Eng. & Technology, Vol.24, No.5, 2005

Dynamic Analysis of Box-Girder Bridges Using a Higher Order Finite Strip Formulation

Dr.A. A. Abdul-Razzak & A. A. Mohammed*

Received on: 28/2/2004
Accepted on:18/5/2005

Abstract

In this paper a higher order finite strip formulation based on the auxiliary nodal line (ANL) technique for both free and forced vibration analysis of box-girder bridges is presented. The free vibration analysis has been performed using the subspace iteration method. The dynamic response of both stiffened plate and box-girder bridge under moving vehicles is investigated.

The vehicle is idealized as a single moving force. Four examples have been studied to show the good performance of the higher order finite strip with one ANL for free and forced vibration analysis of plate, stiffened plate and box-girder bridges. Guyan reduction (mass reduction) technique is adopted to eliminate the auxiliary nodal line parameters for both bending and in-plane actions.

Keywords: *Box-Girder Bridge, Dynamic Response, Finite Strip, Forced Vibration Analysis, Free Vibration Analysis, Moving Force Model.*

التحليل الديناميكي للجسور الرافدية الصندوقية باستعمال صيغة الشرائح المحددة ذات النسق العالي

الخلاصة

في هذا البحث ، تم تقديم صيغة الشريحة المحددة ذات النسق العالي والمعتمدة على تقنية خط عقدي مساعد (ثانوي) لحل الأهتزاز الحر والقسري للجسور الرافدية الصندوقية. تحليل الأهتزاز الحر تم انجازه باستعمال طريقة الفضاء الجزئي التكراريه. تم تقصي الاستجابة الديناميكية للالواح المجسئه والجسور الرافدية الصندوقية تحت عربات متحركة.

العربه تم تمثيلها كقوة متحركه واحدة . أربعة أمثله تمت دراستها لأظهار الفعالية الجيدة للشريحة المحددة ذات النسق العالي مع خط عقدي مساعد (ثانوي) واحد في التحليل الأهتزازي الحر والقسري للألواح والألواح المجسئه والجسور الرافدية الصندوقية. تقنية تقليل كايون (تقليل الكتلة) أستعملت لحذف معاملات الخط العقدي المساعد (الثانوي) لكلا أفعال الأنحناء والأفعال المستوية.

* Dept. of Civil Engineering, University of Mosul.

https://doi.org/10.30684/etj.24.5.10
University of Technology-Iraq, Baghdad, Iraq/2412-0758

الكلمات الدالة: جسر رافدي صندوقي، استجابه ديناميكية، شريحه محدده، تحليل اهتزاز قسري، تحليل اهتزاز حر، موديل القوة المتحركة.

Notation

a	Length of strip
$[A]$	Damping matrix
b	Width of strip
$[B]$	Strain matrix
$[C]$	Coefficient matrix for the displacement function
$[D]$	Rigidity matrix
E	Modulus of elasticity
$\{f\}$	Displacement function
$\{F\}$	Force vector
h	Thickness of strip
k_m	$= m\pi / a$
m	Particular harmonic number
$[M]$	Mass matrix
$\{M\}$	Vector of bending moments
$[N]$	$=[C]\sin k_m y$
r	Specified number of harmonic terms in a solution
$[R]$	Transformation matrix
$[S]$	Stiffness matrix
t	Time
v	Velocity of vehicle
$\{W\}$	Vector of nodal displacements
y	Span-wise direction
Y_m	Harmonic function
β, γ	Parameters of Newmark time integration scheme
$\{\phi\}$	Vector of curvatures
$\{\psi\}$	Eigen vector
ρ	Mass per unit volume
υ	Poisson's ratio

ω Natural frequency (rad/sec)

Introduction

It is well recognized that the dynamic response analysis of box-girder bridges (as a structure of considerable complexity) which exhibit constant material and geometrical properties in the longitudinal direction can be easily simplified by using the finite strip simulation. The basic function of a box-girder bridge is to carry traffic. Bridges are subjected to dynamic loads in the form of vehicular traffic, which causes them to vibrate. The top flange of a box-girder bridge is loaded by groups of either point loads or patch loads, which represents the wheel loading of the moving vehicles. For simplicity, engineers increase the static live load on bridges by a factor called the impact factor (*I*) to account for the dynamic behavior.

The free vibration analysis must be made first before the forced vibration analysis in order to determine the fundamental natural frequency of the bridge to avoid resonance. Moreover, the effect of bridge damping on the dynamic response depends on the assumption used for the bridge damping. Viscous damping usually assumed for bridges and its formulation is based on the determination of the first two natural frequencies of the bridge.

During the past decades, extensive works have been undertaken to study the factors affecting the dynamic behavior of bridges. Hutton and Cheung [1] presented a lower order finite strip solution of the dynamic response of slab-on-girder and box-girder bridges. Canet *et al.* [2] in 1989 used the simple two node linear strip element for dynamic response of thick and thin prismatic shell type structures. Examples for free and forced vibration analysis of the plates and bridges have been presented. In 1997, Senthilvasan *et al.* [3] developed a spline finite strip analysis for dynamic response of curved box-girder bridges. The bridge was considered as an assemblage of curved folded plates discretized by spline finite strip. Natural frequencies of curved box-girder bridge has been calculated and dynamic response to a moving vehicle is also carried out in their study. It was included in their study that the dynamic response increases with the speed of the vehicle; and the dynamic deflections in some cases are lower for higher speed of the vehicle.

An investigation of the effect of dynamic loads on the dynamic amplification factors of an existing continuous bridge has been presented in 1998 by Fafard *et al.* [4]. Experimental testing has been conducted and two vehicle models, having seven and eleven DOF, are considered in the analysis of the vehicle-bridge interaction. A finite element model to analyze the bridge has been developed. It is concluded that the current design codes tend to underestimate dynamic amplification factors, especially for long-span continuous bridges.

In the present study, a higher order finite strip with sixth order bending strips combined with third order in-plane displacement function have been used for the dynamic response of box-girder bridges in both free and forced vibration analysis. The vehicle has been represented by single moving force with the same velocity as the vehicle.

Basic Concepts

Derivation of the Stiffness Matrix

The finite strip method is based on combining Fourier expansions along the longitudinal "prismatic" direction of the structure with polynomials along the transverse direction [2]. Thus, the finite strip method may be thought of as a special form of the finite element method. The procedure assumes that the structure under analysis is divided into a number of narrow longitudinal finite strips, each of which may have independent material and geometric properties [1].

The higher order finite strip with one auxiliary nodal line (ANL) used herein has (6+3) order of polynomial functions which incorporates the 6^{th} order strip for bending combined with an in-plane 3^{rd} order strip.

The 6^{th} order bending displacement function for a simply supported strip with one ANL can be given as [5]

$$w(x,y) = \sum_{m=1}^{r} \left[C^b\right]\left\{W_m^b\right\} \sin k_m y \qquad (1)$$

in which:

$k_m = m\pi / a$

y : Span-wise direction

a : Span length

r : Number of harmonics

$[C^b] = [C_1, C_2, ..., C_7]$ are coefficients (from boundary conditions) and functions of x

only; and can be defined as following:

$$C_1 = 1-39S^2+162S^3-276S^4 +216S^5-64S^6$$
$$C_2 = x(1-8S+25S^2-38S^3 +28S^4-8S^5)$$
$$C_3 = 48S^2-224S^3+432S^4 -384S^5+128S^6$$
$$C_4 = x(-8S+32S^2-40S^3+16S^4)$$
$$C_5 = x^2(2-12S+26S^2-24S^3+8S^4)$$
$$C_6 = -9S^2+62S^3-156S^4 +168S^5-64S^6$$
$$C_7 = x(S-7S^2+18S^3-20S^4+8S^5) \quad (2)$$

in which $S = x/b$ and b is width of the strip.

For an orthotropic strip, the curvature and moment vectors are given by:

$$\{\phi\} = \sum_{m=1}^{r}\left[B_m^b\right]\left\{W_m^b\right\} \quad (3)$$

$$\{M\} = \left[D^b\right]\ \{\phi\} \quad (4)$$

or

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix} \begin{Bmatrix} -\partial^2 w/\partial x^2 \\ -\partial^2 w/\partial y^2 \\ 2\partial^2 w/\partial x\partial y \end{Bmatrix} \quad (5)$$

where $D_x, D_y, \ldots$ etc, are the bending rigidities of the orthotropic plate and $\left[D^b\right]$ is bending rigidity matrix. The total potential energy of a strip under load function $q(x,y)$ can be expressed as:

$$U^p = \frac{1}{2}\int_0^a\int_0^b \{M\}^T\{\phi\}\,dx\,dy - \int_0^a\int_0^b q(x,y)\{W\}^b\,dx\,dy \quad (6)$$

Substituting equations (3)-(5) into (6) and minimizing the resulting expression with respect to all the deformation amplitudes; leads to, and for a particular harmonic m,

$$\left[S_m^b\right]\ \left\{W_m^b\right\} = \left\{F_m^b\right\} \quad (7)$$

where $\left[S_m^b\right]$ and $\left\{F_m^b\right\}$ are the bending stiffness and force matrices, respectively. Using a similar procedure as outlined previously for the plate formation, the in-plane strip stiffness and force matrices can be given:

$$\left[S_m^p\right]\left\{W_m^p\right\} = \left\{F_m^p\right\} \quad (8)$$

The direction cosine matrix will transfer the local force system

$\{F_m\}$ to the global system $\{\overline{F}_m\}$, and couples the bending and in-plane actions, where:

$$\{F_m\}=[R]^T\{\overline{F}_m\} \quad (9)$$

and

$$\{W_m\}=[R]^T\{\overline{W}_m\} \quad (10)$$

but

$$[S_m]\,\{W_m\}=\{F_m\} \quad (11)$$

Substituting equations (9) and (10) into (11) gives

$$[\overline{S}_m]\,\{\overline{W}_m\}=\{\overline{F}_m\} \quad (12)$$

where

$$[\overline{S}_m]=[R][S_m][R]^T \quad (13)$$

is the global stiffness matrix.

Derivation of the Consistent Mass Matrix

The displacement function of any strip has the general form as following [6]:

$$\{f\}=[N]\{W\}=\sum_{m=1}^{r}[N_m]\{W_m\} \quad (14)$$

where $[N_m]$ combines together the series and the shape functions and can be given as:

$$[N_m]=[C]Y_m=[C]\sin k_m y \quad (15)$$

in which $[C]$ represents coefficients matrix for either bending or in-plane displacement functions. Thus the basic unit submatrix in a consistent mass matrix is [6]:

$$[M]_{mn}=\int \rho\, h[N]_m^T\,[N]_n\, d\,(\text{area}) \quad (16)$$

where ρ is the mass per unit volume and h the thickness of strip.

Based on equation (16), the consistent mass matrix of higher order strip in bending can be given as

$$[M^b]_m=\int_0^b [C]^T[C]\int_0^a \rho\, h\sin^2 k_m y\, dx\, dy \quad (17)$$

and the consistent mass matrix of higher order in-plane strip can be derived by the same manner with the following expression of $[N]_m$

$$[N]_m=\begin{bmatrix} C_1Y_m & 0 & C_2Y_m & 0 \\ 0 & \frac{a}{m\pi}C_1Y_m & 0 & \frac{a}{m\pi}C_2Y_m \end{bmatrix}$$

$$\begin{bmatrix} C_3Y_m & 0 & C_4Y_m & 0 \\ 0 & \frac{a}{m\pi}C_3Y_m & 0 & \frac{a}{m\pi}C_4Y_m \end{bmatrix}$$

$$\begin{bmatrix} C_3Y_m & 0 & C_4Y_m & 0 \\ 0 & \frac{a}{m\pi}C_3Y_m' & 0 & \frac{a}{m\pi}C_4Y_m' \end{bmatrix} \quad (18)$$

in which C_1, C_2, C_3 and C_4 are

in which C_1, C_2, C_3 and C_4 are defined as following for the in-plane analysis

$$C_1 = 1 - 5S + 8S^2 - 4S^3$$
$$C_2 = 4S - 4S^2$$
$$C_3 = (-2S + 6S^2 - 4S^3)\, b$$
$$C_4 = S - 4S^2 + 4S^3 \qquad (19)$$

where $S = x / b$

Guyan Reduction

A popular method of reduction is the static condensation method. This method, though simple to apply, is only approximate and may produce relatively large errors when applied to dynamic problems [7]. For dynamic analysis, a similar type of condensation was introduced by Guyan [8].

This method, called "Guyan Reduction" method or "Mass Reduction" method [9], will be used in this study to eliminate the auxiliary nodal line parameters for both bending and in-plane actions. For more details about Guyan reduction, readers are referred to the [8,9]. However, the resulting system matrices are identical to those of a lower order finite strip.

Modeling of the Vehicle.

A moving vehicle represents a highly complex dynamic system. Most heavy vehicles consist of several major components, such as tractors, trailers and suspension systems. Vehicle models with multi- degrees of freedom usually used to study the dynamic behavior of the vehicle system and rider comfort. The vehicle model used in this paper includes moving force travel the bridge by the same velocity of the vehicle and with magnitude represents the vehicle weight. This model is called "Single Moving Force Model" and is used to simplify the dynamic problem in order to understand the dynamic response of the bridges more than that of the vehicles. It is assumed that the moving force will remain in contact with the bridge surface, therefore, no jumps occur between the moving force and the bridge.

Equations of the Dynamic Response

Forced Vibration Analysis

The forced vibration equations defining motion in the mth longitudinal mode may be expressed as:

$$[M_B]_m \{W''\}_m + [A_B]_m \{W'\}_m + [S_B]_m \{W\}_m = \{F_B\}_m$$

(20)

in which $\{W\}_m$ represents the nodal global degrees of freedom including both bending and in-plane effects. $[M_B]$, $[A_B]$ and $[S_B]$ are the mass, damping and stiffness matrices of the structure, respectively; $\{W''\}_m$ and $\{W'\}_m$ are the acceleration and velocity amplitude vectors respectively. The load vector $\{F_B\}_m$ represents the nodal loads (in the mth longitudinal mode) caused by the presence of vehicles upon the deck [1], which varies as the vehicle traverses across the bridge.

Solution of the Equation of Motion

Equation (20) represents the matrix differential equation of system motion. Let $\{\Delta W\}$ denote the increment in $\{W\}$ occurring during the time step from t to $t+\Delta t$. By Newmark's finite difference scheme, the vector $\{W\}$ and its derivatives at the instant $t+\Delta t$ can be related to those at the instant t [10]:

$$\{W''\}_{t+\Delta t} = a_0\{\Delta W\} - a_2\{W'\}_t - a_3\{W''\}_t \quad (21a)$$

$$\{W'\}_{t+\Delta t} = \{W'\}_t + a_6\{W''\}_t + a_7\{W''\}_{t+\Delta t} \quad (21b)$$

and

$$\{W\}_{t+\Delta t} = \{W\}_t + \{\Delta W\} \quad (21c)$$

where the quantities with subscript t are those occurring at time t, assumed to be known. Using Newmark's parameters β and γ, the coefficients and those to be used later can be given as [11]:

$$a_0 = \frac{1}{\beta(\Delta t)^2} \qquad a_1 = \frac{\gamma}{\beta t}$$

$$a_2 = \frac{1}{\beta \Delta t}$$

$$a_3 = \frac{1}{2\beta} - 1 \qquad a_4 = \frac{\gamma}{\beta} - 1 \quad (22)$$

The parametric values of $\beta =$ 0.25 and $\gamma =$ 0.5 are used throughout, implying that the marching scheme is unconditionally stable.

After calculate the initial acceleration vector $\{W''\}_0$ at time $t = 0$:

$$\{W''\}_0 = [M_B]^{-1}(\{F(0)\} - [A_B]\{W'\}_0 - [S_B]\{W\}_0) \quad (23)$$

and forming the effective stiffness matrix $\left[\hat{S}_B\right]$:

$$\left[\hat{S}_B\right] = \left[S_B\right] + a_0\left[M_B\right] + a_1\left[A_B\right] \qquad (24)$$

It will be possible, for each time step, to calculate the dynamic response in the following steps:

1. Calculate the effective force vector at time $t+\Delta t$ as following:

$$\left\{\hat{F}_B\right\}_{t+\Delta t} = \left\{F_B\right\}_{t+\Delta t} + \left[M_B\right] + (a_0\{W\}_t + a_2\{W'\}_t + a_3\{W''\}_t) + \left[A_B\right](a_1\{W\}_t + a_4\{W'\}_t + a_5\{W''\}_t) \qquad (25)$$

2. Solve for the displacements at time $t+\Delta t$:

$$\left[\hat{S}_B\right]\{W\}_{t+\Delta t} = \left\{\hat{F}_B\right\}_{t+\Delta t}$$

$$\{W\}_{t+\Delta t} = \left[\hat{S}_B\right]^{-1}\left\{\hat{F}_B\right\}_{t+\Delta t} \qquad (26)$$

Equation (26) sometimes called the pseudo-static equation.

3. Calculate the accelerations and velocities at time $t+\Delta t$ according to equation (21).

Free Vibration Analysis

Making $\left\{\hat{F}_B\right\}_m$ zero in equation (20), the equation of free vibration of the undamped structure is:

$$\left[M_B\right]\{W''\}_m + \left[S_B\right]_m\{W\}_m = 0 \qquad (27)$$

The standard eigenvalue formulation of the problem is therefore [12]:

$$(\left[S_B\right]_m - \omega_m^2\left[M_B\right])\{\psi\}_m = \{0\} \qquad (28)$$

Where ω_m^2 is an eigenvalue and $\{\psi\}_m$ is the corresponding eigenvector. The subspace iteration algorithm [13] is employed for solving equation (28) to determine the natural frequencies and their corresponding mode shapes.

It has been shown that, for each harmonic term, corresponding elements of the stiffness matrix $\left[S_B\right]_m$ will be different. It follows that equation (28) will have to be solved for each harmonic term in the series [12]. The lowest natural frequency for the (m+1)th term will be higher than the lowest natural frequency of the mth term but will usually be lower than the highest natural frequency of the mth term.

Numerical Examples

In order to demonstrate the capability and efficiency of the formulation presented and the reliability of the higher order finite strip with one ANL in dealing with dynamic problems, four typical examples of free and forced vibration of bridges have been studied.

Example 1: Free Vibration of Straight Slab/Beam Bridge

For the slab/beam bridge shown in Fig. 1 with the geometrical and material properties, the transverse section is discretised using 12 higher order strips. In Fig.2 natural frequencies for the lowest five modes of the first harmonic obtained in the analysis are compared well with the results reported by Hinton [14].

Example 2: Free Vibration of Straight Box- Girder Bridge

A single-cell, simply supported and straight box-girder bridge presented by Huang *et al.* [15] is chosen and shown in Fig.3. This concrete box-girder bridge has a span length of 45.72 m (150 ft) and a roadway width of 9.144 m (30 ft). The density of the material used is 2750 kg/m^3 (i.e., ρ = mass density = 0.262 kg-sec^2/m^4). The first six natural frequencies are plotted in Fig.4, which shows a good agreement with the values obtained by thin-walled beam element in Ref.[15].

Example 3: Plate under Moving Force

In this example, a simply supported square plate is subjected to a 0.907 kg moving force. The moving force traverses along the centerline of the plate. The following data are adopted [16]: modulus of elasticity $E = 2.11\times10^6$ kg/cm^2, length a=10 cm, width b=10 cm, thickness h = 0.25 cm, mass density $\rho = 1.1\times10^{-5}$ kg-sec^2/cm^4, Poisson's ratio υ = 0.3. The dynamic amplification factors or normalized displacements (which is defined as the maximum dynamic deflection divided by maximum static deflection) for the moving force on the plate are given in Fig.5. As can be seen, good agreement has been achieved between the present solution and the finite element method.

It can be observed from Fig.6 that the dynamic amplification factors for various velocities of the moving force are in good agreement with the values obtained by the finite element method.

Example 4: Simply Supported Box-Girder Bridge subjected to a Moving Force.

Consider a simply supported box-girder bridge subjected to a constant force of (P=500 kN) moving at a constant speed of (v=60 mph) (26.82 m/s), as shown in Fig.7. The following data are used in this example: span length = 50 m (164 ft), Young's modulus E = 3.34×10^{10} N/m^2, mass density = 2400 kg/m^3.

The results obtained from the method presented in this paper are compared well with the results obtained from the simple beam theory (where the bridge is simulated as a simply supported beam with 40-element model).

The deflection time-history of the midpoint of the bridge due to moving force for both top and bottom flanges is plotted in Fig.8, along with analytical solution given in [17] based on the simple beam approximation of the box-girder bridge.

The results obtained from Ref. [17] referred to the point at the centriod of the box-girder cross-section due to the simple beam theory used. Therefore, the results by the present method are given for the top and bottom flanges in order to match the case of centriod response as in the beam theory.

As can be seen, good agreement has been achieved between the present solution and the analytical ones. The deflection response at point (span/5) from end is also given in Fig.9, which shows again the good performance of the present formulation.

Conclusions

In this paper a higher order finite strip formulation for dynamic analysis of box-girder bridges has been presented. The method is also extended to the dynamic analysis of flat plates and stiffened panels. A comparison of the present results with those in the literature shows that the subspace iteration technique and the single moving force model can satisfactorily simulate the dynamic behavior of box-girder bridges. The model of the simple moving force (without springs, dampers, etc.) has been easily used to investigate the dynamic deflections and amplifica-tion factors (normalized displacements) of both plates and box-girder bridges. It is also concluded that the dynamic magnification factor increase with the velocity of the moving vehicle up to specific limit then the bridge dynamic response tends to decrease with the vehicle speed increases. The method of Guyan reduction has been used successfully to eliminate deformation parameters at the auxiliary nodal line. Due to the accuracy of the results obtained by this model; it can be concluded that the higher order (6+3) finite strip with one ANL, in conjunction with the single moving force simulation, can be used successfully in the dynamic analysis and design of simply supported box-girder bridges.

References

1. S. G. Hutton and Y. K. Cheung, Dynamic response of single span highway bridges. Earthquake Engineering and Structural Dynamics. Vol. 7, 543-553, 1979.
2. J. M. Canet, B. Suarez and E. Onate, Dynamic analysis of structures using a Reissner-Mindlin finite strips formulation. *Computers & Structures*, Vol.31, No. 6, 967-975, 1989.
3. J. Senthilvasan, G. H. Brameld and D. P. Thambiratnam, Bridge-vehicle interaction in curved box girder bridges. *Microcomputers in Civil Engineering*, Vol. 12, 171-181, 1997.
4. M. Fafard, M. Laflamme, M. Savard and M. Bennur, Dynamic analysis of existing continuos bridge. ASCE, J. bridge engineering, Vol. 3, No. 1, 1998.
5. M. A. Abdullah and A. A. Abdul-Razzak, Finite strip analysis of prestressed box-girders. Computers & Structures, Vol. 36, No. 5, 817-822, 1990.
6. Y. K. Cheung, *The Finite Strip Method in Structural Analysis*. Pergamon Press, Oxford (1969).
7. M. Paz, *Structural Dynamics: Theory and Application*. CBS Publishers and Distributors, Delhi (1987).
8. R. J. Guyan, Reduction of stiffness and mass matrices. AIAA J., Vol. 3, No.2, 1965.
9. W. Weaver and P. R. Johnston, *Structural Dynamics by Finite Elements*. Prentice-Hall, Inc., New Jersey (1987).
10. Y. B. Yang and Y. S. Wu, A versatile element for

analyzing vehicle-bridge interaction response. Engineering Structures, Vol. 23, 452-469, 2001.
11. J. W. Tedesco, W. G. Mcdougal and C. A. Ross, *Structural Dynamics: Theory and Applications*. Addison Wesley Longman, Inc., California, 1999.
12. J. W. Smith, Finite strip analysis of the dynamic response of beam and slab highway bridges. Earthquake Engineering and Structural Dynamics, Vol. 1, 357-370, 1973.
13. K. J. Bathe, *Finite Element Procedures*. Prentice-Hall, Inc., New Jersey, 1996.
14. E. Hinton, *Numerical Methods and Software for Dynamic Analysis of Plates and Shells*. Dotesios Printers Ltd., Wiltshire, 1988.
15. D. Z. Huang, T.L. Wang and M. Shahawy, Vibration of thin-walled box-girder bridges excited by vehicles. ASCE, J. Structural Engineering, Vol. 121, No. 9, 1995.
16. M. R. Taheri and E. C. Ting, Dynamic response of plates to moving loads: finite element method. Computers and Structures, Vol. 34, No. 3, 509-521, 1990.
17. M. J. Inbanathan and M. Wieland, Bridge vibrations due to vehicles moving over rough surface. ASCE, J. Structural Engineering, Vol. 113, No. 9, 1987.

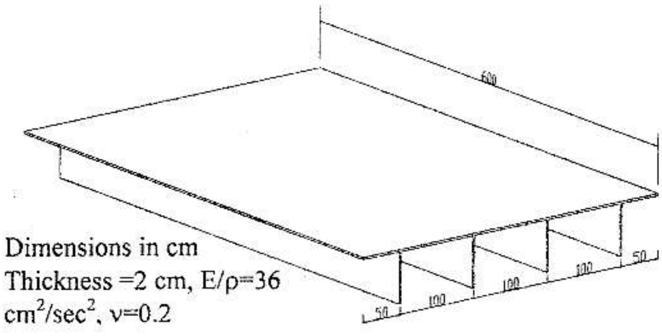


Fig. 1. Straight Slab/Beam Bridge (Example 1)

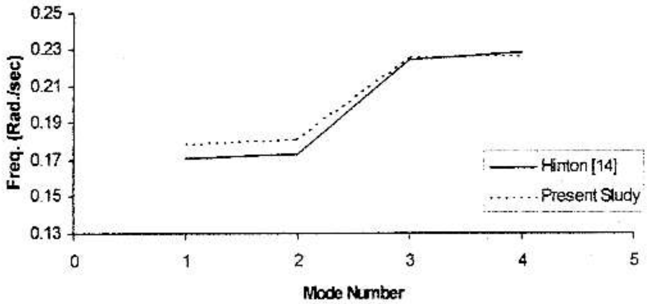


Fig. 2. Natural Frequencies of Straight Slab/Beam Bridge (first mode)

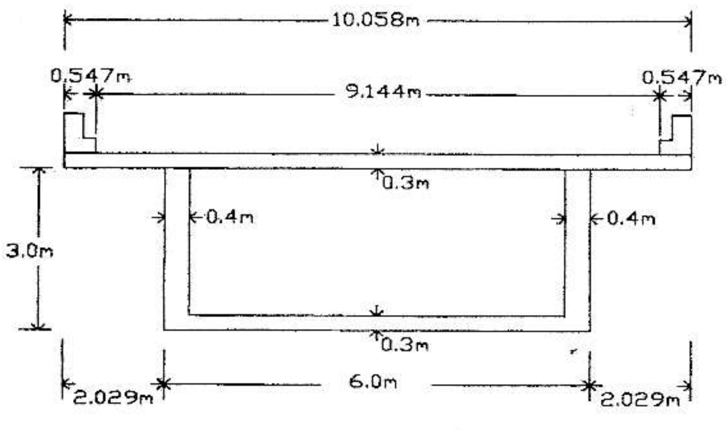


Fig.3. Simply Supported Box-Girder Bridge (Example 2)

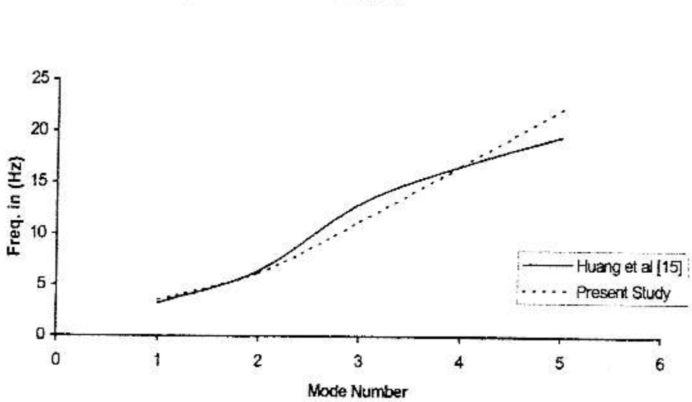


Fig. 4.Natural Frequencies of Simply Supported Box-Girder Bridge

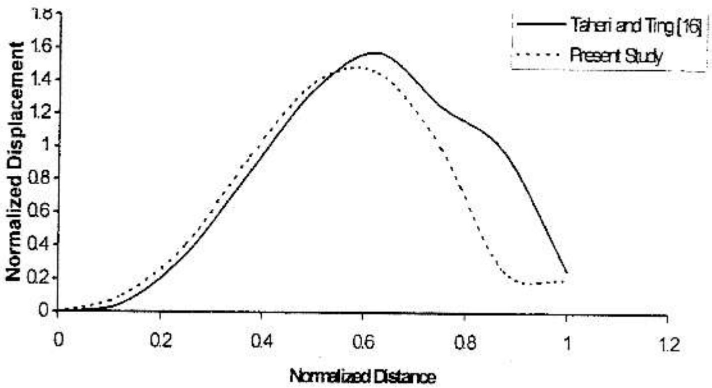


Fig.5. Normalized Displacements of a Square Plate under Moving Force

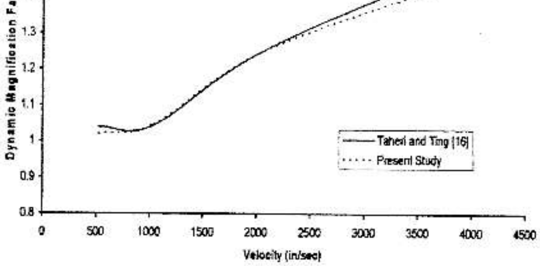
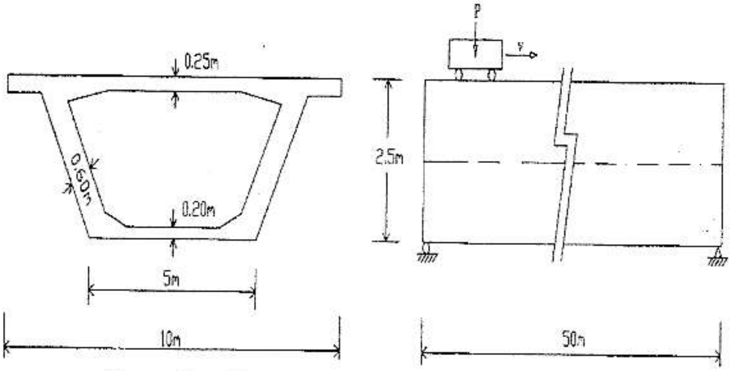


Fig. 6. Dynamic magnification factors for moving force on plate



Cross-Section

Fig. 7. Simply Supported Box-Girder Bridge under Moving Force (Example 4)

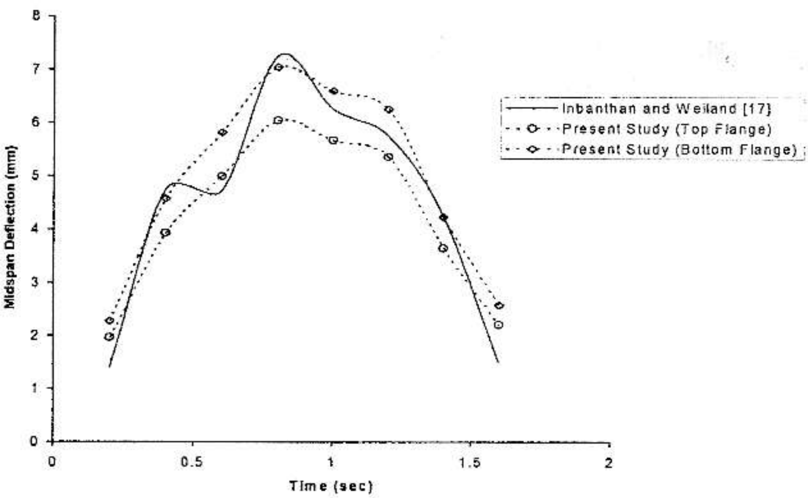


Fig. 8 Deflection Response of Box-Girder Bridge at Midspan due to Moving Force

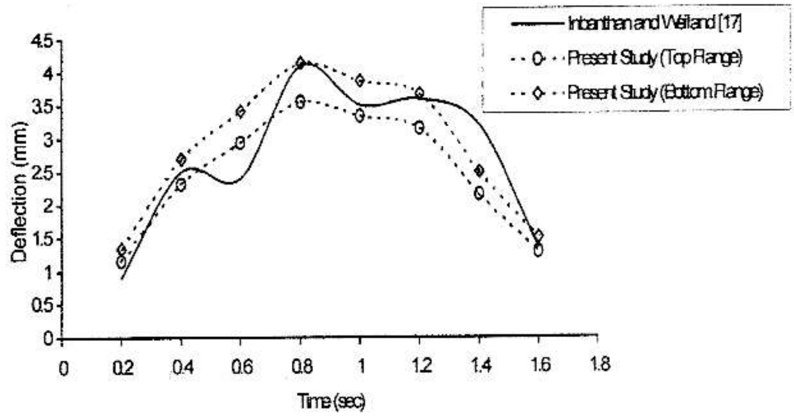


Fig. 9. Deflection Response of Box-Girder Bridge at Point L/5 from End due to Moving Force