), p(fx_n, fx_n)\} = p(fx_n, fx_n)$

$$\begin{aligned} p(fx_n, fx_{n+1}) &\le \alpha p(fx_{n-1}, fx_n) + Lp(fx_n, fx_n) \\ &\le \alpha p(fx_{n-1}, fx_n) + L[p(fx_n, fx_{n+1}) + p(fx_{n+1}, fx_n) - p(fx_{n+1}, fx_{n+1})] \\ &\le \alpha p(fx_{n-1}, fx_n) + L\,p(fx_n, fx_{n+1}) + Lp(fx_{n+1}, fx_n) \\ &\le \frac{\alpha}{1-2L} p(fx_{n-1}, fx_n) \end{aligned}$$

Since, $\beta_2 = \dfrac{\alpha}{1-2L} < 1$. Therefore

$$p(fx_{n+1}, fx_n) \le \beta_2 p(fx_{n-1}, fx_n)$$

(3) If

$\max\{p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1})\} = p(fx_n, fx_{n+1})$, and

$\min\{p(fx_{n-1}, fx_{n+1}), p(fx_n, fx_n)\} = p(fx_{n-1}, fx_{n+1})$

$$\begin{aligned} p(fx_n, fx_{n+1}) &\le \alpha p(fx_n, fx_{n+1}) + Lp(fx_{n-1}, fx_{n+1}) \\ &\le \alpha p(fx_n, fx_{n+1}) + L[p(fx_{n-1}, fx_n) + p(fx_n, fx_{n+1}) - p(fx_n, fx_n)] \\ &\le \alpha p(fx_n, fx_{n+1}) + Lp(fx_{n-1}, fx_n) + Lp(fx_n, fx_{n+1}) \end{aligned}$$

then,

$$p(fx_n, fx_{n+1}) \le \frac{L}{1-(\alpha+L)} p(fx_{n-1}, fx_n)$$

Since, $\beta_3 = \dfrac{L}{1-(\alpha+L)} < 1$. Therefore

$$p(fx_n, fx_{n+1}) \le \beta_3 p(fx_{n-1}, fx_n)$$

(4) If $\max\{p(fx_{n-1}, fx_n), p(fx_n, fx_{n+1})\} = p(fx_n, fx_{n+1})$ and

$\min\{p(fx_{n-1}, fx_{n+1}), p(fx_n, fx_n)\} = p(fx_n, fx_n)$

$$\begin{aligned} p(fx_n, fx_{n+1}) &\le \alpha p(fx_n, fx_{n+1}) + Lp(fx_n, fx_n) \\ &\le \alpha p(fx_n, fx_{n+1}) + Lp(fx_{n-1}, fx_{n+1}) \\ &\le \alpha p(fx_n, fx_{n+1}) + L[p(fx_{n-1}, fx_n) + p(fx_n, fx_{n+1}) - p(fx_n, fx_n)] \\ &\le \alpha p(fx_n, fx_{n+1}) + Lp(fx_{n-1}, fx_n) + Lp(fx_n, fx_{n+1}) \end{aligned}$$

then,

$$p(fx_n, fx_{n+1}) \le \frac{L}{1-(\alpha+L)} p(fx_n, fx_{n-1})$$

Since, $\beta_4 = \dfrac{L}{1-(\alpha+L)} < 1$. Therefore

$p(fx_n, fx_{n+1}) \le \beta_4 p(fx_{n-1}, fx_n)$ Therefore choose $\beta = \max\{\beta_1, \beta_2, \beta_3, \beta_4\}$.

Therefore, $0 \le \beta < 1$.

For each $n \in N$, we have,

$$p(fx_{n+1}, fx_n) \le \beta p(fx_n, fx_{n-1}) \qquad (3.3)$$

For each $n \in N$, we obtain,

$$p(fx_{n+1}, fx_n) \le \beta^n p(fx_1, fx_0) \qquad (3.4)$$

Let $n, m \in N$ with $m > n$. By applying (3.4) we have,

$$p(fx_n,fx_m)\le[p(fx_n,fx_{n+1})+ p(fx_{n+1},fx_{n+2}) +\ldots+p(fx_{m-1},fx_m)] -[p(fx_{n+1},fx_{n+1})+p(fx_{n+2},fx_{n+2}) +\ldots+p(fx_{m-1},fx_{m-1})]$$

$$\le[p(fx_n,fx_{n+1})+p(fx_{n+1},fx_{n+2})+\ldots+ p(fx_{m-1},fx_m)]$$
$$\le\left[\beta^n+\beta^{n+1}+\ldots+\beta^{m-1}\right]p(fx_1,fx_0)$$
$$\le\frac{\beta^n}{1-\beta}p(fx_1,fx_0),\ \beta<1$$

It follow that

$$\lim_{n,m\to\infty} p(fx_m,fx_n)=0 \qquad (3.5)$$

And by (p2) we obtain

$$\lim_{n\to\infty} p(fx_n,fx_n)=0=\lim_{m\to\infty} p(fx_m,fx_m) \quad (3.6)$$

From the definition of d_p and (3.6)

$$d_p(fx_m,fx_n)=2p(fx_m,fx_n)-p(fx_m,fx_m) -p(fx_n,fx_n) \le 2p(fx_m,fx_n)$$

This yields

$$\lim_{n,m\to\infty} d_p(fx_m,fx_n)=0 \qquad (3.7)$$

This implies that $\{fx_n\}$ is a Cauchy sequence in (fX,d_p) and by the completeness of fX, we have $\{fx_n\}$ is convergent to some $u\in X$ that is,

$$\lim_{n\to\infty} fx_n=u \qquad (3.8)$$

Also, the subsequences $\{fx_n(k)\}$ and $\{fx_m(k)\}$ convergent to u. Therefore, there exists $z\in X$ such that $u=fz$.

By lemma (2.3) and (3.8), we obtain that

$$p(fz,fz)=\lim_{n\to\infty} p(fx_n,fz) = \lim_{m,n\to\infty} p(fx_m,fx_n) \qquad (3.9)$$

For (3.5) and (3.9) we get

$$p(fz,fz)=0 \qquad (3.10)$$

Now, we claim that $p(Tz,fz)=0$.

Suppose the contrary, that $p(Tz,fz)>0$.

By (p4) and (3.1) we have,

$$p(fz,Tz)\le p(fz,Tx_{n+1})+p(Tx_{n+1},Tz) -p(Tx_{n+1},Tx_{n+1}) \le p(fz,Tx_{n+1})+p(Tx_{n+1},Tz) \qquad (3.11)$$

Now,

$$p(Tx_{n+1},Tz)\le\alpha M_p(x_{n+1},z)+LN_p(x_{n+1},z)$$
$$\le\alpha\max\{p(fx_{n+1},fz),p(fx_{n+1},Tx_{n+1}), p(fz,Tz),\tfrac{1}{2}[p(fx_{n+1},Tz)+p(fz,Tx_{n+1})]\}+ L\min\{p(fx_{n+1},Tx_{n+1}),p(fz,Tz), p(fx_{n+1},Tz),p(fz,Tx_{n+1})\}$$
$$\le\alpha\max\{p(fx_{n+1},fz),p(fx_{n+1},fx_{n+2}), p(fz,Tz),\tfrac{1}{2}[p(fx_{n+1},Tz)+p(fz,fx_{n+2})]\}+ L\min\{p(fx_{n+1},fx_{n+2}),p(fz,Tz), p(fx_{n+1},Tz),p(fz,fx_{n+2})\}$$
$$\le\alpha\max\{p(fx_{n+1},fz),p(fx_{n+1},fz)+ p(fz,fx_{n+2})-p(fz,fz),p(fz,Tz), \tfrac{1}{2}[p(fx_{n+1},Tz)+p(fz,fx_{n+2})]\}$$
$$\le\alpha\max\{p(fx_{n+1},fz),p(fx_{n+1},fz)+ p(fz,fx_{n+2}),p(fz,Tz), \tfrac{1}{2}[p(fx_{n+1},Tz)+p(fz,fx_{n+2})]\}+ L\min\{p(fx_{n+1},fz)+p(fz,fx_{n+2}), p(fz,Tz),p(fx_{n+1},Tz),p(fz,fx_{n+2})\}$$

Since,

$$p(fx_n,Tz)\le p(fx_n,fz)+p(fz,Tz)-p(fz,fz) \le p(fx_n,fz)+p(fz,Tz)$$

and

$$p(fz,Tz)\le p(fz,fx_n)+p(fx_n,Tz)- p(fx_n,fx_n) \le p(fz,fx_n)+p(fx_n,Tz)$$ we

have, $p(fz,Tz)-p(fz,fx_n)\le p(fx_n,Tz) \le p(fx_n,fz)+p(fz,Tz) \qquad (3.12)$

Taking the limit as $n\to\infty$ in (3.12) we have,

$\lim_{n\to\infty} p(fx_n, Tz) = p(fz, Tz)$.

Therefore, taking the limit as $n \to \infty$ in (3.11) this yields $p(fz, Tz) \le \alpha p(fz, Tz)$.

Since, $\alpha \in [0,1)$, we get $p(fz, Tz) = 0$.

Hence, $fz = Tz = u$

Therefore, T and f have a coincidence point z.

If T and f are weak*compatible, we have $T(fz) = f(Tz)$ since, $fz = Tz = u$ this yields that $fu = Tu = u$.

Thus u is a fixed point.

Now, we claim that T and f have a unique fixed point. If it is not assume that there exist another fixed point w such that $fw = Tw = w$, then

$$p(u,w) = p(Tu, Tw)$$
$$\le \alpha M_p(u,w) + LN_p(u,w)$$
$$\le \alpha \max\{p(fu,fw), p(fu,Tu), p(fw,Tw), \tfrac{1}{2}[p(fu,Tw) + p(fw,Tu)]\} + L\min\{p(fu,Tu), p(fw,Tw), p(fu,Tw), p(fw,Tu)\}$$
$$\le \alpha \max\{p(u,w), 0, 0\} + L.0$$
$$\le \alpha p(u,w),$$

$\alpha \in [0,1)$. Then $p(u,w) = 0$ and $u = w$. This implies that T and f have a unique fixed point.

Corollary 3.2 Let (X, p) be a partial metric space. Let $f, T : X \to X$ be such that $TX \subseteq fX$. Assume that T satisfies the following condition:

$$p(Tx, Ty) \le \alpha p(fx, fy) + Lp(fy, Tx) \quad (3.13)$$

for all $x, y \in X$, where $\alpha \in [0,1)$, $L \ge 0$, with $\alpha + 2L < 1$.

If fX or TX is a complete subspace of X Then f and T have coincidence point.

Further, if f and T are weak*compatible, Then f and T have a unique fixed point.

Proof: It comes from theorem 3.1 with $\min\{p(fx,Tx), p(fy,Ty), p(fx,Ty), p(fy,Tx)\} = p(fy,Tx)$

Remark 3.3

(1) In metric space (X, d) condition (3.1) in [2] is called T generalized condition (B) associated by self map f of X.

(2) In metric space (X, d) condition (3.13) with $f = I_X$, (where I_X the identity map on X) is called almost contraction which introduced by Berinde [4].

Corollary 3.4

(1) Banach type: Let (X, p) be a complete partial metric space $T : X \to X$ be a map such that $p(Tx, Ty) \le \alpha p(x, y)$, for all $x, y \in X$, where $\alpha \in [0,1)$. Then T has a unique fixed point.

(2) Kannan type: Let (X, p) be a complete partial metric space $T : X \to X$ be a map such that $p(Tx, Ty) \le \beta p(x, Tx) + \gamma p(y, Ty)$ for all $x, y \in X$, where $\beta, \gamma \ge 0$ and $\beta + \gamma < 1$. Then T has a unique fixed point.

(3) Ciric type: Let (X, p) be a complete partial metric space $T : X \to X$ be a map such that $p(Tx, Ty) \le \alpha \max\{p(x,y), p(x,Tx), p(y,Ty), \tfrac{1}{2}[p(x,Ty) + p(y,Tx)]\}$ for all $x, y \in X$, where $\alpha \in [0,1)$. Then T has a unique fixed point.

Example 3.5 let $X = \{0,1,2\}$ be endowed with the partial metric $p : X \times X \to \Re^+$ defined by

$p(0,0)=p(1,1)=0$,
$p(0,1)=p(1,0)=\frac{1}{4}$,
$p(2,2)=\frac{1}{3}$,
$p(0,2)=p(2,0)=\frac{2}{5}$,
$p(1,2)=p(2,1)=\frac{13}{20}$ we define $T:X\to X$ by $Tx=\begin{cases}0, & if \quad x\neq 2\\ 1, & if \quad x=2\end{cases}$ and $fx=I_X$

$$M_p(x,y)=\max\{p(x,y),p(x,Tx),p(y,Ty),\tfrac{1}{2}[p(x,Ty)+p(y,Tx)]\}.$$

Now,

(i) if $x=0$, $y=2$, then $p(T0,T2)=p(0,1)=\frac{1}{4}$ and

$$M(0,2)=\max\{p(0,2),p(0,T0),p(2,T2),\tfrac{1}{2}[p(0,T2)+p(2,T0)]\}$$

$$=\max\{p(0,2),p(0,0),p(2,1),\tfrac{1}{2}[p(0,1)+p(2,0)]\}$$
$$=\max\{\tfrac{2}{5},0,\tfrac{13}{20},\tfrac{13}{40}\}=\tfrac{13}{20}$$

$p(T0,T2)=\frac{1}{4}\le \alpha M_p(0,2)=\frac{13}{20}\alpha$, for $\alpha\in[\frac{5}{13},1)$

Similarly,

(ii) If $x=2$, $y=0$, then $p(T2,T0)=p(1,0)=\frac{1}{4}\le\alpha M_p(0,2)$ $=\frac{13}{20}\alpha$ for $\alpha\in[\frac{5}{13},1)$.

(iii) If $x=2$, $y=1$, then $p(T2,T0)=p(1,0)=\frac{1}{4}$ and

$$M_p(2,1)=\max\{p(2,1),p(2,T2),p(1,T1),\tfrac{1}{2}[p(1,T2)+p(2,T1)]\}$$

$$=\max\{p(2,1),p(2,1),p(1,0),\tfrac{1}{2}[p(1,1)+p(2,0)]\}$$
$$=\max\{\tfrac{13}{20},\tfrac{1}{4},\tfrac{1}{2},[0+\tfrac{2}{5}]\}=\tfrac{13}{20}$$
$$p(T2,T1)=\tfrac{1}{4}\le M_p(2,1)=\tfrac{13}{20}$$

For $\alpha\in[\frac{5}{13},1)$. Similarly,

(iv) If $x=1$, $y=2$, ,then $p(T1,T2)=p(0,1)=\frac{1}{4}$ $\le\alpha M_p(2,1)=\frac{13}{20}\alpha$ for $\alpha\in\left[\frac{5}{13},1\right)$.

(v) If $x=2$, $y=2$,then $p(T2,T2)=p(1,1)=0\le\alpha M_p(2,2)$ for $\alpha\in[0,1)$

(vi) If $x,y\in\{0,1\}$ then $p(T0,T1)=p(0,0)=0\le\alpha M_p(0,1)$ for $\alpha\in[0,1)$. this all conditions of corollary (3.4) (Ciric type) are satisfied and $x=0$ is the unique fixed point in X.

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نتائج جديدة للنقطة المتطابقة والنقطة الصامدة لدوال احادية في الفضاءات المترية جزئيا

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المستخلص

في هذا البحث قمنا بدراسة مفهوم (T,f) للشرط المعمم (B) وارتباطة مع مفهوم الفضاء المتري جزئيا وتم برهان بعض النتائج في مثل هذا النوع من الفضائيات . النتائج عممت و وسعت و وحدت العديد من النتائج السابقة مع اعطاء مثال لبيان مدى امكانية تطبيق النتائج.