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Heat Transfer of Fluid Flow by Conduction and Convection in Inclined Cavities

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Abstract

This research dealt with a numerical simulation of the issue of heat transfer by conduction and convection in two dimensions resulting from the flow of fluid in horizontal and inclined cavities. The cavity walls are long H and wide L and consist of a porous material. Ode45 techniques were used to represent the governing equations (volume, momentum, and energy) after converting them to non-dimensional formulas using some appropriate transformations. The results showed the effect of the Reynolds number, Prandtl number, Rayleigh number, Hartmann number, Darcy number, Eckert number, in addition to the angle of inclination, on the problem, as shown in the drawings and illustrations.

Keywords:
Ode45 method, Heat Transfer, Rayleigh number, Prandtl number, Porous medium.

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I. Introduction

The importance of studying heat transfer lies in solutions and solutions related to industrial and environmental issues. It takes into account the effects of heat transfer in energy conversion production processes in the above fields. Many energy sources are used, including self-energy, conventional fossil wisdom, and geothermal warming [1]. The fluid flow and transfer of heat in a wavy-shaped vertical tube with permeable walls were examined by Umavathi J. and Shakar M. [2], and the Darcy-Brinkman concept was applied to explain that flow.

A numerical analysis of the transmission of heat and the flow of liquid in a cavity with a flat upper surface and a wavy lower surface was carried out by Heidary and Kermani.Porous substance appears in the first cavity from the top to the centre of the space. Two parameters, wave number and wave amplitude, were used to study the influence of undulation on the surface for a range of Reynolds numbers, from 100 to 1000 [3]. In addition to thermal radiation, Hamoodat A. and Juma'a A. investigated the flow of a substance at an inclined frequency with porous surfaces when a magnetic field was sheathed to the cavity [4].

The issue of heat transmission by convection from the atmosphere on a square porous continent with angular irradiation and a field of magnets has been researched by Abbas H. and Falah M. [5].In order to model heat transfer by forced convection arising from the flow of a fluid within a surface with semicircular parts and filled with porous mediums, Muwaffaq H. and Amir D. worked out a simulation study [6].The impact of the transfer of masses and absorption characteristics on the movement of a two-dimensional viscous liquid in a horizontal cavity with a radiation source present was demonstrated by Kumari K. and Gayal M. [7].

The movement of fluid in an area under the control of an electrical field (EMF) and the solution of the equations that govern it have been looked at by Hammodat A. and Saleem H. Other than regulating the computer's temperature using tangible values [8].Anandamoy M. and Amar K. explored the effects of changing viscosity on the long-term stability of a gravity-driven Newtonian thin liquid film travelling down a heated, evenly heated substrate over heat flux heat flux (HF) boundaries [9]. The exact measurements of partial Casson fluids along the channel's surface caused by a material with pores and the MHD event are also covered by Sunthrayuth et al. The lower plate's unstable stream activity, which is constrained by alignment with diagonal sidewalls, supports the flow field, as stated in [10].

Heat transport by conduction as well as convection in horizontal and inclined cavities under the influence of a magnetic field was the subject of this experiment, which used a numerical simulation by Ode45. Graphs are used to illustrate the impact of numerous pertinent parameters on temperature and velocity.

II. Mathematical Model

The answers to the governing equations include the equations for mass conservation, conserving momentum, and energy preservation, among others. Because these equations are somewhat complex, several assumptions must be imposed to facilitate the solution. These hypotheses are:

1. The flow is laminar, stable, and fully developed thermally and hydraulically.
2. It has a completely porous and saturated medium.
3. There is no heat generated within the porous medium.
4. The cavity is filled with an incompressible fluid with dispersion of viscosity.
5. Horizontal walls insulate while vertical walls conduct heat. As shown in figure 1

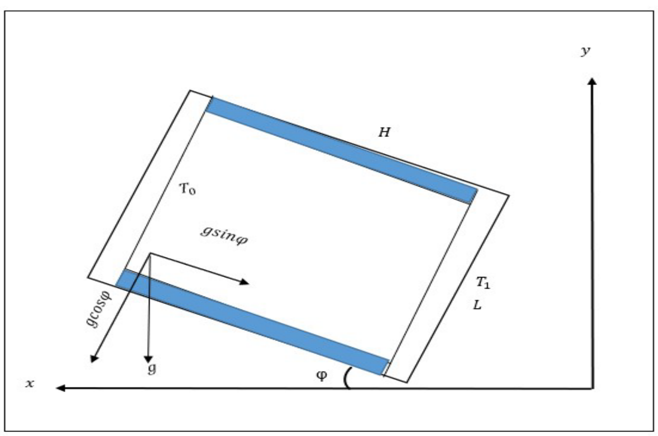

Fig 1. Physical model of the problem.

Thus, the governing equations by the hypotheses are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u-\beta g(T-T_0)cos\varphi + \left(\frac{\sigma B_o^2}{\rho}-\frac{\upsilon}{k}\right)u \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\nabla^2 v+\beta g(T-T_0)sin\varphi + \left(\frac{\sigma B_o^2}{\rho}-\frac{\upsilon}{k}\right)v \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\nabla^2 T+\epsilon\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right] \tag{4}$$

Where u, v are the components of velocity in x, y directions respectively , t is the time and $\rho, \upsilon, \beta, g, T, \sigma, B_o, k, C_p, \epsilon$ are the density, kinematics viscosity, updraft expansion coefficient, gravitational acceleration, temperature, electrically conductive, field of magnetism, Permeability of medium , precise temperature under continuous pressure, The dispersion parameter, focused attention respectively.

With the subsequent restrictions on boundaries,

$$\left.\begin{array}{c} T(0,y)=T_1, T(L,y)=T_0, (\frac{\partial T}{\partial y})_{y=0,H}=0 \\ u(0,y)=v(0,y)=0, u(L,y)=v(L,y)=0 \\ u(x,0)=v(x,0)=u(x,H)=v(x,H)=0 \end{array}\right\} \tag{5}$$

Where non-dimensional variables are defined as follows [11, 12]:

$$\left.\begin{array}{l} u^*=\frac{u}{U} \quad , \quad v^*=\frac{v}{U} \quad , \quad x^*=\frac{x}{L} \quad , \quad y^*=\frac{y}{L} \\ \theta=\frac{T-T_0}{T_1-T_0} \quad , \quad t^*=\frac{tU}{L} \quad , \quad p=p^*\rho U^2 \end{array}\right\} \tag{6}$$

And non- dimensional Parameters [13]:

$$\left.\begin{array}{l} U=\frac{\alpha}{L}\sqrt{RaPr}=\sqrt{g\beta\Delta TL} \ , Ra=\frac{\rho g\beta(T_1-T_0)L^3}{\mu\alpha} \\ Pr=\frac{\upsilon}{\alpha}=\frac{\mu C_p}{k} \ , Da=\frac{k}{L^2} \ , Ec=\frac{u^2}{C_p\Delta T} \ , Re=\frac{Lu}{\upsilon} \\ Ha^2=B_o^2L^2\frac{\sigma}{\mu} \ , \ \epsilon=\frac{\mu}{\rho cp}, \quad Cp=\frac{\sigma}{\alpha} \end{array}\right\} \tag{7}$$

Where $Ra, Pr, Da, Re, Ha\ and\ Ec$ are Rayleigh number, Prandtl number, Darcy number, Reynolds number, Hartmann number and Eckert number respectively.

By substituting non-dimensional values (7), into the equations (1)-(4) and (5) we get:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{8}$$

$$\frac{\partial}{\partial t^*}\left[\frac{\partial v^*}{\partial x^*}-\frac{\partial u^*}{\partial y^*}\right]+\frac{\partial}{\partial x^*}\left[u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\right]-$$

$$-\frac{\partial}{\partial y^*}\left[u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right]$$

$$=\frac{1}{Re}\nabla^2\left[\frac{\partial v^*}{\partial x^*}-\frac{\partial u^*}{\partial y^*}\right]+\sin\varphi\ \frac{\partial\theta}{\partial x^*}+\cos\varphi\ \frac{\partial\theta}{\partial y^*}+$$

$$Ha^2\sqrt{\frac{Pr}{Ra}}\left(\frac{\partial v^*}{\partial x^*}-\frac{\partial u^*}{\partial y^*}\right)-\frac{\sqrt{Pr}}{Da\sqrt{Ra}}\left(\frac{\partial v^*}{\partial x^*}-\frac{\partial u^*}{\partial y^*}\right) \quad (9)$$

But $u^*=\frac{\partial\psi}{\partial y^*}$ and $v^*=-\frac{\partial\psi}{\partial x^*}$ has the function of a stream [14]. Incorporate $\xi=\nabla^2\psi$, then equation (9) becomes:

$$\frac{\partial\xi}{\partial t^*}=\frac{1}{Re}\nabla^2\xi-\sin\varphi\ \frac{\partial\theta}{\partial x^*}-\cos\varphi\ \frac{\partial\theta}{\partial y^*}+\left[Ha^2\sqrt{\frac{Pr}{Ra}}-\frac{\sqrt{Pr}}{Da\sqrt{Ra}}\right]\xi \quad (10)$$

$$\frac{\partial\theta}{\partial t^*}+u^*\frac{\partial\theta}{\partial x^*}+v^*\frac{\partial\theta}{\partial y^*}=\frac{1}{\sqrt{Ra\,Pr}}\left[\frac{\partial^2\theta}{\partial x^{*2}}+\frac{\partial^2\theta}{\partial y^{*2}}\right]+\frac{Pr\,Ec}{\sqrt{Ra\,Pr}}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2+\left(\frac{\partial v^*}{\partial x^*}\right)^2\right] \quad (11)$$

Let $\emptyset=\frac{Pr\,Ec}{\sqrt{Ra\,Pr}}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2+\left(\frac{\partial v^*}{\partial x^*}\right)^2\right]$ is a constant physical quantity, and equation (11) becomes:

$$\frac{\partial\theta}{\partial t^*}+u^*\frac{\partial\theta}{\partial x^*}+v^*\frac{\partial\theta}{\partial y^*}=\frac{1}{\sqrt{Ra\,Pr}}\left[\frac{\partial^2\theta}{\partial x^*}+\frac{\partial^2\theta}{\partial y^*}\right]+\emptyset \quad (12)$$

While the non-dimensional boundary conditions are as follows:

$$\left.\begin{array}{l}\theta(0,y^*)=1,\theta(L,y^*)=0,\left.\frac{\partial\theta}{\partial y^*}\right|_{y^*=0,H}=0\\ u(0,y^*)=v(0,y^*)=0,u(L,y^*)=v(L,y^*)=0\\ u(x^*,0)=v(x^*,0)=0,u(x^*,H)=v(x^*,H)=0\end{array}\right\} \quad (13)$$

III. Numerical Analysis

The discretization of equations (8),(10) and (12), using the ODE45 solver (Runge–Kutta formula) with successive iterations over time is the basis for this method's concept. We attempt to use numerical modelling for $-L\le\bar{\mathcal{X}}\le L$ and $-H\le\bar{\mathcal{Y}}\le H$, where is the calculational field's incident random length located between $\bar{\mathcal{X}}$ and $\bar{\mathcal{Y}}$, to get beyond the energy equation (12), the double vibration equation (10) and the beginning circumstances (13).As previously mentioned, the ODE45 technique is used to solve the core equation of the movement (10) and energy equation (12). We employ: to divide the previously described domain into $N+1$ points in the $\bar{\mathcal{X}}$-direction and $M+1$ in $\bar{\mathcal{Y}}$-direction.

$$\overline{\mathcal{X}\imath}=-L+(i-1)\Delta\bar{\mathcal{X}}\ ,i=1,\dots,N+1,$$
$$\overline{\mathcal{Y}\jmath}=-H+(j-1)\Delta\bar{\mathcal{Y}},j=1,\dots,M+1$$

where $\Delta\bar{\mathcal{X}}=\frac{2L}{N}$ and $\Delta\bar{\mathcal{Y}}=\frac{2H}{M}$. Consider how the primary equation in this study is discretized using the forward and backward finite difference methods [14,15].

IV. Conclusions

The ODE45 method, which we used to solve the governing equations completely without decreasing or changing, is the most significant accomplishment of this work. Based on the results, we conclude that all equations can be solved to a stable state after a number of iterations, and at different angles 15, 45, and 75, the ODE45 method, as shown in the figures, is the most important achievement.

1. The equation of motion's growing Rayleigh number suggests that we are moving farther away from stability at various time steps. But as Figures 2 and 3 show, its impact on the heat equation is negligible.
2. We are getting closer to stability in the equation of motion, the larger the Prandtl number. Figures 4 and 5 illustrate how moving away from stability occurs as the Prandtl number in the energy equation is increased.
3. A wider distance from stability in various steps is implied by the equation of motion's increased Darcy number. But as Figures 6 and 7 show, its impact on the heat equation is negligible.
4. Our approach to stability increases with the Eckert number in the equation of motion. Figures 8 and 9 illustrate how moving away from stability is caused by raising the Eckert number in the energy equation.
5. In various time steps, stability is approached more closely the higher the Hartmann number of the equation of motion. Figures 10 and 11 show how little impact it has on the heat equation.
6. In various time steps, a greater distance from stability is implied by the equation of motion's increasing Reynolds number. Figures 12 and 13, however, show how little of an impact it has on the heat equation.
7. It is implied that the equation of motion is moving farther away from stability in successive time steps by the increasing angle in the equation. However, as Figures 14 and 15 show, its impact on the heat equation is negligible.

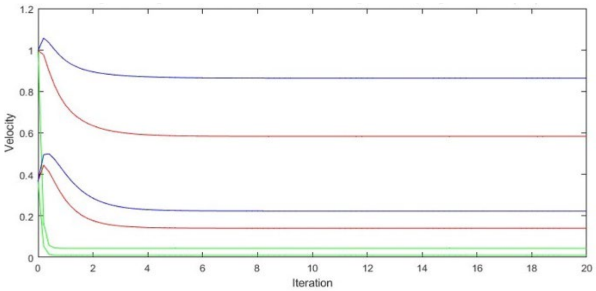

Fig 2. Computation velocity distribution against Rayleigh number (Ra).

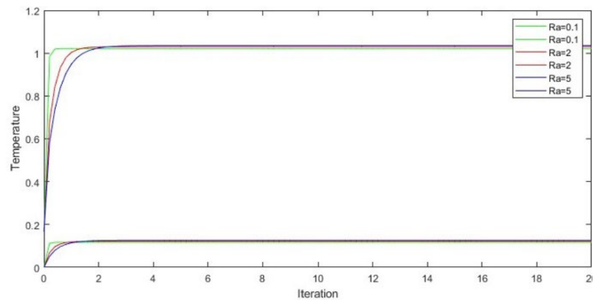

Fig 3. Temperature distribution for Rayleigh number (Ra).

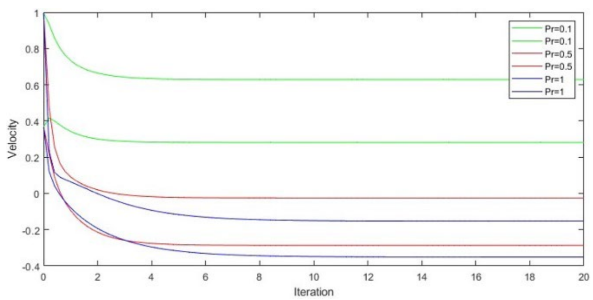

Fig 4. Velocity distribution for Prandtl number (Pr).

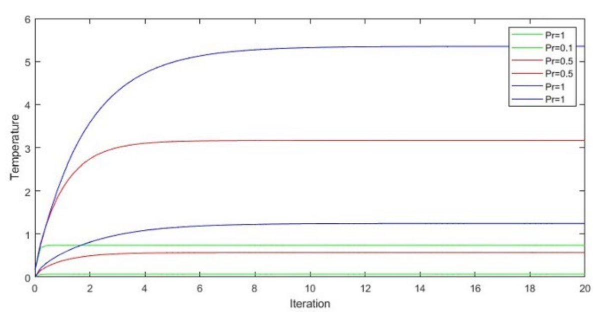

Fig 5. Computation temperature distribution against Prandtl number (Pr).

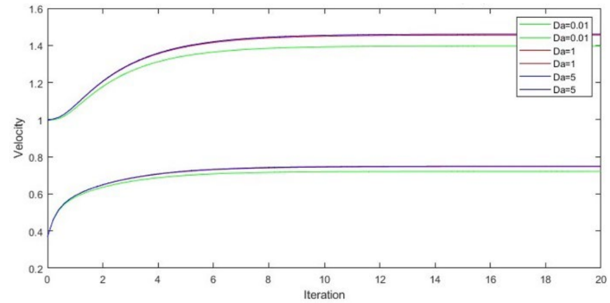

Fig.6. Velocity distribution for Darcy number (Da).

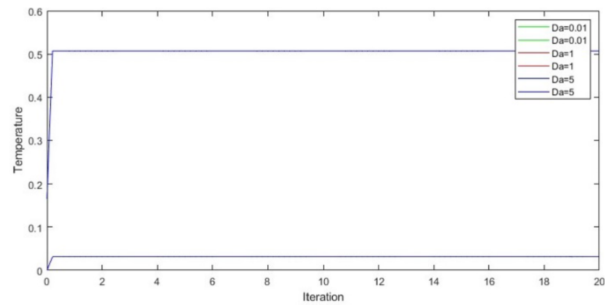

Fig 7. Temperature distribution for various Darcy number (Da).

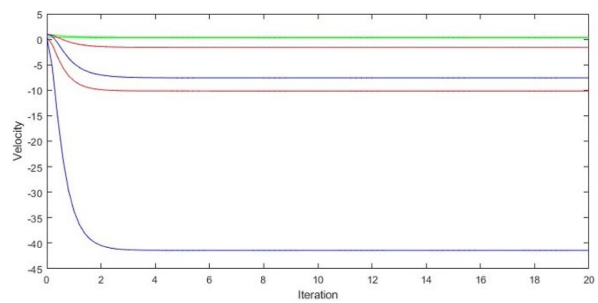

Fig 8. Velocity distribution for various Eckert parameter (Ec).

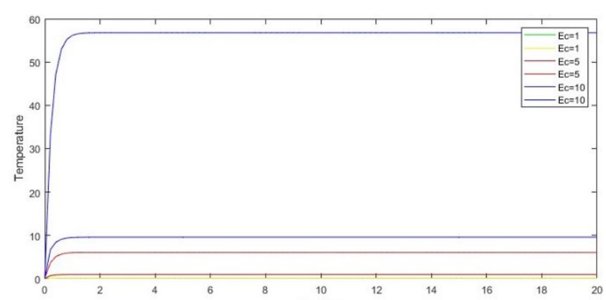

Fig 9. Computation temperature distribution against Eckert number (Ec).

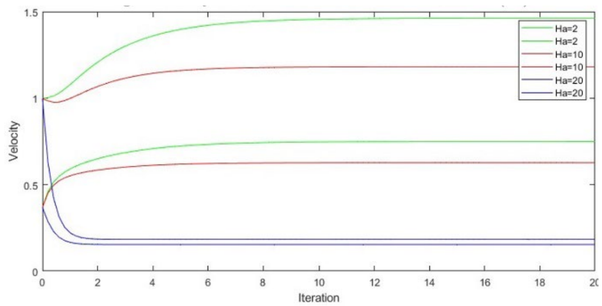

Fig 10. Velocity distribution for different Hartmann number (Ha).

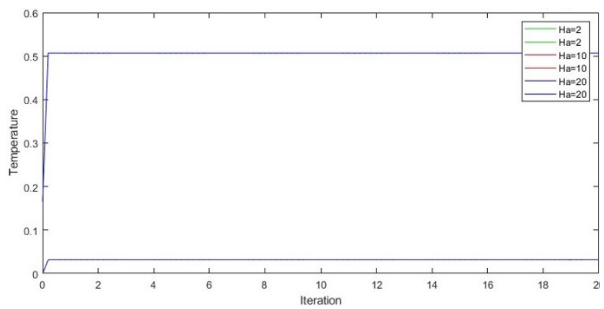

Fig.11. Computation temperature distribution against Hartmann number (Ha).

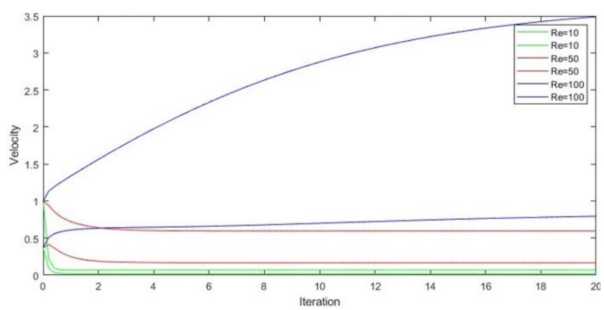

Fig 12. Velocity distribution for different Reynold number (Re).

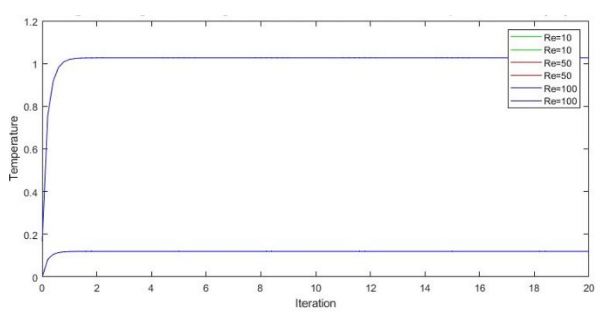

Fig 13. Computation temperature distribution for Reynold number (Re).

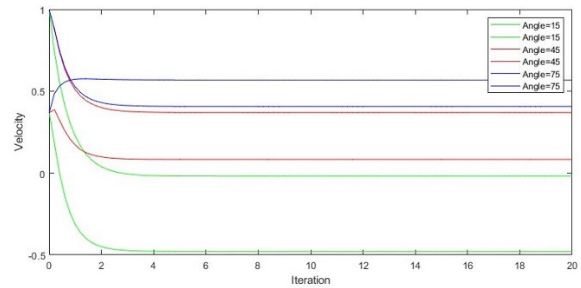

Fig 14. Effect of angle (phi) for velocity Distribution.

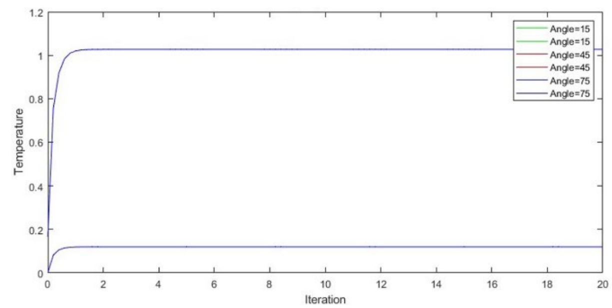

Fig 15. Effect of angle (phi) for temperature Distribution.

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