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Finite Volume and Finite Difference Numerical Methods for Conduction Heat Transfer via an Aluminum Rod

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Abstract

The aim of this study is to investigate a conduction heat transfer phenomenon in Aluminum metal rod. Finite volume method has been used as a numerical technique to simulate the study in addition to finite difference method for more details. Both methods were compared to the exact solution which has been performed and utilized as a base line for the physical case in current study. Due to its widespread applications, Aluminum material has been used as a viable metal for the solid rod in this study. The Aluminum rod length and diameter were (10 cm) and (1 cm) respectively. The Aluminum rod has been set into a computational domain of air for the convection case to be taken into account. The computational domain of the air length and diameter were (12 cm) and (2 cm) respectively. The left side of the Aluminum rod has been set at temperature of (700 K) and the right side at (300 K). The surrounding computational domain of the air has been set at (300 K). Energy equation has been solved numerically to visualize and calculate the temperature distribution along the Aluminum rod. The results showed that there was good approach between finite volume and finite difference methods in one-dimensional temperature distribution. Hence, in addition to the fact that finite volume is crucial technique in all directions and used in more complicated applications, finite difference is still important method in one dimensional case as well. Moreover the results showed that volumetric heat source raises the temperature of the material during heat transfer operation. Also, the three methods, finite volume, finite difference and exact solution have showed good approach.

Keywords: Finite Volume method, Finite Difference method, Conduction Heat Transfer.

Introduction

When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region. We say that the energy is transferred by conduction and that the heat-transfer rate per unit area is proportional to the normal temperature gradient. Thus the temperature difference is the driving force for heat transfer. Furthermore heat transfer plays a major role in the design of many other devices, such as car radiators, solar collectors, various components of power plants, and even spacecraft…etc. Recently most of the researches emphasis on the numerical methods to treat the heat transfer problems. Numerical analysis by finite volume is the analysis of systems involving fluid flow, heat transfer and associated phenomena by means of computer-based simulation. The technique is very powerful and spans a wide range of industrial application areas [1-5]. However other numerical techniques like finite difference is still applicable in some cases.

V. Anjo [6] numerically simulated heat transfer conduction study during the solidification of Aluminum in green sand mould. Two dimensional finite difference analysis had been used. The method includes; the finite difference analysis of the heat conduction equation in steady (Laplace) and transient states. The numerical technique had been used for the thermal flow and cooling curve. The results obtained were considered relevant in the control of the grain size and mechanical properties of the casting.

H. K. Jagdeesh et al. [7] studied the three main concepts of heat transfer, i.e. conduction, convection and radiation to find the heat transfer happening in the exhaust heat shield. The project mainly focused on thermal behavior of the exhaust heat shield. Thermal analysis was carried out with the boundary condition for a realistic working condition. Experimental test was carried out by using the prototype for a prescribed condition (more than room temperature) and the same thing was simulated. The temperature distribution was also shown through the real time prototype demo. The finite element results exhibited the temperature distribution happening in the heat shield. The results were validated through some real time instruments which in turn build the prototype. The dynamic behavior of the component was found by doing modal analysis.

The natural frequency was safe enough to fit in with automobile assembly. H.C. Sun and L.S. Chao [8] performed an experimental investigation to measure the temperature distribution within a casting system comprising a cylindrical aluminum casting and a green-sand mold. The numerical results obtained for the temperature curves in the green-sand mold were found to agree well with the experimental profiles. Finite element simulation was performed to model the evolution of the temperature distribution within the casting during the solidification process. The predicted solidification time is found to be in reasonable agreement with that observed in the experimental casting process.

A. Polozine and L. Schaeffer [9] presented methods for determining the thermal parameters of metals. The study testing was in the range of moderate temperatures (300–500 $^{\circ}$C), and high temperatures (700–950 $^{\circ}$C). The obtained results showed that the accuracy of the approximate method was low in the range of moderate temperatures, because the thermal losses by convection were comparable with the simultaneous thermal losses by radiation in this range. Therefore, any error in determining the convection causes the considerable error for the respective calculated emissivity. Accuracy of the approximate method is satisfactory in the range of high temperatures, because the thermal losses by convection are considerably less than the simultaneous thermal losses by radiation. Therefore, the influence of an uncertain convection value on the calculated emissivity is insignificant. However, the method is too laborious. In view of this fact, the possibility of applying the exact method in the industry depends on the development of relevant software. Both methods can be used to calculate of average values of the mentioned thermal parameters of metals.

Heat transfer is commonly encountered in engineering systems and other aspects of life, and one does not need to go very far to see some application areas of heat transfer.

B. Mezrag et al. [10] performed an investigation for the cold metal transfer process to join Zinc coated Steel with Aluminum alloy by braze-welding. A 4043 filler metal is deposited on the surface of the coated Steel, and the effect of the current waveform on the metal transfer and the heat transferred to the base metal was investigated. They concluded that heat transferred to the base metal, and thus the thickness of inter metallic layer formed at the Al/steel interface, was lower for an equivalent deposit rate when the short-circuit frequency was high.

For heat transfer situation, the general energy equation could be solved using finite volume method. Equation (1) represents the three-dimensional unsteady state (time dependent) energy equation.

It could be noted that in many applications, equation (1) solved for steady state.

$$\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\vec{v}\rho h) = \nabla \cdot (k\nabla T) + S_h \qquad (1)$$

Where ρ, h, S_h, $\vec{v}$ and k are density, enthalpy, volumetric heat source, velocity and thermal conductivity respectively.

Heat transfer field is still wide and vital for different applications and investigations. For the case of a rod positioned between two constant temperatures, the steady state differential equation based on heat conservation [11], can be given as:

$$\frac{d^2T}{dx^2} + \acute{h}(T_a - T) = 0 \qquad (2)$$

Where (T) is the temperature of the Aluminum, (T_a) is the temperature of the air and h' is the heat transfer coefficient.

Equation (2) includes the heat flow through the rod as well as between the rod and the surrounding air. The second part of equation (2) is the steady state form of Newton's law of cooling as it is usually quoted. The bar over h' indicated that it is an average over the surface of the body [12].

An alternative discretization method is based on the idea of regarding the computation domain as subdivided into a collection of finite volumes. In this view, each finite volume is represented by a line in one-dimension, an area in two-dimensions and a volume in three-dimensions. Nodes located inside each finite volume become the locus of computational values. Discretization equations are obtained by integrating the original partial differential equation over the span of each finite volume. The method is easily extended to nonlinear problems [1-3].

Computational Details

In the present study the heat transfer and temperature distribution investigations onto an Aluminum rod have been performed. Finite volume and finite difference have been used and compared with the exact solution. The Aluminum rod length and diameter were (10 cm) and (1 cm) respectively. The Aluminum rod has been set into a computational domain of air for the convection case to be taken into account. The computational domain of the air length and diameter were (12 cm) and (2 cm) respectively. The left side of the Aluminum rod has been set at temperature of (700 K) and the right side at (300 K). The surrounding computational domain of the air has been set at (300 K) as illustrated in Figure 1.

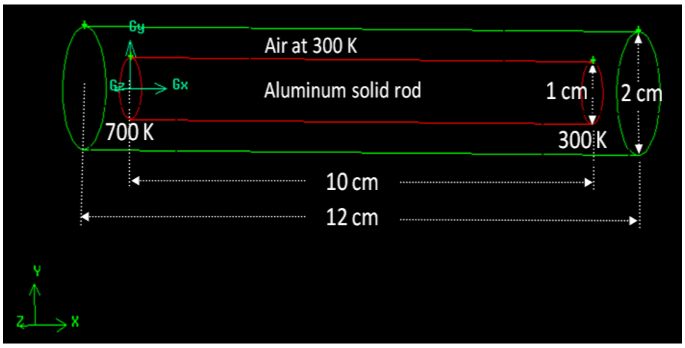

Fig.1. Sketch of the Aluminum rod and the air computational domain.

The computational domain and Aluminum rod undergo meshing process in three dimensions to calculate the temperature distribution at any point on the Aluminum rod using finite volume method. In addition, Aluminum rod of (10 cm) in length has been divided into ten sections of (Δx=1) as illustrated in figure 2 to calculate the temperature distribution at different positions (T_1, T_2….T_9) using finite difference method.

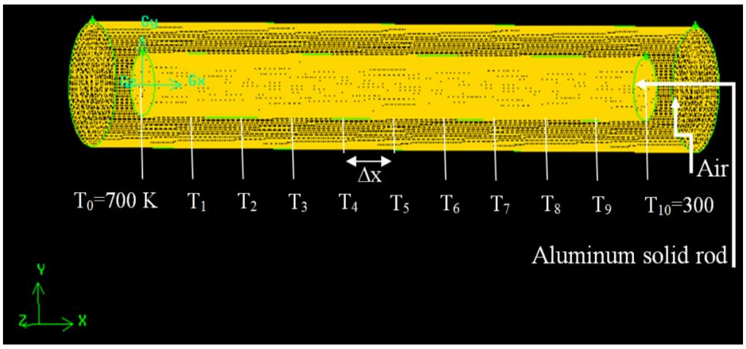

Fig.2. Meshing of the three-dimensional Aluminum rod and the air computational domain.

Results and Discussion

1. Finite Difference Method

The Aluminum rod has been divided into ten sections to calculate the temperature distribution at different positions on the rod assigned as (T_1, T_2….T_9) as mentioned before. The basic idea behind the finite difference method is to replace the various derivatives appearing in the mathematical formulation of the problem by suitable approximations on a finite difference mesh of nodes. Thus rewriting equation (2) numerically gets:

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} + \acute{h}(T_a - T_i) = 0 \qquad (3)$$

After substitution for the boundary conditions of (T_0=700 K), (T_{10}=300 K), (Δx=1) and the typical value of (h'=0.01) in equation (3), all the discretization schemes lead to the following equation:

$$AT = B \qquad (4)$$

In matrix notation, equation (4) could be written as follow:

$$\begin{bmatrix} 2.01, -1,0,0,0,0,0,0,0 \\ -1,2.01, -1,0,0,0,0,0,0 \\ 0, -1,2.01, -1,0,0,0,0,0 \\ 0,0, -1,2.01, -1,0,0,0,0 \\ 0,0,0, -1,2.01, -1,0,0,0 \\ 0,0,0,0, -1,2.01, -1,0,0 \\ 0,0,0,0,0, -1,2.01, -1,0 \\ 0,0,0,0,0,0, -1,2.01, -1 \\ 0,0,0,0,0,0,0, -1, 2.01 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 703 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 303 \end{bmatrix} \qquad (5)$$

Where the matrix A, represents the numbers multiplied by the values of T in the system of linear equations.
Matrix inversion gives the solution:

$$T = A^{-1}B \qquad (6)$$

The values of the calculated temperatures by finite difference method along the Aluminum rod resulting from equation (6) have been depicted in table 1.

The temperature values versus the positions along the Aluminum rod in one dimension have been illustrated in Figure 3.

2. Exact Solution

For comparison purpose, it is more convenient to calculate equation (2) by use of exact method. Fundamentals of differential equations have been utilized in current study to obtain the following solution:

$$T = -62.607e^{0.1x} + 462.607e^{-0.1x} + 300$$

The exact solution of the above equation has been shown in Figure (4).

From the first glance, it could be noted that there is good approach between the exact and finite difference solutions.

Table 1. The calculated temperature values by finite difference method along the Aluminum rod.

T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
700.00	**649.40**	**602.29**	**558.21**	**516.71**	**477.38**	**439.82**	**403.66**	**368.53**	**334.09**	**300.00**

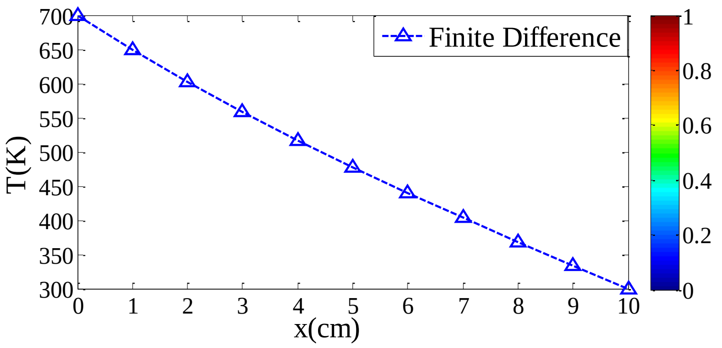

Fig.3. Temperature distribution onto the Aluminum rod by finite difference.

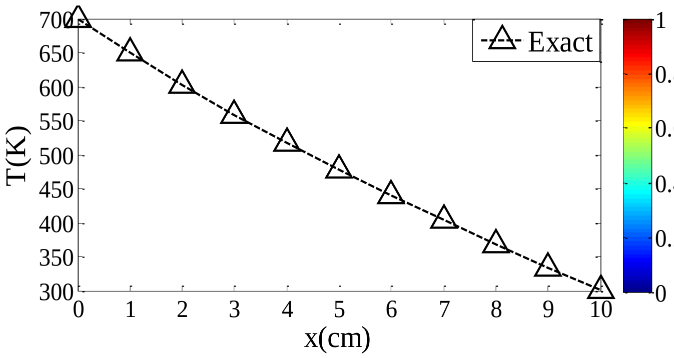

Fig.4. Temperature distribution onto the Aluminum rod by exact solution.

3. Finite Volume

The first step of the finite volume discretization process, the governing equations are integrated over the finite volumes into which the domain has been subdivided, then the Gauss theorem is applied to transform the volume integrals of the convection and diffusion terms into surface integrals. Surface and volume integrals are transformed into discrete ones and integrated numerically through the use of integration points. Equation (1) has been solved for steady state case numerically by means of computer-based simulation. The contour of the three-dimensional temperature distribution of the Aluminum rod has been illustrated in Figure (5). The result shows temperature gradient along the Aluminum rod because of the heat transfer from the high to low temperature regions. The values of the calculated temperatures have been shown in the color map of Figure (5).

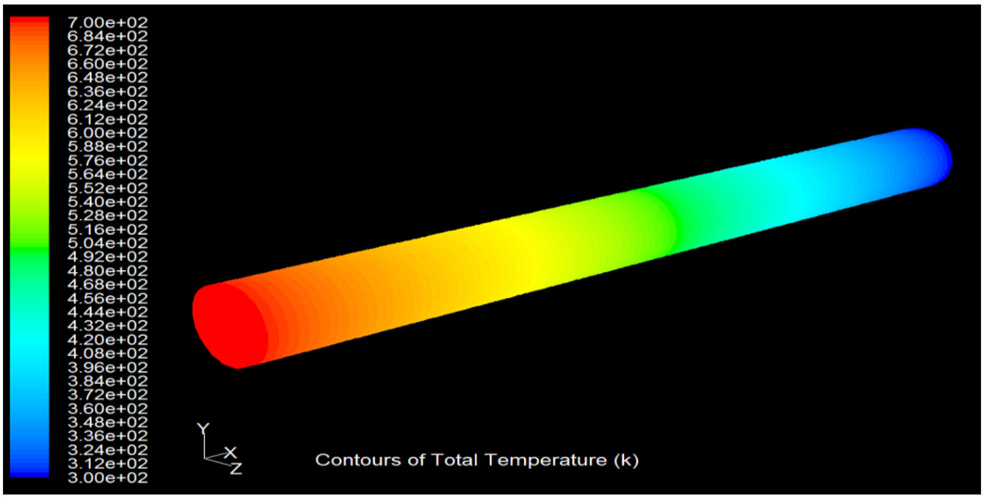

Fig.5. Contour of the three-dimensional temperature distribution of the Aluminum rod.

It is well known that finite volume method is more powerful than any other method. Similar to other numerical methods developed for the simulation, the finite volume method transforms the set of partial differential equations into a system of linear algebraic equations. Thus, the discretization procedure used in the finite volume method is distinctive and involves two basic steps. In the first step, the partial differential equations are integrated and transformed into balance equations over an element. This involves changing the surface and volume integrals into discrete algebraic relations over elements and their surfaces using an integration quadrature of a specified order of accuracy. The result is a set of semi-discretized equations. In the second step, interpolation profiles are chosen to approximate the variation of the variables within the element and relate the surface values of the variables to their cell values and thus transform the algebraic relations into algebraic equations. For comparison with the results obtained by finite difference and exact solution, a plot of the total temperature versus the position along the Aluminum rod in one dimension has been performed by finite volume method and illustrated in Figure (6). Furthermore the visualization of heat transfer and temperature distribution along the Aluminum rod has been illustrated in Figure (7). For more details, the three different methods, finite difference, finite volume and the exact solution have been illustrated in Figure (8). It could be emphasize that there is good approach between finite volume and finite difference especially in the regions near the boundaries of the Aluminum rod. At the midpoints, the finite volume curve is slightly higher than finite difference because of the effect of the volumetric heat source (S_h) in equation (1) which is taken into account in case of finite volume method. Thus it is distinguishable that the volumetric heat source which is responsible for the heat generated inside the material during heat transfer operation can raise the temperature of the material. Also, it should be noted that the thermal conductivity of the Aluminum is highly effective in thermal conductivity situation.

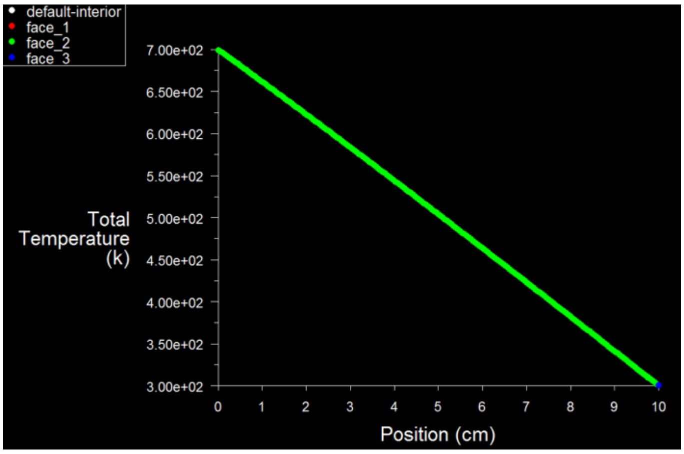

Fig.6. Plot of total temperature versus the position along Aluminum rod obtained by finite volume method.

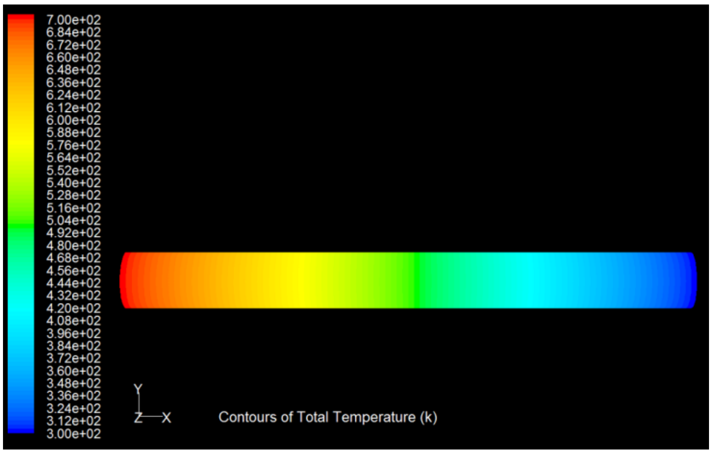

Fig.7. Visualization of heat transfer and temperature distribution along the Aluminum rod obtained by finite volume method.

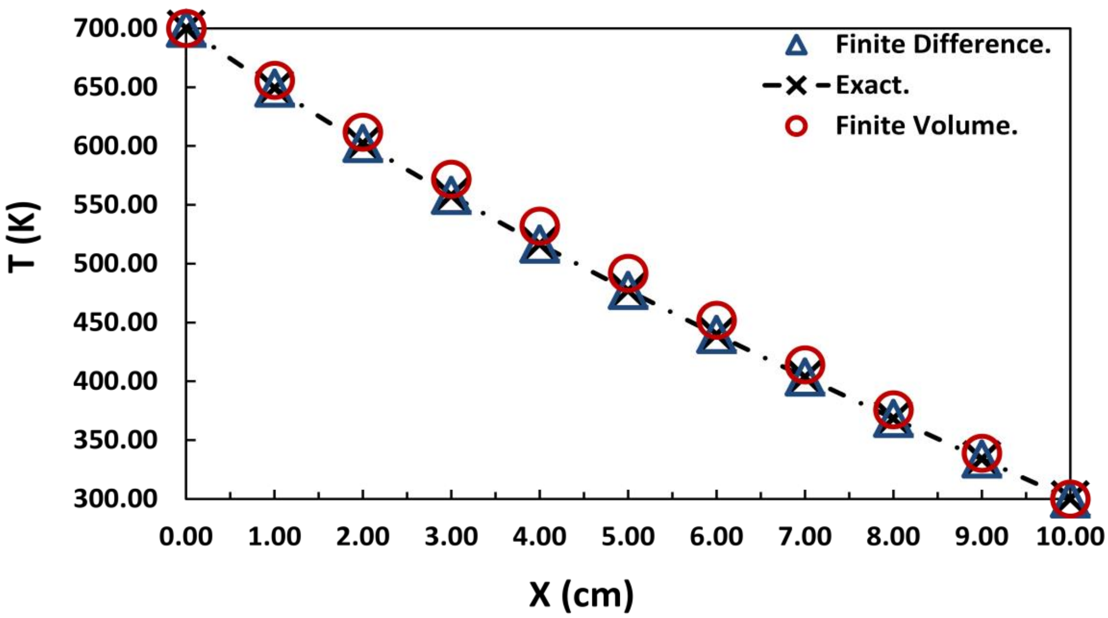

Fig.8. Comparison of temperature distribution along the Aluminum rod for three kinds of solution, finite difference, exact and finite volume.

Conclusion

In the present study, investigations for heat transfer phenomena in a solid Aluminum metal rod have been performed numerically. , both methods have been compared with the exact solution which has been performed in this study. Energy equation has been solved numerically to visualize and calculate the temperature distribution along the Aluminum rod. It is well known that finite volume provides highly sophisticated technique to solve wide variety of applications by means of computer-based simulation. Furthermore in the finite volume methods it is quite easy to implement a variety of boundary conditions, since the unknown variables are evaluated at the centre of the volume elements, not at their boundary faces. These characteristics have made the finite volume method quite suitable Finite volume and finite difference methods have been used as numerical techniques. For more details

for the numerical simulation of a variety of applications involving fluid flow, heat and mass transfer. The results showed that there was good approach between finite volume and finite difference methods in one-dimensional case. Hence, it could be said in addition to the fact that finite volume is crucial technique in all directions of the case under the study, finite difference is still active method in one-dimensional case. Also, it could be concluded that the volumetric heat source, which is responsible for the heat generation inside the material during heat transfer operation can raise the temperature of the material.

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الخلاصة

الهدف من هذا البحث هو دراسة ظاهرة الانتقال الحراري بطريقة التوصيل لقضيب معدني من الالمنيوم. تم استخدام تقنية الحجم المتناهي كطريقة للتحليل العددي. كما تم استخدام طريقة الفرق المتناهي كطريقة اخرى للتحليل العددي من اجل المزيد من التفاصيل. كلتا الطرقتين تم مقارنتهما مع الحل الواقعي الذي تم اجراءه في هذا البحث للحالة الفيزيائية. تم استخدام معدن الالمنيوم في هذا البحث وذلك لاهميته في كثير من التطبيقات. ان طول وقطر قضيب الالمنيوم هما (10 cm) و (1 cm) على التوالي. تم وضع قضيب الالمنيوم داخل مجال من الهواء لاخذ ظاهرة الحمل الحراري في نظر الاعتبار. ان طول وقطرالمجال الهوائي هما (12 cm) و (2 cm) على التوالي. الجانب الايسر للقضيب وضع بدرجة حرارة (700 K) بينما وضع الجانب الايمن بدرجة حرارة (300 K). وضع المجال الهوائي المحيط بدرجة حرارة (300 K). تم حل معادلة الطاقة عدديا باستخدام طريقة الحجم المتناهي و طريقة الفرق المتناهي بالاضافة الى الحل الواقعي من اجل حساب واظهار توزيع درجات الحرارة على طول قضيب الالمنيوم. اظهرت نتائج الحل العددي وجود تقارب كبير بين تقنيتي الحجم المتناهي والفرق المتناهي. اظهرت النتائج ان مصدر الحرارة الحجمي يرفع درجة الحرارة للمادة اثناء عملية الانتقال الحراري. كما اظهرة النتائج تقارب جيد بين الطرق الثلاث المستخدمة, طريقة الحجم المتناهي ,طريقة الفرق المتناهي و الحل الواقعي للحالة الفيزيائية .

الكلمات الدليلية: طريقة الحجم المتناهي , طريقة الفرق المتناهي, انتقال الحرارة بالتوصيل.