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The Caputo-Fabrizio New Fractional Derivation Applied to the Fisher Emission Linear Equation

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Abstract

An analysis and paper were provided for Fisher's reaction-diffusion equation using the time-frictional Caputo-Fabrizio equation. In addition, we offered the modified problem's iterative method solution. We demonstrated the stability of the approach using fixed-point theory. These operators, however, are limited in their ability to mimic physical situations and have a power law kernel. Recently, Caputo and Fabrizio presented an alternative fractional differential operator with an exponentially decaying kernel in order to get around this problem. The Caputo-Fabrizio (C-F) operator is a revolutionary method for fractional derivatives that has drawn the attention of several researchers because of its non-singular kernel. Additionally, the C-F operator works best when representing a certain class.

I. Introduction

One of the most widely used concepts in Applied Mathematics is the derivation concept, which is introduced to describe the rate of change of a given function and is widely used in the modeling of real-world equations of this concept. Despite all the advantages of derivation in real-world modeling, it has been observed over time that differential equations with ordinary derivations cannot meet the needs of scientists due to the complexity that exists in real-world problems.[1,2] So the concept of fractional derivations with the initial questions of the great scientist Leibniz entered the arena then a scientist named Euler took the first step in generalizing the symbolism for the derivation of arbitrary functions taking into account the fractional values for the order of the derivation, years later live Ville also pointed out the existence of the left and right derivations by proposing new ideas and then defined the fractional integral operator in the continuation of the activities of these scientists a scientist named Riemann used the Taylor series generalization to obtain a formula for the order integral used a deficit. Since fractional derivations are one of the favorite topics of scientists in all sciences, useful studies have been conducted in this field such as [3, 10, 11, 13]. On the other hand, the Fisher equation is one of the equations of Engineering, Chemistry and physics, so there has been valuable research on it [1, 4, 5,]. In recent years, a derivation called the Caputo-Fabrizio fractional derivation has been proposed [6, 7, 8, 9] and scientists have used this derivation to solve differential equations according to analytical methods, in the meantime, the Sumudu method is a very effective method in solving these types of Equations [2, 7-11]. The thesis also uses the Caputo-Fabrizio fractional derivation in solving the nonlinear Fisher reaction -diffusion equation according to the Sumudu Method [5]. The most popular operators for calculating energy acceleration, midpoint fracturing for non-elastic medium, and reevaluating porosity in dispersion in leaking media are fractional derivatives. Additionally, they are broadly congruent with the theories of thermodynamics that followed.

Additionally, they are accurate and well-designed enough to match conventional derivations of the same order in derivations of the same order, in addition to closely matching a set of observable facts. However, [2,3,4] if different derivation operators or even simpler ones are employed, these cells have little to do with the impact they characterize in empirical reality or conjecture; in the absence of this feature, one may be able to obtain counseling responses from fractional.

II. Preliminaries

Definition 2.1: For $0 < \alpha < 1$ and $f \in H^1(0,b), b > 0$ the Caputo fractional derivation of the order α for f is defined as: $^{C}D^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-x)^{-\alpha} f'(x)dx, t > 0$. (1) [4]

By changing the nucleus $(x-s)^{-\alpha}$ with the function $\frac{1}{\sqrt{2\pi(1-\alpha^2}}$ and $\frac{1}{\Gamma(1-\alpha)}$ and $\exp[-\alpha \frac{t-x}{1-\alpha}]$ we come to the new definition recently presented by Caputo and Fabrizio:

Definition 2.2:let $f \in H^1(a,b), b > a, a \in [a,b]$ then the new caputo_Fabrizio derivative of fractional order is defined as:$D_t^{\alpha}\big(f(t)\big) = \frac{M(\alpha)}{1-\alpha} \int_{\alpha}^{t} f'(x) \exp\left[-\alpha \frac{t-x}{1-\alpha}\right] dx$ where $M(\alpha)$ is normalization function such that $M(0) = M(1) = 1$. But, if the function does not belong to $H^1(a, b)$ then, the derivative can be reformulated as .

$D_t^{\alpha}\big(f(t)\big) = \frac{M(\alpha)}{1-\alpha} \int_{\alpha}^{t} (f(t) - f(x)) \exp\left[-\alpha \frac{t-x}{1-\alpha}\right] dx$ (2)

[5]

Definition 2.3:let $u \in H^1(0,b), b > 0, 0 < \gamma < 1$, then the time fractional Caputo_Fabrizio fractional differential Operator (C-FFDO) is defined as .

$$^{CF}D_t^{\gamma} u(t) = \frac{(2-\gamma)M(\gamma)}{2(1-\gamma)} \int_0^t exp\left[-\frac{\gamma(t-s)}{1-\gamma}\right] u'(\tau)d\tau$$

$t \geq 0, 0 < \gamma < 1$, (3)

With a normalization function M(γ) which is depending on $\gamma \ni M(0) = M(1) = 1$.

[5]

Definition 2.4: the C-FFDO of order 0< $\gamma < 1$ *is given by*

$^{CF}D_t^{\gamma} u(t) = \frac{2(1-\gamma)}{(2-\gamma)M(\gamma)} u(t) + \frac{2\gamma}{(2-\gamma)M(\gamma)} \int_0^t u(\tau)d\tau$ (4)

Like the usual Caputo derivative, this new operator gives $^{CF}D_t^{\gamma} u(t) = 0$, if u is a constant function.

The main Advantage of the Caputo-Fabrizio operator over the old operator of Caputo is that there is no singular for t=s in the new kernel.

[5]

2.5 Analyzing by using(nonlinear reactive Fisher's diffusion equation) :

we present an analysis of the nonlinear reactive Fisher's diffusion equation with the time derivation of the Caputo-Fabrizio fraction. The equation is given as follows:

$$^{CF}_{0}D_t^{\gamma} u(x,t) = \alpha \frac{\partial^2 u(x,t)}{\partial x^2} + \beta u(x,t)\big(1 - u^m(x,t)\big),$$

$m > 0, \ 0 < \gamma < 1$ (5)

[7]

The above equation has the following primary conditions:

$$u(x,0) = 0 \quad a \leq x \leq b$$

III. Methodology

3.1 Iterative method for obtaining solution

Theorem 3.1: Suppose f (t) is a function that Caputo-Fabrizio derivation has. So, the Sumudu transform for the Caputo-Fabrizio fractional derivation of the function f (t) is as follows:

$S(^{CF}_{0}D_t^{\alpha}) f(t)(u) = M(\alpha) \frac{F(u) - F(0)}{1-\alpha+\alpha u}$ (6)

In which F (u)= S (f (t)) (u). [12]

In this section, we get the private solution of equation

(1) in an iterative way. According to theorem 3.1 we know that:

$$S(^{CF}_{0}D_t^{\gamma})\big(u(x,t)\big)(s) = M(\gamma) \frac{S\big(u(x,t)\big)(s) - u(x,0)}{1-\gamma+\gamma s}$$

(7)

So, by applying Sumudu transform to the sides of equation (1)(3) we have:

$$M(\gamma) \frac{S\big(u(x,t)\big)(s) - u(x,0)}{1-\gamma+\gamma s} = S\left\{\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \beta u(x,t) (1 - u^m(x,t))\right\}$$

$\frac{M(\gamma)S(u(x,t))(s)}{1-\gamma+\gamma s} - \frac{M(\gamma)u(x,0)}{1-\gamma+\gamma s} = S\left\{\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \beta u(x,t)\big(1 - u^m(x,t)\big)\right\}$ (8)

Through changing the two sides we will have:

$$\frac{M(\gamma)}{1-\gamma+\gamma s} S\big(u(x,t)\big)(s) = \frac{M(\gamma)}{1-\gamma+\gamma s} u(x,0) + S\left\{\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \beta u(x,t)(1 - u^m(x,t)\right\} \quad (9)$$

And through dividing the sides by $\frac{M(\gamma)}{1-\gamma+\gamma s}$we have:

$S\big(u(x,t)\big)(s) = u(x,0) + \frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \beta u(x,t)(1 - u^m(x,t))\right\}$ (10)

Now, by applying the inverse Sumudu transform to the two sides of equation (10) we will have:

$\big(u(x,t)\big)(s) = u(x,0) + s^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \beta u(x,t)(1 - u^m(x,t))\right\}\right\}$ (11)

Then we have the following recursive relationship:

$$u(x,0) = u(x,t)$$
$$u_{n+1}(x,t) = u_n(x,t) + s^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 u_n(x,t)}{\partial x^2} - \beta u(x,t)(1-u_n^m(x,t))\right\}\right\} \quad (12)$$

Then the answer to (10) equation will be as follows:

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

$$u(x,0) = 0, \qquad a \le x \le b$$

3.2 Stability analysis using fixed point theorem Method

Definition 3.2: Let's assume that $(x, ||.||)$ is a Banach space and H is a self-portrait of X. Suppose that $y_{n+1=}g(H, y_n)$ is a special recursive relation. Also suppose that F (H) is the set of fixed points of the mapping H with at Least one member and the sequence $y_n$converges to a point like P ∈ H. Let's assume$\{x_n\} \subseteq X$and define:

$e^n = \|x_{n+1} - g(H, x_n\|$.

If $\lim_{n\to\infty} e_n = 0$ leads to $\lim_{n\to\infty} x_n = p$, then the iteration method of$y_{n+1=}g(H, y_n)$ is called stable -H.

Without detracting from the totality since convergence is not possible otherwise, we have to assume that the sequence $\{x_n\}$ has an upper bound. After all these prerequisites for the so-called Picard repetition $(y_{n+1} = Hy_n)$ have been established, the repeated procedure H -will be stable.[5].

IV. Main results

Theorem 4.1: Let's assume $(X, \|.\|)$ is a Banach space and H is a self-portrait. Suppose that for every x, x,y ∈, there are constants 0 ≤ C , 0 ≤ c < 1 so that

$$\|H_x - H_y\| \le C\|x - H_x\| + C\|x - y\| \qquad (13)$$

Therefore, the mapping of H will be H -stable Picard. [16].

Proof:

Now we consider the sequence related to nonlinear reactive-diffusion equation with fractional derivation

$$u_{n+1}(x,t) = u_n(x,t) + S^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \beta u(x,t)(1-\widehat{u^m(x,t)})\right\}\right\} \qquad (14)$$

In which $\frac{1-\gamma+\gamma s}{M(\gamma)}$ is the fractional Lagrange Coefficient and $\widehat{u_n^m}$ is the Variable Limits result to $\delta \widehat{u_n^m = } 0$

Theorem 4.2:. Let's assume that T is a defined self-Portrait as follows:

$$T(u_n(x,t)) = u_{n+1}(x,t) = u_n(x,t) + S^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 u_n(x,t)}{\partial x^2} - \beta u(x,t)(1-u_n^m(x,t))\right\}\right\}$$

So, this mapping in$-T, L^2(a,b)$ is stable whenever

$$\left\{\frac{\alpha\beta_1\beta_2 + (C+A)^m\beta}{M(\gamma)}\gamma + \frac{\gamma}{M(\gamma)}\right\} < \beta \quad (21)$$

[2]

Proof: The first step of the proof is assigned to show that T has a fixed point. To achieve this, we find the following values for every $(n,k) \in N \times N$:

$$\|T(u_n(x,t) - T(u_k(x,t)\| = \left\| u_n(x,t) - u_k(x,t) + S^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 u_n(x,t)}{\partial x^2} - \beta u_n(x,t)(1-u_n^m(x,t)) - S^{-1}\{\frac{1+(s-1)\gamma}{M(\gamma)} S\{\alpha \frac{\partial^2 u_k(x,t)}{\partial x^2} - \beta u_k(x,t)(1-u_k^m(x,t))\right\}\right\}\right\| \qquad (22)$$

Considering the linearity of the Sumudu transform inverse, we will have:

$$\|T(u_n(x,t) - T(u_k(x.t)\| = \left\| u_n(x,t) - u_k(x,t) + S^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 (u_n(x,t) - u_k(x,t))}{\partial x^2} - \beta(u_n(x,t) - u_k(x,t) + \beta(u_n^{m+1}(x,t))\right\}\right\}\right\| \quad (23)$$

Using a triangular inequality for norms we will have:

$$\|T(u_n(x,t)) - T(u_k(x,t))\| \le \|u_n(x,t) - u_k(x,t)\| + \left\| S^{-1}\left\{\frac{1-\gamma+\gamma s}{M(\gamma)} S\left\{\alpha \frac{\partial^2 (u_n(x,t) - u_k(x,t))}{\partial x^2} - \beta(u_n(x,t) - u_k(x,t) + \beta(u_n^{m+1}(x,t) - u_k^{m+1}(x,t))\right\}\right\}\right\| \quad (15)$$

Using the norm and integral displacement property, the above expression is as follows:

$$\|T(u_n(x,t) - T(u_k(x,t)\| \le \|u_n(x,t) - u_k(x,t)\| \quad (16)$$

$$+S^{-1}\left\{\frac{1+(s-1)\gamma}{M(\gamma)}S\left\{\left\|\alpha\frac{\alpha\partial^2(u_n(x,t)-u_k(x,t)}{\partial x^2}\right\|\right\}\right\}+$$
$$S^{-1}\left\{\frac{1+(s-1)\gamma}{M(\gamma)}S\{\|-\beta\{u_n(x,t)-u_k(x,t)\}\|\}\right\}$$
$$+S^2\left\{\frac{1+(s-1)\gamma}{M(\gamma)}S\{\|\beta\{u_n^{m+1}(x,t)-u_k^{m+1}(x,t)\}\|\}\right\}$$
(17)

We investigate the (17) equation point by point as follows:

$$\left\|\alpha\frac{\partial^2\{u_n(x,t)-u_k(x,t)\}}{\partial x^2}\right\| \leq \alpha\beta_1\beta_2\|u_n(x,t)-u_k(x,t)\| \qquad (18)$$

Also, we have:

$$\|\beta\{u_n^{m+1}(x,t)-u_k^{m+1}(x,t)\| \leq \left\|\sum_{j=0}^{m}C_m^j\left(u_n(x,t)\right)^j(u_k(x,t))^{m-j-1}\right\|\,\|\{u_n(x,t)-u_k(x,t)\}\| \qquad (19)$$

Since $u_k(x,t) and\ u_n(x,t)$ are bound; therefore, there are different positive fixed numbers of A and C so that for each (x,t) we have:

$$\|u_n(x,t)\| < C\,, \|u_k(x,t)\| < A\,, (u,k) \in N\times N$$

So, using the triangular inequality with the above inequality, the inequality of (3)(6) will be as follows:

$$\|\beta\{u_n^{m+1}(x,t)-u_k^{m+1}(x,t)\| \leq (C+A)^m\|\{u_n(x,t)-u_k(x,t)\}\| \quad (20)$$

By putting (18)and(19) in(20) we have:

$$\|T(u_n(x,t)-T(u_k(x,t)\| \leq \{1-\beta\frac{\alpha\beta_1\beta_2(C+A)^m\beta}{M(\gamma)}\gamma + \frac{\gamma}{M(\gamma)}\}\|\{u_n(x,t)-u_k(x,t)\}\|$$

(24)

In which

$\{\frac{\alpha\beta_1\beta_2(C+A)^m\beta}{M(\gamma)}\gamma+\frac{\gamma}{M(\gamma)}\}<\beta$

As a result, -T is a nonlinear self-portrait with a fixed point.

We show that T satisfies the definition 1. Suppose? be established. So, we put

$$d=0, D=\left\{1-\beta\frac{\alpha\beta_1\beta_2(C+A)^m\beta}{M(\gamma)}\gamma+\frac{\gamma}{M(\gamma)}\right\} \quad (25)$$

It shows that the conditions of the theorem 4.1 are true for the nonlinear mapping of T. Therefore, since all conditions in the theorem 4.1 are true for the defined nonlinear mapping T, then T is the Picard constant T.

V. Conclusion

It is evident that popular topical theories cannot oversee heterogeneities and Simple configurations with diverse scales, which is one of the intriguing aspects of the novel fractional derivative provided by Caputo and Fabrizio. Nonlocal connections between atoms, which are recognized as significant Features of materials, provide a supplementary application for understanding the microscopic behavior of specific materials. We have provided helpful tools. For new fractional order derivation with these features. We have adjusted Fisher's diffusion equation using this new approach. By using the Sumudu approach. In relation to the fractional Lagrange coefficient, we were able to achieve the private solution. We have employed the idea of T stable mapping and Theorem to demonstrate the iterative process's stability and the existence and uniqueness requirements for nonlinear fractional differential equations employing the Caputo-Fabrizio differential operator are presented in this paper. In addition, we have created ILTM to effectively get approximate Solutions to the fractional differential equations (Caputo-Fabrizio). The approximations are contrasted with exact answers and other solutions that Have already been found using different techniques. It is noted that the first Three terms' approximate series solutions are extremely accurate and converge quickly to the solutions of actual physical issues. The suggested Method for locating approximations to several fractionally-order nonlinear reaction-diffusion equations is dependable, straightforward, and efficient.

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