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# A modification of Dai and Yuan Method for Solving Unconstrained Optimization Problems

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**Abstract**

The Conjugate Gradient (CG) method is a widely used and efficient approach for solving large-scale unconstrained optimization problems due to its simplicity and low memory demands. Despite its success, ongoing research seeks to enhance its performance and robustness. This paper proposes a novel parameter modification to the classical Dai-Yuan CG formula, aiming to improve numerical behavior, reduce iteration counts, and boost convergence speed and stability. Theoretical analysis confirms that the new method ensures strong global convergence and descent properties.

***Keywords***:
Conjugate gradient method, global convergence, sufficient descent condition.

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## I. Introduction

Conjugate gradient (CG) method is a typical method to solve large scale unconstrained optimization problems in the form:

$$\min f(x)_{x\in R^n} \quad (1)$$

Where $f: R \rightarrow R^n$ is differentiable and continuously. Using the iterative form:

$$x_{k+1} = x_k + \propto_k d_k, k = 0,1,2,\ldots \quad (2)$$

Where $x_k$ is the $k_{th}$ iterative point and $d_k$ is the search direction, $\propto_k > 0$ represented the step length and $d_k$ is calculated by:

$$d_{k+1} = \begin{cases} -g_{k+1} & if\ k = 0 \\ -g_{k+1} + \beta_k d_k & if\ k > 0 \end{cases} \quad (3)$$

$\beta_k \in R$ Defined as a coefficient for the CG method [1,2], Note that they are of various formats and have undergone many and various modifications. Here we mention the most famous of them: Fletcher-Reeves (FR)[3],Polark-Ribi`ere-Polyak(PRP)[4],Hestenes-Stiefel (HS)[5],conjugate-Descent(CD)[6],Liu-Storrey(LS)[7], and Dai-Yuan (DY)[8].

$$\beta_k^{DY} = \frac{\|g_{k+1}\|^2}{d_k^T y_k},$$

Where $g_k$ is the derivative of $f(x)\ at\ x_k$ and

$y_k = g_{k+1} - g_k$ .

Let us now turn our attention to the line search. An efficient strategy. studied by wolf [9], consists in accepting a positive step length $\alpha$ is it satisfies:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \rho \alpha_k g_k^T d_k \quad (4)$$

$$g_{k+1}^T d_k \geq \sigma g_k^T d_k \quad (5)$$

Where $0 < \rho < \sigma < 1$. Often the strong – wolfe line search is used in the implementation of conjugate gradient methods. These are given by (5) and:

$$|g_{k+1}^T d_k| \leq -\sigma g_k^T d_k \quad (6)$$

The global convergence of the $DY$ method has been proven. Noting that it was used many different lines search as exact line search and standred-wolfe line search (SWP) line search.

In our method, we used robust strong –Wolfe line search [11, 12]. The numerical experiments presented demonstrated good efficiency and competitiveness of the new method. The rest is divided as follows: In Section 2 we write the new formulation and algorithm based on modifications of the DY method. In Section 3 we prove the regression and global convergence properties of the proposed method. The numerical results and conclusions will be in Sections 4 and 5, respectively.

## II. The new algorithm

Dai and Yuan [6] proved the global convergence of the conjugate gradient algorithm:

$$|\beta_k| = \theta\beta^{DY} \tag{7}$$

Where $\theta \in [-(\frac{1-\sigma}{1+\sigma}),1]$.on the other hand Dai and Liu in [DL] modified the conjugate condition to the

$y_k^T d_{k+1} = -t s_k^T g_{k+1}$ where $t > 0$ $and$ $s_k = x_{k-1} - x_k$ . Based on the eq. (7) and conjugated condition by Dai-Liu with suitable $t$ we get:

$\theta = \left(\frac{y_k^T g_{k+1}}{\|g_{k+1}\|^2} - \rho\frac{|y_k^T g_k| s_k^T g_{k+1}}{\|g_{k+1}\|^2 s_k^T y_k}\right)$ $where$ $\rho > 0$ and

$\beta^{HDY} = \theta\beta^{DY}$

Hence, we can write $d_k$

$$d_{k+1} = \begin{cases} -g_{k+1} + (\theta\beta^{DY} s_k) \; if \; \theta \in (0,1) \\ -g_{k+1} + \beta^{DY} s_k \quad\quad O.w \end{cases} \tag{8}$$

In this section we suggest a new algorithm by modified (*DY*) method as follows:

Consider $\beta_k^{DY} = \frac{\|g_{k+1}\|^2}{y_k^T s_k}$ the proposed $\beta_k^{HDY}$ will be as follows:

$$\beta_k^{HDY} = \left[\frac{g_{k+1}^T y_k}{y_k^T s_k} - \frac{\rho|y_k^T g_{k+1}| s_k^T g_{k+1}}{(y_k^T s_k)^2}\right] \tag{9}$$

Where $\rho > 0$ , and $d_{k+1} = -g_{k+1} + \beta^{HDY} s_k$

The proposed algorithm will be as follows:

***2.1 Algorithm:***

Step (1): Choose an initial point $x_0$ and $\varepsilon$ sufficiently small. $Set$ $d_0 = -g_0$ $and$ $k = 0$.
Step (2): Test a criterion for stopping the iterations. If this test is satisfied, then stop; otherwise, continue with step (3).

Step (3): Using the Wolfe line search conditions, determine the step size $\alpha_k$ and Compute

$x_{k+1} = x_k + \alpha_k d_k$

Step (4): Compute the conjugate gradient parameter $\beta_k^{HDY}$by eq. (9)

Step (5): Compute the search direction

$d_{k+1} = -g_{k+1} + \beta_k d_k$

Step (6): Restart criterion. $f$ $|g_{k+1}^T g_k| >$ $0.2\|g_{k+1}\|^2$ $or$ $k = n$ then set

$d_{k+1} = -g_{k+1}$ and go to step (2)
Step (7): Set $k = k + 1$ and go to step(3)

## III. Global Convergence Properties

To prove the global convergence of the suggested conjugate gradient method, we will assume the following.

***Assumption 3.1:(see*** [10])

the set $\Omega = \{x \in R^n, f(x) \leq f(x_0)\}$ is bounded if x is the initial point. For some neighborhood ***N*** of the ***set Ω***, assume ***g*** be Lipchitz continuous on **Ω**, i.e., ∃ a positive constant ***L* > 0**, such that:

$$\|g(x) - g(y)\| \leq L\|x - y\|, \forall x, y \in \Omega \tag{10}$$

In addition to this assumption, we may deduce that a positive constant $r$ exists , such that:

$$\|g(x)\| \leq r, \forall x \in \Omega \; and \; \beta > 0, |x| \leq \beta \tag{11}$$

Moreover, when the function is uniformly convex, the following relationship will hold:

$$y_k^T s_k \geq \mu\|s_k\|^2 \tag{12}$$

***Lemma 1***. If we presume that $\alpha_k$ satisfies Strong Wolfe-Powell (SWP), then the

direction of the search,$d_{k+1}$ , which is produced by:

$$d_{k+1} = -g_{k+1} + \beta_k^{HDY} s_k \tag{13}$$

Satisfy the condition:

$$d_{k+1}^T g_{k+1} \leq -c_1\|g_{k+1}\|^2 \tag{14}$$

***Proof:*** from (13), we get:

$$d_{k+1} = -g_{k+1} + \frac{g_{k+1}^T y_k s_k}{s_k^T y_k} - \frac{\rho|g_{k+1}^T y_k| s_k^T g_{k+1}}{(s_k^T y_k)^2} s_k \tag{15}$$

by multiplying through (15) by ($g_{k+1}^T$) gives:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \frac{g_{k+1}^T y_k s_k^T g_{k+1}}{s_k^T y_k} - \frac{\rho|g_{k+1}^T y_k|(s_k^T g_{k+1})^2}{(s_k^T y_k)^2}$$

Since $s_k^T g_{k+1} \leq s_k^T y_k, g_{k+1}^T y_k \leq \|g_{k+1}\|.\|y_k\|$ , we get :

$$\leq -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(s_k^T y_k)}{s_k^T y_k} - \frac{\rho|g_{k+1}^T y_k|(s_k^T y_k)^2}{(s_k^T y_k)^2}$$

$$\leq -\|g_{k+1}\|^2 + \|g_{k+1}\|\|y_k\| - \rho\|g_{k+1}\|\|y_k\|$$

$$\leq -\left(1 - \frac{\|y_k\|}{\|g_{k+1}\|} + \frac{\rho\|y_k\|}{\|g_{k+1}\|}\right)\|g_{k+1}\|^2$$

Since $0 < \frac{\|y_k\|}{\|g_{k+1}\|} < 1$, let $c_1 = \left(1 - \frac{\|y_k\|}{\|g_{k+1}\|} + \frac{\rho\|y_k\|}{\|g_{k+1}\|}\right)$

$\therefore d_{k+1}^T g_{k+1} \leq -c_1\|g_{k+1}\|^2$

***Theorem 1:*** *Assuming assumption (3.1) is valid,* $\alpha_k$

*satisfies Strong Wolfe-Powell (SWP) conditions, and Lemma 1 holds, Then,*

$$\lim_{k\to 0}\|g_k\| = 0 \ ,\forall k \ge 0 \qquad (16)$$

Proof: Reformulating the equation for determining the search direction (13) by

$$d_{k+1} = -g_{k+1} + \beta_k^{HDY} s_k \qquad (17)$$

*Where* $\beta_k^{HDY}$ *is defined by eq. (9), we get*

$$d_{k+1} = -g_{k+1} + \frac{g_{k+1}^T y_k s_k}{s_k^T y_k} - \frac{\rho|g_{k+1}^T y_k| s_k^T g_{k+1} s_k}{s_k^T y_k s_k^T y_k}$$

*Since* $0 < \beta < 1 \to \beta^{HS} = c_2$, *we get*

$$\|d_{k+1}\| \le \|g_{k+1}\| + c_2\|s_k\| - \rho c_2 \frac{s_k^T y_k}{s_k^T y_k} s_k$$

**Table 1.** Comparison concerning (NOI and NOF) for dimensions n=1000, 5000, and 10000.

| Function | N | Problem | Alg. DY | | Alg. CD | | Alg. PRP | | Alg. HDY | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Iter. | Fg. | Iter. | Fg. | Iter. | Fg. | Iter | Fg. |
| 1 | 1000 | Freudenstein & Roth | 15 | 33 | 23 | 39 | 14 | 30 | 35 | 584 |
| | 5000 | | 13 | 27 | 1003 | 27820 | 21 | 43 | 35 | 485 |
| | 10000 | | 13 | 28 | 525 | 15034 | 20 | 42 | 1003 | 28183 |
| 2 | 1000 | Extended Rosenbrock SROSENBR (CUTE) | 41 | 82 | 40 | 75 | 44 | 89 | 35 | 73 |
| | 5000 | | 41 | 90 | 47 | 91 | 41 | 87 | 36 | 79 |
| | 10000 | | 40 | 86 | 43 | 97 | 41 | 87 | 36 | 78 |
| 3 | 1000 | Extended White & Holst | 35 | 69 | 41 | 87 | 35 | 72 | 32 | 68 |
| | 5000 | | 34 | 72 | 34 | 69 | 41 | 91 | 36 | 80 |
| | 10000 | | 34 | 72 | 38 | 81 | 38 | 81 | 27 | 55 |
| 4 | 1000 | Extended Beale BEALE (CUTE) | 15 | 28 | 15 | 29 | 15 | 28 | 14 | 27 |
| | 5000 | | 13 | 24 | 17 | 31 | 16 | 30 | 15 | 28 |
| | 10000 | | 14 | 27 | 14 | 26 | 16 | 31 | 15 | 28 |
| 5 | 1000 | Hager | 29 | 51 | 32 | 53 | 28 | 46 | 27 | 44 |
| | 5000 | | 121 | 2379 | 198 | 5105 | 139 | 3139 | 301 | 8555 |
| | 10000 | | 499 | 14538 | 595 | 17952 | 451 | 13310 | 289 | 7748 |
| 6 | 1000 | Generalized Tridiagonal 1 | 23 | 45 | 23 | 45 | 23 | 45 | 28 | 187 |
| | 5000 | | 24 | 49 | 29 | 140 | 25 | 49 | 26 | 48 |
| | 10000 | | 30 | 130 | 64 | 1236 | 33 | 125 | 29 | 52 |
| 7 | 1000 | Extended Three Expo Terms | 20 | 122 | 11 | 17 | 9 | 14 | 15 | 24 |
| | 5000 | | 26 | 408 | 13 | 21 | 8 | 14 | 21 | 32 |
| | 10000 | | 13 | 75 | 105 | 2927 | 38 | 515 | 18 | 29 |
| 8 | 1000 | Extended Himmelblau HIMMELBC (CUTE) | 11 | 20 | 11 | 20 | 11 | 20 | 11 | 20 |
| | 5000 | | 23 | 36 | 12 | 22 | 11 | 20 | 11 | 20 |
| | 10000 | | 23 | 36 | 23 | 36 | 12 | 22 | 12 | 22 |

$$\le \omega + c_2\|s_k\| + \rho c_2\|s_k\| = D_2 \qquad (18)$$

$$\sum_{k\ge 1}^{\infty} \frac{1}{\|d_{k+1}\|^2} \ge \frac{1}{D_2^2} \sum_{k\ge 1}^{\infty} 1 = \infty$$

## IV. Numerical results

In this section, we presented our numerical results that explain the new algorithm specified by equation (9) is compared to the classical algorithm CD [6]as well as algorithm PRP [4] and algorithm DY [8]. The numerical results showed clear superiority over other algorithms with regard to function evaluations and number of iterations. Many researchers have adopted the principle of modification as well as hybridization to obtain more accurate results. The specific parameters are as follows: $\rho = 0.5$ ,$\varepsilon = 10^{-5}$, and the stopping condition is $\|g_{k+1}\| \le 10^{-6}$. The results are shown in **Table 1**.

| Function | N | Problem | Alg. DY | | Alg. CD | | Alg. PRP | | Alg. HDY | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Iter. | Fg. | Iter. | Fg. | Iter. | Fg. | Iter | Fg. |
| 9 | 1000 | Extended Powell | 403 | 715 | 433 | 739 | 1003 | 1489 | 309 | 539 |
| | 5000 | | 1003 | 1323 | 410 | 749 | 1003 | 1166 | 528 | 911 |
| | 10000 | | 357 | 639 | 502 | 826 | 1003 | 1096 | 651 | 1275 |
| 10 | 1000 | Extended PSC1 | 81 | 153 | 84 | 160 | 65 | 124 | 77 | 143 |
| | 5000 | | 77 | 137 | 78 | 146 | 72 | 132 | 92 | 173 |
| | 10000 | | 99 | 182 | 70 | 129 | 72 | 145 | 89 | 165 |
| 11 | 1000 | Extended Maratos | 72 | 158 | 78 | 174 | 67 | 157 | 68 | 162 |
| | 5000 | | 64 | 139 | 106 | 251 | 79 | 182 | 113 | 349 |
| | 10000 | | 98 | 228 | 120 | 402 | 74 | 165 | 65 | 150 |
| 12 | 1000 | Full Hessian FH1 | 12 | 30 | 11 | 29 | 1003 | 1036 | 12 | 31 |
| | 5000 | | 13 | 27 | 13 | 26 | 17 | 33 | 10 | 23 |
| | 10000 | | 17 | 50 | 14 | 34 | 16 | 39 | 16 | 49 |
| 13 | 1000 | Full Hessian FH2 | 81 | 177 | 86 | 186 | 86 | 186 | 80 | 175 |
| | 5000 | | 81 | 176 | 87 | 187 | 86 | 185 | 78 | 170 |
| | 10000 | | 81 | 174 | 91 | 193 | 86 | 183 | 81 | 174 |
| 14 | 1000 | Full Hessian FH3 | 94 | 195 | 94 | 145 | 96 | 148 | 100 | 152 |
| | 5000 | | 258 | 399 | 254 | 396 | 245 | 391 | 224 | 346 |
| | 10000 | | 366 | 578 | 34 | 535 | 341 | 540 | 30 | 471 |
| 15 | 1000 | Tridiagonal White & Holst (c=4) | 25 | 52 | 29 | 61 | 27 | 58 | 24 | 55 |
| | 5000 | | 72 | 163 | 76 | 182 | 75 | 197 | 72 | 219 |
| | 10000 | | 38 | 91 | 37 | 88 | 37 | 88 | 36 | 86 |
| 16 | 1000 | BDQRTIC (CUTE) | 16 | 31 | 12 | 22 | 12 | 23 | 12 | 23 |
| | 5000 | | 13 | 25 | 12 | 23 | 13 | 25 | 13 | 25 |
| | 10000 | | 16 | 32 | 13 | 26 | 17 | 33 | 15 | 29 |
| 17 | 1000 | FLETCHCR (CUTE) | 86 | 134 | 80 | 123 | 78 | 123 | 83 | 129 |
| | 5000 | | 184 | 284 | 220 | 346 | 163 | 259 | 17 | 286 |
| | 10000 | | 225 | 352 | 250 | 394 | 239 | 373 | 253 | 393 |
| 18 | 1000 | ENGVAL1 (CUTE) | 88 | 137 | 91 | 136 | 8 | 139 | 81 | 119 |
| | 5000 | | 188 | 292 | 199 | 316 | 197 | 317 | 13 | 345 |
| | 10000 | | 263 | 426 | 72 | 454 | 227 | 376 | 33 | 394 |
| 19 | 1000 | DENSCHNA (CUTE) | 31 | 48 | 33 | 51 | 30 | 52 | 29 | 53 |
| | 5000 | | 41 | 65 | 39 | 65 | 42 | 68 | 40 | 64 |
| | 10000 | | 38 | 63 | 34 | 64 | 40 | 67 | 35 | 61 |

| Function | N | Problem | Alg. DY | | Alg. CD | | Alg. PRP | | Alg. HDY | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Iter. | Fg. | Iter. | Fg. | Iter. | Fg. | Iter | Fg. |
| 20 | 1000 | DENSCHNB (CUTE) | 105 | 162 | 99 | 151 | 104 | 163 | 105 | 159 |
| | 5000 | | 246 | 384 | 222 | 349 | 229 | 373 | 209 | 322 |
| | 10000 | | 336 | 533 | 375 | 595 | 324 | 519 | 298 | 469 |
| 21 | 1000 | DENSCHNC (CUTE) | 107 | 167 | 103 | 161 | 102 | 164 | 99 | 157 |
| | 5000 | | 244 | 387 | 258 | 404 | 288 | 456 | 232 | 363 |
| | 10000 | | 356 | 547 | 344 | 546 | 330 | 525 | 299 | 474 |
| 22 | 1000 | QP1 Extended Quadratic Penalty | 16 | 27 | 15 | 27 | 17 | 30 | 14 | 26 |
| | 5000 | | 10 | 20 | 10 | 20 | 10 | 20 | 10 | 20 |
| | 10000 | | 14 | 27 | 14 | 25 | 13 | 25 | 13 | 25 |
| 23 | 1000 | PRODcos | 36 | 61 | 34 | 59 | 32 | 55 | 35 | 56 |
| | 5000 | | 36 | 60 | 32 | 57 | 36 | 61 | 36 | 58 |
| | 10000 | | 36 | 56 | 34 | 57 | 36 | 59 | 39 | 64 |
| 24 | 1000 | Extended Trigonometric ET1 | 35 | 97 | 30 | 62 | 25 | 84 | 20 | 36 |
| | 5000 | | 76 | 121 | 73 | 113 | 71 | 114 | 64 | 106 |
| | 10000 | | 39 | 66 | 36 | 65 | 39 | 66 | 30 | 46 |
| 25 | 1000 | Perturbed Quadratic | 124 | 187 | 113 | 172 | 110 | 152 | 99 | 148 |
| | 5000 | | 253 | 404 | 267 | 395 | 246 | 387 | 230 | 377 |
| | 10000 | | 420 | 486 | 437 | 597 | 445 | 701 | 346 | 547 |
| 26 | 1000 | Raydan 1 | 76 | 117 | 85 | 131 | 80 | 126 | 70 | 110 |
| | 5000 | | 238 | 407 | 268 | 440 | 675 | 991 | 207 | 330 |
| | 10000 | | 506 | 952 | 452 | 880 | 638 | 981 | 407 | 755 |
| 27 | 1000 | TR-SUMMBEALE | 315 | 728 | 470 | 1126 | 305 | 6678 | 287 | 437 |
| | 5000 | | 213 | 324 | 217 | 331 | 216 | 330 | 115 | 218 |
| | 10000 | | 164 | 327 | 184 | 426 | 178 | 331 | 137 | 328 |
| 28 | 1000 | Diagonal 1 | 65 | 107 | 69 | 111 | 59 | 106 | 55 | 104 |
| | 5000 | | 140 | 233 | 157 | 259 | 152 | 250 | 122 | 206 |
| | 10000 | | 214 | 350 | 219 | 352 | 220 | 374 | 197 | 353 |
| 29 | 1000 | Extended Tridiagonal 1 | 11 | 22 | 11 | 22 | 14 | 27 | 11 | 22 |
| | 5000 | | 12 | 23 | 12 | 23 | 12 | 23 | 12 | 23 |
| | 10000 | | 16 | 30 | 14 | 27 | 14 | 27 | 13 | 26 |
| 30 | 1000 | Tridiagonal Double Borded Arrow Up | 15 | 29 | 14 | 27 | 11 | 20 | 9 | 18 |
| | 5000 | | 11 | 22 | 11 | 22 | 10 | 20 | 8 | 16 |
| | 10000 | | 19 | 38 | 17 | 34 | 15 | 30 | 11 | 22 |

| Function | N | Problem | Alg. DY | | Alg. CD | | Alg. PRP | | Alg. HDY | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Iter. | Fg. | Iter. | Fg. | Iter. | Fg. | Iter. | Fg. |
| 31 | 1000 | Extended Cliff | 50 | 89 | 54 | 92 | 55 | 96 | 46 | 80 |
| | 5000 | | 110 | 202 | 135 | 236 | 110 | 192 | 103 | 202 |
| | 10000 | | 156 | 273 | 181 | 317 | 158 | 278 | 191 | 342 |
| 32 | 1000 | Quadratic Diagonal Perturbed | 30 | 60 | 36 | 59 | 30 | 56 | 28 | 56 |
| | 5000 | | 25 | 49 | 28 | 54 | 25 | 49 | 21 | 46 |
| | 10000 | | 27 | 52 | 28 | 50 | 34 | 64 | 26 | 51 |
| 33 | 1000 | Diagonal Double Borded Arrow Up | 112 | 174 | 119 | 179 | 116 | 181 | 103 | 161 |
| | 5000 | | 282 | 435 | 278 | 429 | 1003 | 1170 | 238 | 372 |
| | 10000 | | 382 | 601 | 408 | 649 | 395 | 625 | 341 | 539 |
| 34 | 1000 | TRIDIA (CUTE) | 3 | 6 | 3 | 6 | 3 | 6 | 3 | 6 |
| | 5000 | | 3 | 6 | 3 | 6 | 3 | 6 | 3 | 6 |
| | 10000 | | 3 | 6 | 3 | 6 | 3 | 6 | 3 | 6 |
| 35 | 1000 | ARWHEAD (CUTE) | 41 | 61 | 40 | 67 | 39 | 65 | 38 | 61 |
| | 5000 | | 35 | 53 | 48 | 71 | 48 | 74 | 58 | 547 |
| | 10000 | | 39 | 56 | 78 | 1313 | 63 | 519 | 41 | 69 |
| 36 | 1000 | Extended Hiebert | 9 | 17 | 10 | 37 | 9 | 17 | 20 | 58 |
| | 5000 | | 10 | 76 | 9 | 54 | 8 | 14 | 10 | 76 |
| | 10000 | | 18 | 178 | 9 | 57 | 26 | 260 | 7 | 14 |
| 37 | 1000 | Almost Perturbed Quartic | 17 | 37 | 15 | 31 | 13 | 26 | 11 | 22 |
| | 5000 | | 35 | 70 | 31 | 65 | 22 | 46 | 15 | 26 |
| | 10000 | | 36 | 78 | 34 | 69 | 41 | 82 | 30 | 68 |
| 38 | 1000 | DENSCHNF (CUTE) | 27 | 48 | 26 | 49 | 26 | 46 | 25 | 46 |
| | 5000 | | 40 | 335 | 85 | 1987 | 109 | 2651 | 26 | 68 |
| | 10000 | | 41 | 280 | 34 | 233 | 40 | 211 | 62 | 127 |
| 39 | 1000 | HIMMELBG (CUTE) | 421 | 665 | 442 | 703 | 470 | 748 | 403 | 723 |
| | 5000 | | 411 | 630 | 387 | 590 | 361 | 560 | 323 | 525 |
| | 10000 | | 390 | 467 | 456 | 826 | 395 | 733 | 423 | 759 |
| 40 | 1000 | HIMMELBH (CUTE) | 18 | 34 | 18 | 34 | 15 | 31 | 22 | 41 |
| | 5000 | | 20 | 44 | 21 | 46 | 24 | 53 | 20 | 44 |
| | 10000 | | 32 | 67 | 19 | 41 | 20 | 45 | 21 | 46 |
| **Total** | **NOI** | | **15518** | | **13463** | | **17567** | | **11788** | |
| | **NOF** | | **81677** | | **52343** | | **79940** | | **36731** | |

## V. Conclusion

In his paper, the (Dai-Yuan, 1999) algorithm was modified to solve unconstrained optimization problems. This modified proved its effectiveness through numerical results, and the regression and global convergence of the proposed method was proven.

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## References

[1] G. H. Yu, Nonlinear self-scaling conjugate gradient methods for large-scale optimization problems, Doctorial thesis, Sun Yat-Sen University, Guangzhou, China, 2007.

[2] G. Yuan and Z. Wei, “New line search methods for unconstrained optimization,” Journal of the Korean Statistical Society, vol. 38, no. 1, pp. 29–39, 2009.

[3] R. Fletcher and C. M. Reeves, “Function minimization by conjugate gradients,” The Computer Journal vol. 7, pp. 149–154, 1964.

[4] E. Polak and G. Ribiere, “Note sur la convergence de m ` ethodes de directions conjugu ´ ees,” ´ Revue Francaised Informatiquet de Recherche Operiionelle, vol. 3, no. 16, pp. 35–43, 1969.

[5] M. R. Hestenes and E. Stiefel, “Methods of conjugate gradients for solving linear systems,” Journal of Research of the National Bureau of Standards, vol. 49, pp. 409–436, 1952.

[6] R. Fletcher, Practical Methods of Optimization. Vol. 1: Unconstrained Optimization, John Wiley & Sons, New York, NY, USA, 2nd edition, 1997

[7] Y. Liu and C. Storey, “Efficient generalized conjugate gradient algorithms. I. Theory,” Journal of Optimization Theory and Applications, vol. 69, no. 1, pp. 129–137, 1991.

[8] Y. H. Dai and Y. Yuan, “A nonlinear conjugate gradient method with a strong global convergence property,” SIAM Journal on Optimization, vol. 10, no. 1, pp. 177–182, 1999.

[9] P.Wolf : Convergence Conditions for ascent methods. SIAM Rer.,11 ( 1969).

[10] H. Yabe and N. Sakaiwa, Journal of the Operations Research Society of Japan 48, 284–296 (2005).

[11] Mahdi, M. M. and Shiker, M. A. K., (2020), Three- Term of New Conjugate Gradient Projection Approach under Wolfe Condition to Solve Unconstrained Optimization Problems, Journal of Advanced Research in Dynamical and Control Systems, 12: 7, pp. 788- 795.

[12] Wasi, H. A. and Shiker, M. A. K., (2020), A new hybrid CGM for unconstrained optimization problems, “in press”, accepted paper for publication in IOP Science, 1st International Virtual Conference on Pure Sciences (IVCPS)- Iraq.