Journal of Kerbala University , Vol. 8 No.4 Scientific . 2010

EFFECTS OF THE NUMBER OF CYLINDERS ON CRANKSHAFTS INERTIAS TORTIONAL VIBRATIONS

SALAH NOORI ABBOOD
University of Kerbala, Kerbala, Iraq.
jousef_salah@yahoo.ca

ABDULJALEEL HUSSAIN ABD
University of Kerbala, Kerbala, Iraq.
abo_ferasan@yahoo.com

Abstract

Many researches that deal with the torsional vibrations phenomenon in crankshafts are considered constant inertias connected to sections of mass less shafting taking into account variation effects of torques only on crank units during running. In reality, crank mechanism with slider piston vibrates with varying inertias and this will lead to additional torsional vibrations depending on the number of cylinders in internal combustion engines.

In this research, inertias of crank mechanisms with different number of cylinders are studied. By solving the ordinary differential equation of torsional vibrations caused by inertia continuous deviation using mathematica and matlab computer programs, graphs of crankshafts torsional oscillation was sketched to compare among torsional vibrations of different internal combustion engines with different number of cylinders.

المستخلص

العديد من البحوث التي تهتم بدراسة ظاهرة الاهتزاز الالتوائي للأعمدة المرفقية تفترض قيم ثابتة لعزوم القصور مرتبطة بعمود مهمل الكتلة وتأخذ بنظر الاعتبار فقط التغير الدوري لعزوم التدوير أثناء العمل. من الناحية العملية يهتز نظام المجموعة المرفقية بعزم قصور ذاتي متغير وهذا التغير الدوري في عزم القصور الذاتي يقود الى اهتزازات التوائية اضافية تعتمد على عدد الأسطوانات في محركات الاحتراق الداخلي.

في هذه الدراسة تم تحليل عزم القصور الذاتي لمجموعات مرفقية بأعداد مختلفة من اسطوانات المحرك. وبحل المعادلة التفاضلية التي تصف الاهتزاز الالتوائي الناتج من التغير الدوري لعزم القصور الذاتي باستخدام برنامج حاسوبي متخصص تم التوصل الى رسم المخططات الخاصة بالاهتزاز الالتوائي لمحركات مختلفة بأعداد اسطوانات مختلفة.

Key words: torsional vibration, crankshafts, cranks inertia.

Nomenclature:

I	Moment of inertia of rotating parts (kg.m^2)
I_1	Moment of inertia flywheel (kg.m^2)
I_R	Ratio of the reciprocating parts inertia to the total inertia of the system
k_t	Torsional stiffness of crankshaft (N.m/rad)
m_r	Mass of reciprocating parts (kg)
R	Crank radius (m)
r	The ratio of the crankshafts angular velocity to the systems natural frequency.
T	Kinetic energy (W)
V	Potential energy (W)
ω	Angular velocity of crankshaft (rad/s)
ω_n	Natural frequency of the system (neglecting the effect of inertias variation).
θ	Angular displacement of the crank unit (rad)

β Angular displacement of flywheel (rad)

α Angle of crank twist (rad)

1- Introduction:

Vibrations in machines and structures should be analyzed and controlled if they have undesirable effects such as noise, unpleasant motions, or fatigue damage with potentially catastrophic consequences.

A crankshaft is subjected to many periodical dynamic loads causing vibrations, consequence stresses and noise which are drastically increased from the engine block, especially from crankcase [1]. Nowadays, due to increased demands for comfort and reliability in modern vehicles, well-developed solutions have been required. One of the most common solutions is that which deals with torque generated by the engines and as a result angle of twist of their crankshafts. Thus, the study of torsional vibrations in the internal combustion engines is of great significance.

Torsional vibrations are the characteristics of nearly all rotational machines and devices. In the case of reciprocating internal combustion engines the use of a viscous TVD assembled to the crankshaft or a double mass rubber damper reduce the torsional amplitudes [2]. However, the most common sources of this kind of vibrations are:

- Variable gas pressure in the cylinder of an engine which can be estimated by measuring the vibration amplitude at the housing of the main bearing of crankshaft [3].
- Inertia forces of a crank mechanism.

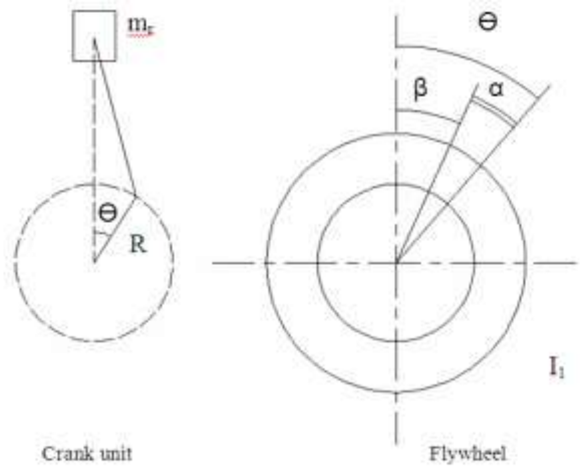
Measuring the torque generated in reciprocating engines is not possible so the theoretical approach is used to determine the dynamic torque [4]. Gas pressure torque acting on the crankshaft changes constantly because of the intermittent pressure on the piston and the dependency of the geometry (connecting rod, crank) on the crank angle. So the shaft alternately winds and unwinds to a small extent with a certain natural frequency. Length and mass of the crankshaft are most significant factors that affect the crank natural frequency. A crankshaft with large length and mass will be rotate with lower value of natural frequency and it may be so low as to equal to the frequency of the combustion impulses at some particular engine speed [5]. Resonance will then be occurred between the forced impulses and the natural frequency of the shaft causing dangerous vibrations during running. So, a cast ductile iron crankshaft with suitably designed damper is more convenient than forged steel crankshafts [6]. Nowadays, it is well known that a number of engines are built with large pistons so there are critical running speeds of running at which the torsional vibrations of the crankshaft become large in amplitude and introduce an element of danger into the system.

The torsional vibration phenomenon in reciprocating engine is usually simulated by a series of discs with constant inertias connected to a mass less shaft. While in reality, the slider crank mechanism is a vibrating system with varying inertia because the effective inertia of the total oscillating mass of each crank assembly varies twice per revolution of the crankshaft. A large variation in inertia torques leads to a rising in the phenomenon of secondary resonance in torsional vibration of modern diesel engines.

The demand of a uniform turning moment and a comfort dynamic balancing has led to the production of six, eight, and even twelve-cylinder engines where questions of first cost are not of primary importance.Twisting here will change linearly along engines crankshafts [7]. This study is to compare among torsional vibrations (caused by moment of inertias) of internal combustion engines with different number of cylinders.

2- Mathematical model:

The simplest way of calculating crankshafts torsional oscillations can be obtained by assuming a reciprocating mass moves with simple harmonic motion. The kinetic energy (T) of the system is given by:



$$T = \frac{1}{2}\theta^{02}\left(I + m_r R^2 \sin^2\theta\right) + \frac{1}{2} I_1 \beta^{o^2} \quad \text{....... (1)}$$

The potential energy (V) of the system is:

$$V = \frac{1}{2} k_t (\theta - \beta)^2 \quad \text{............................ (2)}$$

The Lagrange's equation for the co-ordinate θ is:

$$\frac{d}{dt}\left(\frac{\partial (T+V)}{\partial \theta}\right) = 0$$

So, the equation of motion will be:

$$\theta^{oo}\left(I + m_r R^2 \sin^2\theta\right) + \theta^{o2} m_r R^2 \sin\theta\cos\theta + k_t \alpha = 0 \quad \text{............ (3)}$$

Making use of; ($\beta = \omega t$) and ($\theta = \omega t + \alpha$) then:

$$\theta^{o} = \omega\left(1 + \frac{d\alpha}{d\theta}\right);$$

$$\theta^{oo} = \omega^2 \frac{d^2\alpha}{d\theta^2}$$

So equation (3) could be reformed as followings:

$$\left(I + m_r R^2 \sin^2\theta\right)\omega^2 \frac{d^2\alpha}{d\theta^2} + m_r R^2 \sin\theta\cos\theta\left[\omega^2 + 2\omega^2 \frac{d\alpha}{d\theta} + \omega^2\left(\frac{d\alpha}{d\theta}\right)^2\right] + k_t \alpha = 0$$

The term $\left(\frac{d\alpha}{d\theta}\right)^2$ can be neglected because of its limited value and the previous equation will take the following form:

$$\left(I+\frac{1}{2}m_r R^2(1-\cos 2\theta)\right)\omega^2\frac{d^2\alpha}{d\theta^2}+m_r R^2\omega^2\sin 2\theta\frac{d\alpha}{d\theta}+k_t\alpha=-\frac{1}{2}m_r R^2\omega^2\sin 2\theta \quad \text{.... (4)}$$

Make use of:

$$I_R=\frac{\frac{1}{2}m_r R^2}{\left(I+\frac{1}{2}m_r R^2\right)} \quad \text{.......... (5)}$$

Where, (I_R) represents the ratio of the equivalent moment of inertia of the reciprocating part to the total equivalent moment of inertia of the system.

And make use of ($r = \omega / \omega_n$) which represents the ratio of the angular velocity of the shaft to the natural frequency of the crank system where:

$$\omega_n=\sqrt{\frac{k_t}{\left(I+\frac{1}{2}m_r R^2\right)}} \quad \text{(Rad/s)} \quad \text{.......... (6)}$$

Then equation (4) will take the following final form:

$$(1-I_R\cos 2\theta)\frac{d^2\alpha}{d\theta^2}+2I_R\sin 2\theta\frac{d\alpha}{d\theta}+\frac{1}{r^2}\alpha=-I_R\sin 2\theta \quad \text{.......... (7)}$$

3- Theoretical calculations:

Equation (7) gives a direct relationship between the angle of twist and the angle of rotation for the crankshafts, so it was selected in this research using ($\varepsilon = 0.34$) and (r=1/12) [8]. Solutions for this equation must be obtained firstly before applying for any reciprocating engine with a different number of cylinders. Runge-Kutta method is the most widely used method of solving differential equations with numerical methods. Mathematica can efficiently solve a set of ordinary differential equations based on the Runge-Kutta method by a function called NDSolve [9]. Comparing with the mass-spring system, the initial conditions necessary to solve this ODE are as follows [8, 10]:

1. The angle of twist caused by inertia variation (γ) at (θ_1=0) is negligible.
2. Derivative of (γ) with respect to (θ_1) is equal to zero at (θ_1=0).

The phase difference between consequence ignition processes depends on the number of cylinders and the arrangement of them. In this research, (In-line) arrangement was studied with number of cylinders of (1, 2, 3, 4, and 6). Thus the angles of phase for the tested engines will be as in table-1.

Table-1 Phase angles for the tested engines

Engine	System of arrangement	No. of cylinders	Phase angle
1	In-line	1	4π
2	In-line	2	2π
3	In-line	3	1.333π
4	In-line	4	π
5	In-line	6	0.667π

4- Results and discussion:

The use of the matlab program (listed in appendix I) and the mathematica program (listed in appendix II) with phase angles illustrated in (table-1) gives the same results. Results are interpolating functions with interval of (0 - 4π) radians which represent two revolutions of crankshafts (four strokes) so that solutions of equation (7) repeated themselves every (4π) radians for all five selected engines Those solutions were visualized as diagrams of angles of crank twist against angle of crank rotation as in figures (1, 2, 3, 4, and 5) for engines with a number of cylinders of (1, 2, 3, 4, and 6) respectively (Y-axis represents angle of crank twist while x-axis represents crank rotation angle). Table-2 illustrates numerical values of angles of twist for every (0.5 rad) of rotation.

Most researches that deal with analyzing crankshaft torsional vibrations consider a constant inertia for each crank unit and study only the torsional vibrations caused by the main torque during running (i.e. it represents crankshaft as a number of high inertia discs connecting with a mass less shaft). Gregory [8] carried out further investigations by constructing solutions of the non-linear equation for an idealized single cylinder engine system and improved that the above assumption is wrong since the inertia of each crank unit of crankshaft will be a function of crank angle of rotation.

Our results agree with Gregory results for a single cylinder engine and show that crankshafts will be twisted regardless of the main effect of pressure variation inside cylinders due to the effect of the cyclic variation of inertia of the reciprocating parts. The use of programming in solving equation (7) gives us a good ability to deal with multi cylinder reciprocating engines. Results show that the number of cylinders is a very effective parameter in this process. In the case of (In-line) reciprocating engines, increasing the number of cylinders does not mean always that an increasing in crank twist will be occurred for the same dimensions of pistons, connecting rods, and crank radius. That was because we took into account the variation of crank units inertia during running (increasing of inertia of any crank unit will be consequent with decreasing of inertia of the next unit and vise versa). That was very significant from figures (3 and 5). Maximum amplitude occurred with an engine of 4-cylinders while the minimum amplitude occurred with the single cylinder engine. That was because in case of the crankshaft of a 4-cylinder engine and for every angle of rotation there are two pistons have the same location between top and bottom dead centre while in the opposite side (single cylinder engine) there are only one piston at each location inside the cylinder so it will have half amplitude of that of 4-cylinder engines.

Engines with (3 and 6) cylinders will oscillate with an equal frequency because they have the same angular arrangement of their crank units which is larger than of oscillation of engines with single cylinder. And an engine with a single cylinder will oscillate with a similar frequency that of engines of (2 and 4) cylinders.

Table-2 A sample of angle of twist for the tested engines via angle of rotation

θ1(rad)	γ (rad)				
	Engine with single cylinder	Engine with two cylinders	Engine with three cylinders	Engine with four cylinders	Engine with six cylinders
0	0	0	0	0	0
0.2 π	-0.002	-0.004	0.0015	-0.008	0.003
0.4 π	-0.0015	-0.003	0.0023	-0.0062	0.0045
0.6 π	0.0013	0.0026	-0.0047	0.0052	-0.0094
0.8 π	0.002	0.004	0	0.008	0
π	0	0	0	0	0
1.2 π	-0.002	-0.004	0.002	-0.009	0.004
1.4 π	-0.0017	-0.0034	-0.0018	-0.007	-0.0037
1.6 π	0.0012	0.0024	-0.0037	0.0048	-0.0074
1.8 π	0.002	0.004	0.0016	0.008	0.0032
2 π	0	0	0	0	0
2.2 π	-0.0025	-0.005	0.0008	-0.01	0.0016
2.4 π	-0.0017	-0.0034	-0.0047	-0.0068	-0.0094
2.6 π	0.0013	0.0025	0	0.005	0
2.8 π	0.0023	0.0047	0.0021	0.009	0.0041
3 π	0	0	0	0	0
3.2 π	-0.0026	-0.0051	-0.0009	-0.01	-0.0018
3.4 π	-0.0015	-0.003	-0.0042	-0.0061	-0.0083
3.6 π	0.0015	0.003	0.0038	0.006	0.0076
3.8 π	0.0025	0.005	0.001	0.01	0.002
4 π	0	0	0	0	0

5- Conclousions:

1. Reciprocating engines vibrates with varying inertias and this variety in inertias will leads to additional torsional vibrations depending on the number of cylinders.
2. Increasing the number of cylinders of a reciprocating internal combustion engine does not mean always that an increasing in angle of twist (caused by inertia forces) will be occurred.
3. An engine of 4-cylinders has maximum inertias torsional vibrations amplitude while the minimum amplitude occurred with the single cylinder engine.
4. Inertias torsional oscillation of six-cylinder engines is equal to that of three-cylinder engines and more than of (2 and 4) cylinders engines.

References

[1] **Mohtasebi S. S., Ghasemi-Varnamkhasti M., Mohtasebi S. M. and Zabolestani M. ,** " Investigation of internal combustion engine vibrations in order to reduce noise using a developed model", Journal of Food, Agriculture & Environment, Vol.7 (3&4), 2009, page 915.

[2] Meirelles P. S., Zampieri D. E., Mendes A.S., " Mathematical Model for Torsional Vibration Analysis in Internal Combustion Engines", 12th IFToMM World Congress, France, June18-21, 2007

[3] موفق علي توفيق و احسان عبد الحسين باقر، "تخمين ضغط اسطوانة محرك الاحتراق الداخلي الآني باستخدام تحليل الاهتزازات لعمود المرفق"، مجلة الهندسة والتكنولوجيا، المجلد ٢٦ ، العدد3 ، 2008، صفحة 62.

[4] Gyan P., Ginoux S., Champoussin J.C., “Crank angle based torque estimation: mechanistic/stochastic”, SAE technical paper, 2000-01-0559, March 2000

[5] Steeds W. and others, “The motor vehicle”, thirteenth edition, Butterworth, 2001, page 31.

[6] Alan P. Druschitz and Robert J. Warrick, "Influence of Crankshaft Material and Design on the NVH Characteristics of a Modern, Aluminum Block, V-6 Engine", Society of Automotive Engineers, Michigan, 1999.

[7] Bernard C., “Diesel engine reference book”, second edition, Linacre House, 1999, page 240

[8] Pasricha M.S. & Hashim F.M., “Effect of the reciprocating mass of slider-crank mechanism on torsional vibrations of diesel engine systems”, AJSTD, Vol.23 Issues 1&2, 2006, page 71-81.

[9] Pao Y. C., “Engineering Analysis: Interactive Methods and Programs with FORTRAN, Quick BASIC, MATLAB, and Mathematica”, CRC press, 2001, page 228.

[10] Haberman R., "Mathematical models", Society for industrial and applied mathematics, Philadelphia, 1998, page 20.

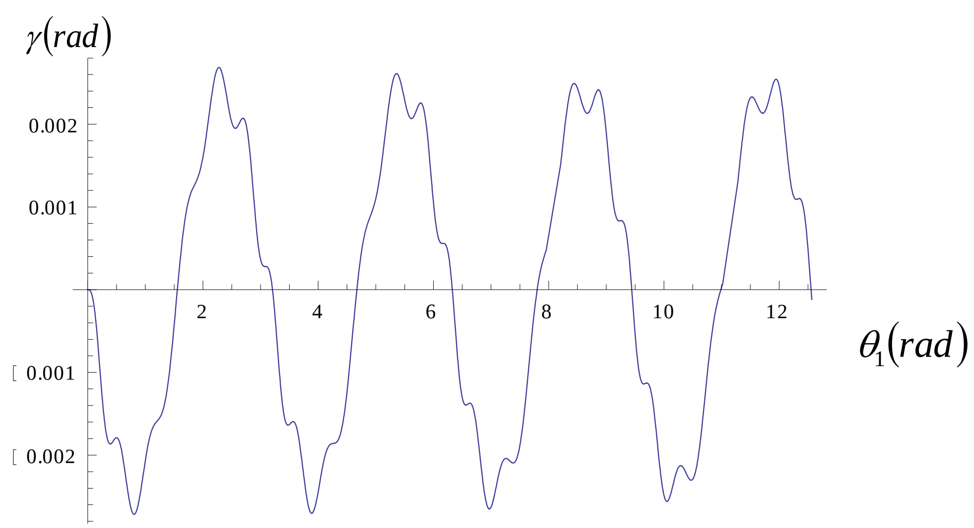


Fig. (1) crank twist against angle of rotation for engine with single cylinder

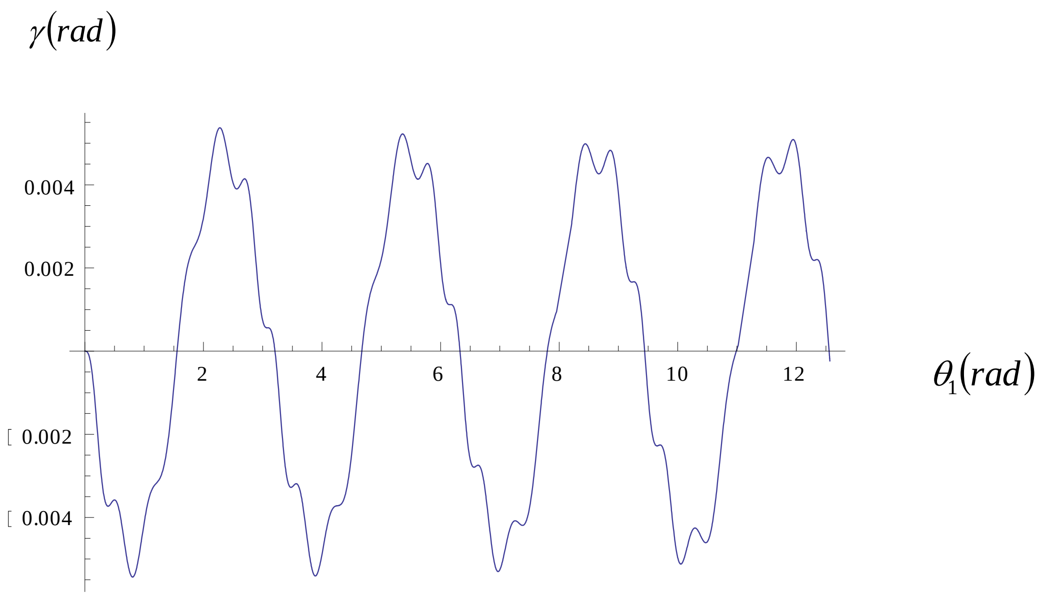


Fig. (2) crank twist against angle of rotation for engine with 2-cylinder

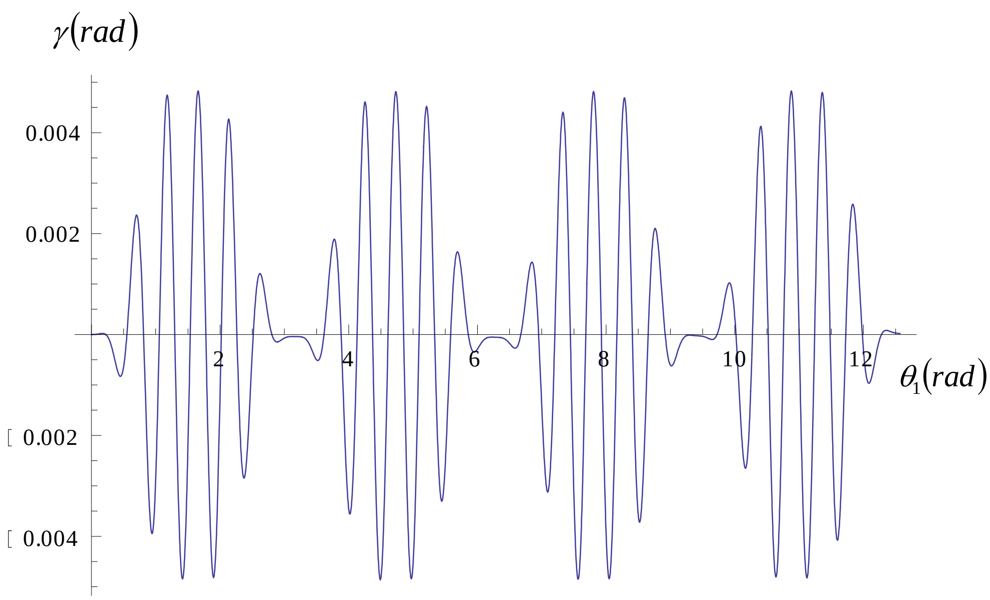


Fig. (3) crank twist against angle of rotation for engine with 3-cylinder

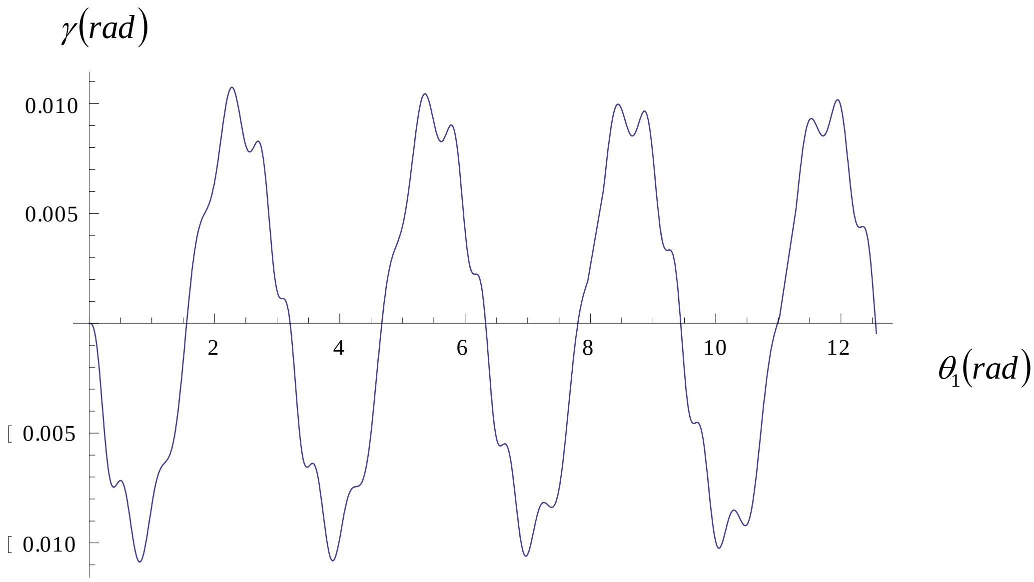


Fig. (4) crank twist against angle of rotation for engine with 4-cylinder

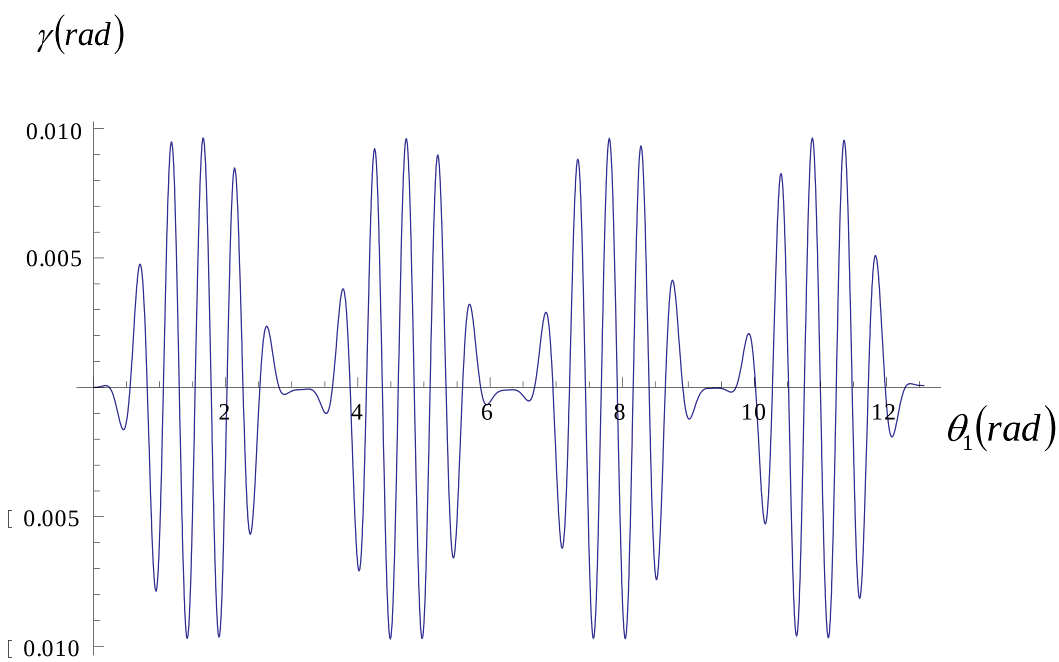


Fig. (5) crank twist against angle of rotation for engine with 6-cylinder

Appendix I

(Matlab program used in solving equation 5)

```
function [ y , z , x ] = webren4 ( f , g , y0 , z0 , a ,b , n )
h = ( b – a ) / n ;
x ( 1 ) = a ;  y ( 1 ) = y0 ; z ( 1 ) = z0 ;
for I = 1 : n
k11 = h * feval ( f , x (i) , y (i) , z (i) );
k21 = h * feval ( g , x (i) , y (i) , z (i) );
k12 = h * feval ( f , x (i) + (1/2) * h , y (i) + (1/2) * k11 , z (i) + (1/2) * k21);
k22 = h * feval ( g , x (i) + (1/2) * h , y (i) + (1/2) * k11 , z (i) + (1/2) * k21);
k13 = h * feval ( f , x (i) + (1/2) * h , y (i) + (1/2) * k12 , z (i) + (1/2) * k22);
k23 = h * feval ( g , x (i) + (1/2) * h , y (i) + (1/2) * k12 , z (i) + (1/2) * k22);
k14 = h * feval ( f , x (i) + h , y (i) + k13 , z (i) + k23);
k24 = h * feval ( g , x (i) + h , y (i) + k13 , z (i) + k23);
y ( i + 1 ) = y (i) + (1/6) * ( k11 + 2 * ( k12 + k13 ) + k14 );
z ( i + 1 ) = z (i) + (1/6) * ( k21 + 2 * ( k22 + k23 ) + k24 );
x ( i + 1 ) = x (i) + (0.2) * pi * h;
end
disp ( ' y(i)              z(i)           x(i) ' )
disp('------------------------------------------')
```

Appendix II

(Mathematica program used in solving equation 5)

```
a=NDSolve[{(1-0.34 Cos[2*θ1]) γ''[θ1]+(0.68 Sin[2*θ1])
γ'[θ1]+(144+0.68 Cos[2*θ1]) γ[θ1]□-
0.34*Sin[2*θ1],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
b=NDSolve[{(1-0.34 Cos[2*(θ1+2 π)]) γ''[θ1]+(0.68 Sin[2*(θ1+2 π)])
γ'[θ1]+(144+0.68 Cos[2*(θ1+2 π)]) γ[θ1]□-0.34*Sin[2*(θ1+2
π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
d=NDSolve[{(1-0.34 Cos[2*(θ1+1.33 π)]) γ''[θ1]+(0.68 Sin[2*(θ1+1.33
π)]) γ'[θ1]+(144+0.68 Cos[2*(θ1+1.33 π)]) γ[θ1]□-0.34*Sin[2*(θ1+1.33
π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
e=NDSolve[{(1-0.34 Cos[2*(θ1+2.67 π)]) γ''[θ1]+(0.68 Sin[2*(θ1+2.67
π)]) γ'[θ1]+(144+0.68 Cos[2*(θ1+2.67 π)]) γ[θ1]□-0.34*Sin[2*(θ1+2.67
π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
```

```
g=NDSolve[{(1-0.34 Cos[2*(θ1+π)]) γ''[θ1]+(0.68 Sin[2*(θ1+π)])
γ'[θ1]+(144+0.68 Cos[2*(θ1+π)]) γ[θ1]□-
0.34*Sin[2*(θ1+π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
i=NDSolve[{(1-0.34 Cos[2*(θ1+3 π)]) γ''[θ1]+(0.68 Sin[2*(θ1+3 π)])
γ'[θ1]+(144+0.68 Cos[2*(θ1+3 π)]) γ[θ1]□-0.34*Sin[2*(θ1+3
π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
k=NDSolve[{(1-0.34 Cos[2*(θ1+0.67 π)]) γ''[θ1]+(0.68 Sin[2*(θ1+0.67
π)]) γ'[θ1]+(144+0.68 Cos[2*(θ1+0.67 π)]) γ[θ1]□-0.34*Sin[2*(θ1+0.67
π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Clear[θ1]
o=NDSolve[{(1-0.34 Cos[2*(θ1+3.33 π)]) γ''[θ1]+(0.68 Sin[2*(θ1+3.33
π)]) γ'[θ1]+(144+0.68 Cos[2*(θ1+3.33 π)]) γ[θ1]□-0.34*Sin[2*(θ1+3.33
π)],γ[0]□0,γ'[0]□0},γ,{θ1,0,4 π}]
Plot[Evaluate[γ[θ1]/.a],{θ1,0,4 Pi}]
Plot[Evaluate[{γ[θ1]/.a}+{γ[θ1]/.b}],{θ1,0,4 Pi},PlotRange→All]
Plot[Evaluate[{γ[θ1]/.a}+{γ[θ1]/.d}+{γ[θ1]/.e}],{θ1,0,4
Pi},PlotRange→All]
Plot[Evaluate[{γ[θ1]/.a}+{γ[θ1]/.g}+{γ[θ1]/.b}+{γ[θ1]/.i}],{θ1,0,4
Pi},PlotRange→All]
Plot[Evaluate[{γ[θ1]/.a}+{γ[θ1]/.k}+{γ[θ1]/.d}+{γ[θ1]/.b}+{γ[θ1]/.e}+{γ
[θ1]/.o}],{θ1,0,4 Pi},PlotRange→All]
Do[Print["θ1",θ1,"γ1",γ[θ1]/.a,"γ2",{γ[θ1]/.a}+{γ[θ1]/.b},"γ3",{γ[θ1]/.a
}+{γ[θ1]/.d}+{γ[θ1]/.e},"γ4",{γ[θ1]/.a}+{γ[θ1]/.g}+{γ[θ1]/.b}+{γ[θ1]/.i}
,"γ6",{γ[θ1]/.a}+{γ[θ1]/.k}+{γ[θ1]/.d}+{γ[θ1]/.b}+{γ[θ1]/.e}+{γ[θ1]/.o}]
,{θ1,0,4 Pi,0.2Pi}]
```