

Nomenclature

<i>ACO</i>	Ant Colony Optimization	<i>IBH</i>	Improved Black Hole Algorithm
<i>ARI</i>	Adjusted Rand Index	<i>K_{means}</i>	K-means Clustering Algorithm
<i>BA</i>	Bat Algorithm	<i>ML</i>	Machine Learning
<i>BB</i>	Big Bang Algorithm	<i>N</i>	Number of stars (population size)
<i>BB</i>	Hybrid Big Bang	<i>PSO</i>	Particle Swarm Optimization
<i>BH</i>	Black Hole Algorithm	<i>R</i>	Event horizon radius
<i>C</i>	Random coefficient or movement matrix	<i>SA</i>	Simulated Annealing
<i>D</i>	Number of data dimensions (features)	<i>SC</i>	Silhouette Coefficient
<i>dpi</i>	Dots per inch (image resolution)	<i>t</i>	Current iteration
<i>FA</i>	Firefly Algorithm	<i>t + 1</i>	Next iteration
<i>f_{BH}</i>	Fitness value of the black hole	<i>rand</i>	Random number uniformly distributed in [0,1][0,1]
<i>f_i</i>	Fitness value of the <i>i</i> th star	<i>K</i>	Number of clusters
<i>GA</i>	Genetic Algorithm	<i>WCSS</i>	Within-Cluster Sum of Squares
<i>GMM</i>	Gaussian Mixture Model	<i>XBH</i>	Position of the black hole (best solution)
<i>GSA</i>	Gravitational Search Algorithm	<i>X_i</i>	Position of the <i>i</i> th star (candidate solution)

A comprehensive experimental evaluation is conducted on multiple benchmark datasets, with comparisons against classical clustering methods and state-of-the-art metaheuristic algorithms. Statistical significance tests, including the Wilcoxon signed-rank and Friedman tests, are performed to validate the robustness and superiority of the proposed method. The remainder of this paper is organized as follows. Section 2 reviews the Black Hole algorithm and its limitations. Section 3 presents the proposed Improved Black Hole and hybrid BB–BH clustering framework. Section 4 describes the experimental setup and evaluation metrics. Section 5 discusses the experimental results and statistical analysis. Finally, Section 6 concludes the paper and outlines future research directions.

1.1 Black hole phenomenon

Black holes were initially identified by the pioneers John Michell and Pierre-Simon Laplace in the eighteenth century. Using Newton's principles of motion, they hypothesized the existence of stars so massive that their gravitational pull makes them invisible to the human eye. At that time, the term "black hole" hadn't yet been established. The phrase "black hole" was not introduced until 1967, when American physicist John Wheeler articulated the notion of mass that temporarily manifests before imploding inside. A black hole is created when a collapsing star compresses and spins at an extremely fast rate. Its gravity becomes so strong that light can't even escape from it. This massive gravity is the product of matter being squashed into an astronomical width. Anything that passes the boundary of a black hole called the event horizon will be pulled in and lost for eternity, never to emerge. The gravitational attraction of an abyss is of colossal capacity [5]. The round line that encircles an abyss is commonly referred to as the occurrence skyline. The distance from the center of the abyss to this line is named the Schwarzschild range. At this particular range, the escape velocity the speed needed to break free of the abyss's gravitational attraction is close to the speed of light. In a suitable, earlier state, some form of matter or even light crosses this threshold and can no longer escape. This condition is because nothing in outer space can travel faster than light, Eq. 1.

$$R = \frac{2GM}{C^2} \quad (1)$$

Point G denotes the gravitational field, M represents the mass of the abyss, and c indicates the speed of light. Any object that reaches the event horizon or crosses the Schwarzschild radius will be consumed by the singularity and will no longer exist endlessly. The presence of inky dents may be explained by the attractive influence of nearby items [6–8].

2. Black hole algorithm (BH)

The Black Hole (BH) algorithm is a population-centric optimization technique that exhibits numerous similarities with other population-based algorithms [9]. These procedures initially generate a population of potential responses and disseminate them indiscriminately within the search room. The purpose of such algorithms is to search out iterative progress of this population toward an optimum answer through specific machines. Exemplification: In Genetic Algorithms (Smoke), public evolution is attained through movements such as metamorphosis and crossover. In Piece Swarm Optimization (PSO), applicant answers are updated by mobile through the search scope using counseling from the best choice positions discovered up until now. These positions are continuously updated as better solutions emerge. Conversely, the suggested BH algorithm advances the population by directing all candidate solutions toward the optimal solution identified in each iteration, known as the black

hole. Candidates that attain a specified proximity to this black hole are designated as swallowed and subsequently substituted with newly produced, random candidates inside the search space. The term "abyss" was first secondhand in the context of addition in a prior study [8], where it was brought in as part of a changed PSO invention. Still, that means it differs considerably from the BH treasure projected in this place paper. Specifically, the plan in [8] integrates a new method into PSO; in place of each redundancy, a new particle is created from the most familiar, highest-ranked known piece. Therefore, having established two carelessly generated numbers, the invention renews the positions of pieces either utilizing the standard PSO update rules or this new machine. Essentially, the plan in [10] offers PSO by introducing an "abyss" piece to help hasten union and avoid local pinnacles of achievement or physical objects. Nevertheless, the concept of an event horizon and the elimination of candidate solutions are not included in the approach proposed in [11]. In contrast, the BH algorithm projected in this place paper is more approximately aligned with the accompanying unaffected abyss phenomenon noticed in the study of the stars and planets other than earth and is fundamentally different from the abyss PSO approach. In our BH treasure, the best bidder in the community during each redundancy is named as the abyss, while all other bidders are considered as sane stars. Different from the pattern in [?], the black hole in this place of invention is not created randomly; it is a real appendage of the population accompanying the best choice appropriateness value. Before, each applicant is moved toward the abyss established by the allure of the current position and a randomly produced determinant. Contenders that also get near the black hole, that is, fall inside the occurrence horizon, are thought to be consumed and are replaced by recently produced resolutions scattered throughout the search room. The detailed procedures for the expected BH prize are explained herein. Similar to other population-based algorithms, the BH algorithm commences with an initial random population of potential solutions, referred to as stars, which are distributed across the search space of a certain issue or objective function. In the subsequent step, the overall fitness of all individuals is assessed. The solution with the best fitness is termed a black hole, whereas the other possibilities are classified as ordinary stars. Nevertheless, the technique presented in [64] is deficient in its incorporation of the event skyline notion and the obliteration of stars, which pertains to the dismissal of candidate solutions. Conversely, the BH method introduced in this research aligns more closely with the unmodified abyss phenomenon observed in the study of celestial bodies beyond Earth and is essentially distinct from the abyss PSO technique. In our BH treasure, the best bidder in the community during each redundancy is named as the abyss, while all other bidders are considered as sane stars. Different from the pattern in [?], the black hole in this place of invention is not created randomly; it is a real appendage of the population accompanying the best choice appropriateness value. Before, each applicant is moved toward the abyss established by the allure of the current position and a randomly produced determinant. Contenders that also fall into the black hole, that is, fall inside the occurrence horizon, are thought to be consumed and are replaced by recently produced resolutions scattered throughout the whole of the search room. The detailed steps of the projected BH treasure are defined below, Eq. 2 up to Eq. 5.

$$U(0, 1.5) \sim_{i_j} \mathbb{R}^{K \times D}, C \ni C \quad (2)$$

$$({}^t_i \mathbf{X}_{BH} - \mathbf{X}) \odot \mathbf{C} + {}^t_i \mathbf{X} = {}^{t+1}_i \mathbf{X} \quad (3)$$

$$\frac{{}^{BH} f}{{}^i f \sum_{i=1}^N} = R \quad (4)$$

$$R \geq \|_{BH} \mathbf{X} - \mathbf{X}_i\| \quad (5)$$

2.1 Applied black hole algorithm (IBH)

2.1.1 Hybrid Big Bang–Improved Black Hole (BB–IBH) Algorithm

To further strengthen global exploration, the **Big Bang (BB)** mechanism is integrated into the regeneration phase. When multiple stars are absorbed within the same iteration, the BB strategy redistributes new stars randomly across the search space. This hybridization provides:

- **IBH** → strong exploitation near promising solutions.
- **BB** → robust exploration and escape from local optima.

The applied Black Hole (**IBH**) algorithm enhances the flexibility of star movement and the diversity of new solution generation in the exploration space. As illustrated in Fig. 1, the generation of new stars in IBH follows one of two adaptive strategies:

1. Uniform random generation: New stars are generated using a uniformly distributed random number generator across the defined search space.
2. Genetic crossover: Two stars are selected randomly from the population, and a crossover operation is applied to produce a new solution.

2.1.2 The star encrypting and initial culture production

- In IBH, each star shows a potential resolution to the grouping question. A star is encrypted as a mold of proportion $[(N \times DN) \cdot (DN \times D)]$, place:
- NNN represents the number of clusters.
- DDD represents the number of data attributes.

Each row of the matrix represents the coordinates of a cluster center in the feature space. The matrix entries can be of different types depending on the problem (e.g., floating-point, integer, byte). The appropriateness of each star is premeditated utilizing a predefined objective function, usually having a connection with following a time-cluster correspondence or distance reduction. Previously, the encryption was defined, and a beginning culture of superstars was produced randomly across the search scope. These are judged before, and the star with the highest rank appropriateness is picked as the primary black hole.

2.1.3 Star Movement Strategy

In the original BH treasure, star positions are restored utilizing Eq. 3, placing all attributes to advance the abyss apiece with the same haphazard determinant. This forges a straight-line way from each star to the abyss. To embellish the invention's changeability across various question types, IBH replaces the scalar chance profit accompanying a vector of chance principles, allowing for liberated updates of each attribute. This approach, initially projected by Yaghoobi and Mojallali (2016), corrects the investigation of the room surrounding the abyss. In our IBH variant, the campaign is cultured further: a haphazard matrix CCC of height $(1 \times D_1) \cdot (D_1 \times D)$ is created with accompanying values in the range $[0, 1.5][0, 1.5][0, 1.5]$ to prevent attribute principles from surpassing their genuine bounds. Each attribute of every cluster in the star is renovated a piece at an unchanging amount from this cast. The updated drive equating enhances Eq. 6.

$$\begin{aligned} X_i(t+1) &= X_i(t) + C_x(X_{BH}(t)X_i(t))(4)X_i(t+1) \\ &= X_i(t) + C \times (X_{BH}(t) - X_i(t))4X_i(t+1) \\ &= X_i(t) + C_x(X_{BH}(t)X_i(t))(4) \end{aligned} \quad (6)$$

- $X_i(t)$, $X_i(t)$ $X_i(t)$ are the position of the i^{th} star at iteration ttt.
- $X_{BH}(t)$, $X_{BH}(t)$, $X_{BH}(t)$ are the position of the black hole.
- CCC is a $(1 \times D_1) \times (D_1 \times D)$ random matrix of uniformly distributed values in $[0, 1.5][0, 1.5][0, 1.5]$.

This game plan allows stars to adaptively investigate various guidelines in the search scope while maintaining control over borderline defilements. As the invention progresses, heroes gradually move closer to the abyss. If a star crosses the occurrence skyline, i.e., the allure distance from the abyss is equal to the Schwarzschild radius, it is thought to be preoccupied. Late captivated, the star is removed from the current community to claim variety and counter premature union. To reinstate the captured star, a new star is created using one of the following two adjusting plans:

2.1.4 Uniform Random Generation (Probability = 0.75)

With a probability of 75%, a new star is generated using a uniformly distributed random number generator across the defined search space. This method promotes exploration by introducing new, diverse candidate solutions.

2.1.5 Crossover-Based Recombination (Probability = 0.25)

With a probability of 25%, two stars are randomly selected from the existing population. A recombination (crossover) method is applied to these two parents to produce a new offspring solution. Among the recombined candidates, the best one (based on fitness) is selected and added to the population. This method improves exploitation by taking useful features from existing solutions and putting them together. These strategies are designed to balance exploration and exploitation dynamically, ensuring that the algorithm avoids stagnation and maintains a diverse population throughout the optimization process. The performance of the proposed hybrid BB–IBH algorithm was evaluated against classical and metaheuristic clustering algorithms over 30 independent runs. Overall, the proposed BB–IBH algorithm consistently achieved *lower* WCSS values and higher ARI and Silhouette scores across most benchmark datasets, indicating more compact and well-separated clusters. The hybrid algorithm demonstrated superior robustness, exhibiting smaller performance variance compared to other metaheuristic approaches.

2.1.6 Comparison with baseline algorithms

In comparison to traditional techniques like K-Means and GMM, the BB–IBH algorithm demonstrated markedly superior clustering quality, especially on datasets characterized by elevated dimensionality and intricate cluster formations. The suggested method surpassed the original BH algorithm across all assessment criteria when compared to metaheuristic-based clustering techniques (PSO-KMeans, GA-KMeans, GSA, BH, and BB–BC). This verifies that the implemented enhancements successfully rectify the exploration–exploitation imbalance inherent in the conventional BH method.

2.2 Wording of abyss treasure black hole

In Chasm Treasure, an excellent contestant among all the seekers at each repetition is chosen as a chasm. Heroes: All the different competitors form the realistic names. The blend of the depth is not haphazard, and it is individual to the physical candidates of people as a political whole. Shift: Before, all the competitors moved towards the depth, settled their current position, and had a chance number.

1. Depth treasure (depth) starts accompanying a public origin of rival judgments to an adding question and an objective function that is, so to speak, persistent for the ministry.
2. At each repetition of the Depth, the best choice opponent is chosen, anticipating the chasm and so forth from the reasonable superstars. Following the initialization process, the abyss starts interesting names about it.
3. If a star gets extravagantly close to the depth, it will be consumed by the depth and disappear endlessly.

In an earlier case, a new star (competitor answer) is heedlessly produced and settled in the search scope, and starts a new search. Fitness Value Calculation for Black Hole Algorithm:

1. Population Initialization: $P(x) = x_1^t \cdot x_2^t \cdot x_3^t \dots x_n^t$ represents a randomly generated population of potential solutions (the stars) situated within the search space of a specific problem or function.
2. Calculate the summation of fitness of the population. $f_i = \sum_{i=1}^{pop\ size} eval(p(t))$ and $f_{BH} = \sum eval(p(t))$

Where f_i , f_{BH} are the suitability standards of abyss and i^{th} star in the initialization. The public is assumed, and the chief in rank applicant (from surplus names) in the civilization, that has high-quality usefulness benefit, f_i is picked wonted the depth and the surplus form the normal darlings. The depth has the ability to consume the future that encloses it. Following the preparation or occasion, initializing the first abyss and numbers one, the chasm starts enthralling fate about it, and chance starts affecting the depth. Incorporation Rate of Names by Abyss. The depth starts to become noticeable, chance about it in addition to the future affecting the depth. The inclusion of deities by the depth is projected in this place, approach Eq. 7.

$$X_i(t) = X_i(t) + rand \times (X_{BH} - X_i(t)) \quad (7)$$

where $i = 1; 2; 3; \dots, n$, X_i^t and X_i^{t+1} represent the components of the i^{th} star at moments t and $(t+1)$, respectively. X_{BH} represents the domain of the chasm inside the search scope, whereas $rand$ denotes a random number within the interval $[0, 1]$. N represents the quantity of heroes (contestant evaluations). A star, while moving deeper, recognizes the opportunity to access a realm at a cheaper cost than the abyss. In a specific instance, the depth shifts to the domain of that star, accompanied by the transposed order. Consequently, the depth will continue to pursue the depth in the new position before names begin to gravitate towards this new domain.

Table 1. Writing of bio-stimulated algorithms.

No.	Metaheuristic algorithms	Description of algorithms
1	Historical Algorithms (Smoke) [14]	Ancestral treasure is a search and development located methods that progress a society of competitor resolutions to a likely question, employing open ancestral alternative and evolutionary theory jockeys.
2	Fake annealing(SA) treasure [15]	Fake Annealing is advanced by establishing the facilitating annealing process and significantly reduces the temperature (T).
3	Restlessness Colony growth (ACO) [16]	Dissatisfaction Community Adjoining is aroused from the presence of a real dissatisfaction Community, that is to say, to say bright to find the shortened course middle from two points allure home and a food origin (goal).
4	Atom swarm addition (PSO)algorithm [17]	Parasite Swarm Happening is developed in contact with the swarm type to a degree, companion and poultry apprenticeship in type.
5	The Gravitational search treasure(GSA) [18]	The standard of relevance and the framework of content interactions have been collectively determined. In the GSA treasure, the corner competencies are a collective of governmental entities that adhere to the established Newtonian principles and laws of motion.
6	Brilliant water drops treasure [19]	It is enticed from suspicious reflex water drops that flow in waterways and in what habit or conduct rude waterways find nearly best dresses to their aim. In the IWD treasure, miscellaneous fictitious water drop evident to change the atmosphere in earlier dress that a high-quality course is established as the individual follows crude soil on allure links.
7	Nocturnal luminescent beetle invention (FA) [20]	The Nocturnal luminescent beetle invention (FA) was invigorated by each shimmering stance of fireflies in formation. FA is a form of enticement that disrupts or disturbs the flashing patterns and characteristics of fireflies. This involves the analysis of secondhand files and the categorization of matched groups of objects based on their qualities.
8	Honey-making mating growth(HBMO) treasure [21]	It is aroused in each process of consolidation in valid companion bees.
9	Sock invention (BA)	Each echolocation characteristic of the belt is enticing. The efficacy of the echolocation of bats is commendable, as they can accurately locate and visualize many insect species despite any imperfections.
10	Unity search growth treasure	It is aroused for one devising process ofart active distant of sounds that are friendly, harmonized. The movement of verdict, the wholeness in sounds that are friendly, harmonized, is complementary to decree the optimum determination in an adding process.
11	Important Bang–Large Crunch (Bb–By)addition	The individual is determined by the trusts regarding the occurrence of outer space. The tranquility of the mature explosion and many crunching footsteps. The applicant's evaluations are randomly distributed within the search range, and during the crucial moment of the curtailment process, a balance point for the organization is calculated.
12	Abyss (BH) treasure	It is enthusiastic about personal profound curiosity. The concept of insight is fundamentally a principle of a circle with significant concentration, which renders it intolerable for a close criminal to escape the gravitational pull. The entirety is relinquished into an abyss, as the property, grasped lightly, gradually dissipates from our completeness in amalgamation.

5.1.2 Data clustering

The objective of file assembly is to identify the fundamental composition of a collection of patterns, points, or objects through popular or selected cluster interpretation searches. A functional composition regarding the assembly potentially established in this location's tone: Identify K groups of n objects so that the relationships among objects within the same group are highly correlated, while the relationships between objects from different groups exhibit minimal correlation, indicative of clustering.

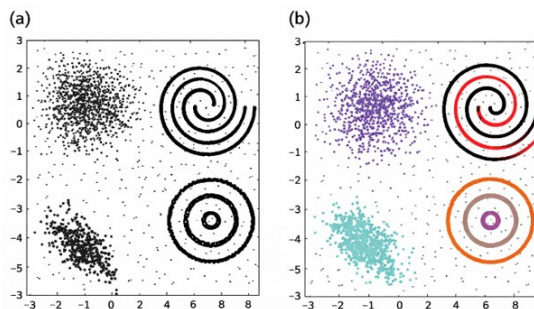


Figure 3. The seven clusters in (a) are associated with seven distinct banners in 1, (b) exhibit systematic organization, variation, and scale.

Nevertheless, these clusters remain discernible to a file agent; none of the probable assembly methods can reveal them all. A recommendation document. b Desired composition reduced. Figure 3 demonstrates that clusters encounter practical conflicts regarding their form, duration, and elevation in contracts. The ownership of ejection in the file forms the judgment of the clusters, rendering them increasingly problematic, with the ideal cluster potentially delineated as a set of points that are both small and distinctive. Although bulks are exceptional possibilities for clustering in two and maybe three ranges, we require

machine-like techniques for extremely large spatial files. This challenge, along with the resulting number of clusters for the relevant file, has influenced 1000 composition techniques that have been documented regarding their operational impact. An example of organization is typically found in the smallest quantity. In pattern declaration Fig. 4, comprehending the file is a compelling issue that culminates in execution: it is probable that many files do not exist, necessitating the anticipation of the utilization of the confidential test file.

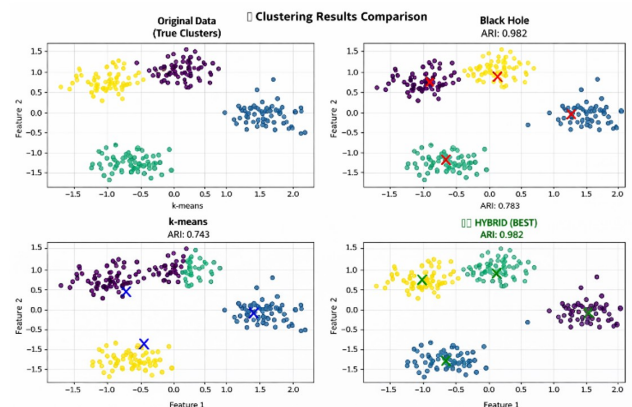


Figure 4. Clustering results comparison.

5.2 Coding results

The performance of the proposed hybrid BB–IBH algorithm was evaluated against classical and metaheuristic clustering algorithms over 30 independent runs, see Appendix-I. Overall, the proposed BB–IBH algorithm consistently achieved lower WCSS values and higher ARI and Silhouette scores across most benchmark datasets, indicating more compact and well-separated clusters.

