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Stability of the universality class of phase transitions against spin interaction variations in the clean two-dimensional square-lattice Ising chain

Khansaa Y. Taha[1], **Ahmed K. Ibrahim**[1*]

[1]Department of Physics, College of Science, University of Anbar, Ramadi, Anbar, Iraq

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Corresponding author
Ahmed K. Ibrahim
akibrahim@uoanbar.edu.iq

ABSTRACT

We perform a comprehensive numerical study of how variations in microscopic coupling strength affect critical behavior in the two-dimensional Ising model. Using extensive Monte Carlo simulations based on the Wolff cluster algorithm, we investigate the clean Ising model on a square lattice for interaction strengths in the range $0.80 \leq J \leq 1.20$. While the exact solution at $J = 1$ has long established the model's critical properties, a systematic numerical assessment of universality under continuous coupling rescaling has not been performed. Here, we address this gap by analyzing systems of linear size $\mathrm{L} = 20-500$, allowing for high-precision finite-size scaling of thermodynamic observables. For each coupling, the critical temperature $T_C(J)$ is determined via finite-size extrapolation of the susceptibility maxima, yielding results in quantitative agreement with Onsager's exact prediction. At the extrapolated critical points, we extract the critical exponents ν, β, γ, and α through detailed scaling analyses. These exponents remain consistent with the exact Ising benchmarks across the full range of interaction strengths, confirming that the universal aspects of the phase transition are preserved. Our findings demonstrate that modifying the microscopic coupling strength rescales the critical temperature without altering the asymptotic critical behavior, thereby providing a clear and quantitative confirmation of the robustness of the two-dimensional Ising universality class.

Keywords: *Ising Model, Phase Transitions, Universality Class*

1 INTRODUCTION

Phase transitions and critical phenomena are central subjects in statistical physics, as they are key to explaining the collective behavior of a large family of many-body systems [1]. One of the most remarkable findings in this context is the universality principle, which holds that systems with different microscopic features may exhibit similar critical behavior, described by universal critical exponents and scaling functions [2]. The universality class of a phase transition is largely determined by global properties such as the dimensionality of systems, symmetry of the order parameter, and range of interactions, but not by locally microscopic details of the Hamiltonian. This allows not only for a simplified theoretical study of intricate systems but also gives physicists the possibility of anticipating critical behavior in many different physical situations [3].

The Ising model is one of the simplest mathematical models in statistical mechanics and serves as a benchmark for a phase transition [4, 5]. The original application of this model was to describe ferromagnetism. However, since Onsager's remarkable solution [6] exactly solves the nearest-neighbor problem on a two-dimensional square lattice, this model and its phase transition have been accurately studied. Despite the relative simplicity of its definition in two dimensions, the Ising model has a critical point at a critical temperature, separating the low-temperature ordered ferromagnetic and high-temperature disordered paramagnetic phases [7, 8]. The critical properties of the two-dimensional Ising model were thoroughly studied using analytical calculations and numerical simulations; therefore, the so-called universality and the influence of small perturbations in the system can be most generally determined [9, 10].

While the universality of the 2D Ising model with nearest-neighbor interactions is well recognized, a significant inquiry in contemporary statistical physics revolves around the robustness of this universality class when interaction parameters are varied. In particular, one may ask whether the critical exponents can be modified when we consider small or systematic changes in the spin interaction strengths with nearest neighbors, or whether these can drive systems out of their universality class [6, 11]. This question becomes all the more relevant in real physical systems. Furthermore, such a perturbation can be studied by computer simulations, and such quantitative information for experiments with magnetic materials and cold atoms, or any other setup where the interaction strength is tunable, can be useful

In this respect, Monte Carlo simulations are a standard framework for the task. By realizing a numerical sampling of the ensemble of the spin configurations, Monte Carlo methods allow one to read off fundamental thermodynamic quantities like energy, magnetization, susceptibility, and specific heat, and critical properties, most importantly, critical exponents and correlation lengths [12, 13]. However, near the critical point, traditional local-update algorithms exhibit critical slowing down, which makes most statistical methods inefficient near criticality. Cluster algorithms, such as the Wolff algorithms, have been developed that perform several flips within a cluster, ensuring that the number of flipped spins equals the total number of spins [14].

In this work, we investigate the stability of the clean 2D square-lattice Ising model universality class by tuning nearest-neighbor spin interactions via large-scale Monte Carlo simulations. We perform the simulations using Wolff's cluster algorithm across varying system sizes, enabling us to conduct an exhaustive finite-size scaling analysis. We hope to probe whether the Ising universality class is robust to these perturbations by continuously tuning the spin interactions and observing how the critical behavior evolves along this line. This study improves our understanding of the link between microscopic interaction details and macroscopic criticality near the transition, thereby further consolidating universality principles in statistical physics. Our findings are expected to confirm that changes in spin interactions do not affect the critical exponents, consistent with the predictions of renormalization group theory, which suggests that irrelevant perturbations do not alter the universality class [15, 16]. This research offers an extensive computational viewpoint on the robustness of universality in the standard 2D Ising model by integrating large-scale numerical simulations with rigorous finite-size scaling analysis.

1.1 Theoretical background

We study the clean two-dimensional square-lattice Ising model with nearest-neighbor ferromagnetic interactions as a test bed for the stability of the universality class of phase transitions to perturbations of the spin-spin interactions. This model consists of a square lattice with N = L×L sites, and on each i site, a classical spin variable is located wit, equals, plus or minus 1, and devoid of any external magnetic field. The Hamiltonian representing the system can be expressed as [6]:

$$H = -J \sum_{<ij>} S_i S_j \tag{1}$$

where $J > 0$ is the nearest-neighbor exchange interaction, and the sum is over nearest-neighbor pairs $\langle i, j \rangle$. This model undergoes a second-order phase transition at a finite critical temperature T_C, below which spontaneous magnetization emerges and above which the system becomes disordered. In this study, we systematically vary J to investigate whether fluctuations in interaction strength affect the universality class of the phase transition in the system. Based on Renormalization Group (RG) theory, irrelevant perturbations like minor, short-range changes in interaction strength are not expected to modify these exponents [15,16]. Conversely, relevant perturbations (for instance, long-range interactions or disorder exceeding specific thresholds) may push the system into a different universality class. This kind of examination assesses the robustness of critical exponents and scaling functions in response to minor alterations in the energy scale of interactions.

In the specific scenario of an infinite clean square lattice with uniform J, Onsager [6] delivered an exact analytical solution for the free energy per spin, enabling the derivation of all thermodynamic properties. For the isotropic case, the critical temperature is expressed as:

$$T_C = \frac{2J}{K_B \ln(1+\sqrt{2})} \tag{2}$$

where K_B Boltzmann's constant. For $J = 1$ (in units where $k_B = 1$), the exact critical temperature is $T_C \approx 2.269$. In fact, in the theory of critical phenomena, systems that have the same dimensions, symmetries, and interaction range fall into a single universality class characterized by identical critical exponents such as α, β, γ, and ν, which account for the long-term scaling properties near the critical temperature. For the clean 2D Ising universality class, the critical exponents for these are predicted by Onsager's exact solution, the following exponent values: Magnetization exponent $\beta = 1/8$, susceptibility exponent $\gamma = 7/4$, correlation length exponent $\nu = 1$, and specific heat exponent $\alpha = 0$ (logarithmic divergence) [6, 17, 18].

2 COMPUTATIONAL METHOD

Monte Carlo simulations offer a powerful computational tool for the study of statistical mechanics systems that lack exact analytical solutions, or are only possible in a specific context [19–21]. For the 2D Ising model, although Onsager's exact solution is known at $J = 1$. Numerical simulations remain crucial for the study of finite-size effects, universality checks, and generalizations to variation in interaction strength. At temperatures close to the critical point, algorithms that flip single spins, like the Metropolis method [22], experience critical slowing down, resulting in divergent correlation times and the system being stuck in local spin states. To address this issue, we utilize the Wolff cluster algorithm [14, 23], which collectively updates groups of correlated spins, considerably decreasing autocorrelation times. This research utilizes extensive Monte Carlo simulations alongside the Wolff cluster algorithm to explore the critical behavior of the clean 2D square-lattice Ising model for nearest-neighbor interactions. We simulate systems with sizes ranging from 20^2 to 500^2 sites with periodic boundary conditions, and adjust the interaction strength within the range $J \in [0.8, 1.2]$. For every system size L and J, multiple simulations are conducted at various temperatures near the anticipated critical point. Thermodynamic properties like magnetization M, energy E, magnetic susceptibility χ, Specific heat C_v, and Binder cumulant U have been computed from the following quantities [21, 24–27]:

$$\langle M \rangle = \frac{1}{N} \sum_{i}^{N} S_i \tag{3}$$

$$\langle E \rangle = -J \frac{1}{N} \sum_{\langle ij \rangle} S_i S_j \tag{4}$$

$$\chi = \frac{1}{N K_B T} \left(\langle M^2 \rangle - \langle M \rangle^2 \right) \tag{5}$$

$$C_v = \frac{1}{N K_B T^2} \left(\langle E^2 \rangle - \langle E \rangle^2 \right) \tag{6}$$

$$U = 1 - \frac{\langle M^4 \rangle}{3 \langle M^2 \rangle^2} \tag{7}$$

To extract critical exponents, we employ the finite-size scaling theory. Near criticality, the correlation length behaves as $\xi \sim |T - T_C|^{-\nu}$. For a finite system of size L, the maximum possible correlation length is limited by L. At pseudo-criticality, we can match them $L \sim \xi \sim |T - T_C|^{-\nu}$, rearranging gives:

$$\frac{1}{L} \sim |T - T_C|^{\nu} \tag{8}$$

Also, for thermodynamic quantities such as the magnetization M , susceptibility χ, and specific heat C_v show singular behavior near the critical temperature T_C [9, 28–30]:

$$M(t, L) \sim L^{\frac{-\beta}{\nu}} \tag{9}$$

$$\chi(t, L) \sim L^{\frac{\gamma}{\nu}} \tag{10}$$

$$C_v(t, L) \sim \ln L \tag{11}$$

where $t = (T - T_C)/T_C$ is the reduced temperature.

All outcomes are averaged across multiple independent Monte Carlo runs. We perform 200 complete Monte Carlo sweeps to equilibrate each sample and conduct 1000 sweeps to measure observables (with one measurement taken per sweep).

3 RESULTS AND DISCUSSION

We begin by examining the key characteristics of the clean two-dimensional square-lattice Ising model for nearest-neighbor spin interactions, where the spin-spin interaction strength is set to $J = 1$. This is achieved through the use of the extensive Monte Carlo method, applied to systems with sizes varying from 20 to 500, while employing periodic boundary conditions. The critical temperature T_c is first estimated from the behavior of the magnetization M, susceptibility χ, and specific heat C_v. Figure 1 demonstrates the influence of finite-size effects. The phase transition is indicated by a smooth decline in magnetization and the presence of finite, broadened peaks in both susceptibility and specific heat. Because these quantities are calculated on finite lattices of size L, the observed transition points (the turns in the magnetization curve or the locations of the peaks in χ and C_v) are dependent on L and do not match the exact critical temperature. As the lattice size increases, the peaks gradually move closer to the exact Onsager value $T_C = 2.269J/K_B$, but complete convergence is only reached in the thermodynamic limit ($L \to \infty$). Consequently, while the graphs verify the occurrence of the phase transition, they are insufficient for pinpointing an exact determination of T_C; rather, methods of finite-size scaling and extrapolation of the pseudocritical temperatures $T_C(L)$ are necessary to obtain the exact critical temperature [11].

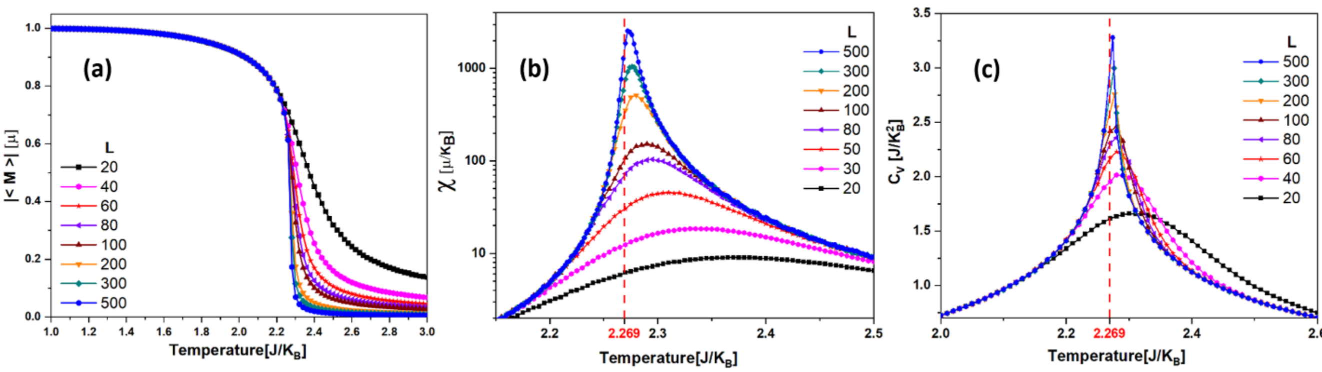

Fig. 1 Finite-size effects in: (a) magnetization M (Eq. 3), (b) susceptibility χ (Eq. 5), and (c) specific heat C_v (Eq. 6) as functions of temperature for different lattice sizes L. In all cases, the transition becomes sharper with increasing L, but finite-size effects cause rounding of the magnetization and systematic shifts of the susceptibility and specific-heat peaks toward the exact critical temperature $T_C = 2.269J/K_B$.

To address the finite-size discrepancies, we utilize the Binder cumulant approach [11]. The Binder cumulant, described in Eq. 7, is computed for various system dimensions L. The point where the $U(T)$ curves intersect for increasing values of L provides a precise and size-independent estimate of the critical temperature ($T_C = 2.269J/k_B$), as shown in Figure 2a. This method greatly diminishes the uncertainty related to identifying T_C based solely on the peaks in susceptibility and heat capacity. After acquiring approximate crossing points, we refine the estimation of T_c through finite-size extrapolation. In particular, we plot the pseudo-critical temperatures $T_C(L)$, derived from the maxima of susceptibility, against $1/L$. Extrapolating to the thermodynamic limit $L \to \infty$ results in a consistent critical temperature $T_C = 2.2690(1)$ that aligns with the findings from the Binder cumulant analysis within numerical errors and agrees with the Onsager value, as shown in Figure 2b. The integration of Binder cumulant crossings and extrapolation thus facilitates a dependable assessment of the critical temperature, bypassing the constraints imposed by finite lattice sizes.

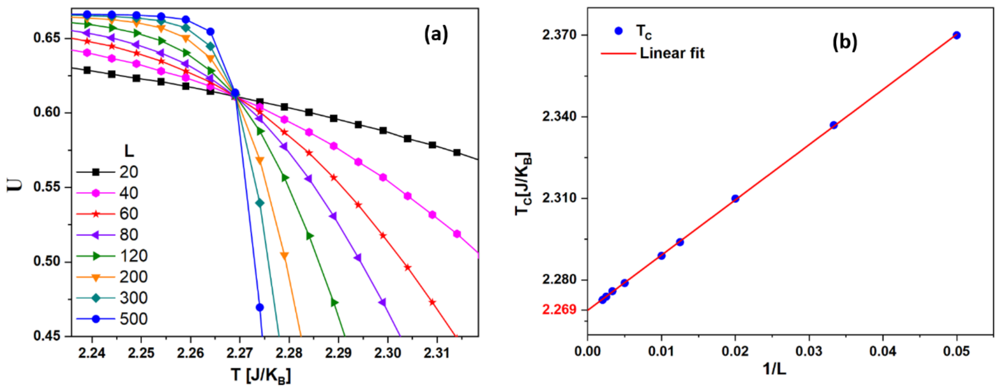

Fig. 2 (a) Binder cumulant U (Eq. 7) as a function of temperature for different system sizes L. The crossing of the curves provides a size-independent estimate of the critical temperature $T_C = 2.2690$. (b) Finite-size extrapolation of the pseudo-critical temperatures $T_C(L)$, obtained from susceptibility maxima, plotted as a function of $1/L$. Extrapolation to $L \to \infty$ yields a consistent value of $T_C = 2.2690(1)$.

With T_C accurately determined, we continue to derive the critical exponents using finite-size scaling, confirming the system's universality class. The finite-size scaling analyses used to determine the correlation-length, magnetization, susceptibility, and specific-heat critical exponents are presented in Figure 3.

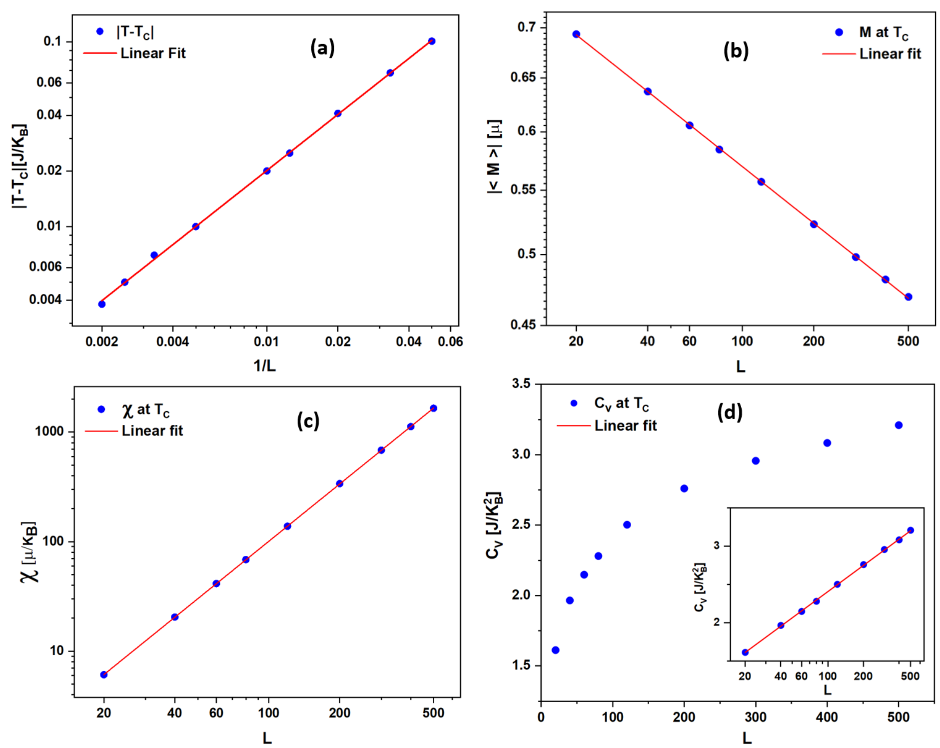

Fig. 3 Finite-size scaling analyses of the critical exponents: (a) correlation-length exponent ν (Eq. 8), (b) magnetization exponent β (Eq. 9), (c) susceptibility exponent γ (Eq. 10), and (d) power-law scaling of the specific-heat exponent α in the main panel and semi-logarithmic scaling according to Eq. 11 in the inset.

3.1 Critical exponents

Upon acquiring a dependable estimate of the critical temperature $T_{C'}$, we moved on to derive the critical exponents that define the universality class of the two-dimensional Ising model. Based on finite-size scaling theory, the characteristics of observables close to the critical point are dictated by power laws in relation to the system size L [11].

The shift of the pseudocritical temperatures with system size provided an independent determination of the correlation length exponent. Fitting the relation in Eq. 8, we obtained $\nu \approx 1.0$, in full agreement with the exact solution, see Figure 3a. The magnetization critical exponent β can be assessed through finite-size scaling of the magnetization close to the critical temperature T_C. At criticality, the magnetization depends on the system size as described in Eq. 9. By plotting the magnetization at the critical point against the system size using a double-logarithmic scale and fitting a linear regression, the slope reveals $-\beta/\nu$, allowing us to calculate β once the correlation length exponent ν is determined. When we apply these methods to our Monte Carlo data, we find $\beta = 0.122(3)$, which closely aligns with the exact value for the 2D Ising model of $1/8 = 0.125$, thereby confirming the validity of our finite-size scaling analysis, as shown in Figure 3b.

The susceptibility critical exponent γ can also be determined through a finite-size scaling analysis of susceptibility at the critical temperature. At $T_{C'}$, the susceptibility exhibits a scaling behavior with system size as described in Eq. 10, thus a $\log - \log$ plot of χ against L provides the slope γ/ν, allowing the calculation of γ using the known correlation length exponent ν, as shown in Figure 3c. When applying this technique to our data, we find $\gamma = 1.74(1)$, which aligns remarkably well with the precise value for the 2D Ising model, $7/4 = 1.75$. The specific-heat exponent α indicates how the heat capacity behaves as it approaches T_C. In finite-size scaling, the peak value at Tc behaves like $C_\nu\,(T_{C'}L) \sim L^{\alpha/\nu}$ when $\alpha > 0$ [11]. For the 2D Ising universality class, $\alpha = 0$, which means the divergence occurs logarithmically instead of a power law; thus, one anticipates this in Eq. 11 as shown in Figure 3d. Hyperscaling connects exponents through the relationship $\alpha = 2 - d\nu$ [31], and with $d = 2$ and $\nu = 1$, this results in $\alpha = 0$, which aligns with the logarithmid behavior.

3.2 Effect of varying interaction strength *J* on critical behavior

The strength of interaction J is essential in influencing the collective dynamics of spin systems, such as the 2D Ising model [32]. It establishes the energy scale for the alignment of spins and directly impacts the critical temperature $T_,$ at which the system experiences a phase transition from an ordered ferromagnetic phase to a disordered paramagnetic phase. By altering J, one can systematically explore how the stability of the ordered phase reacts to variations in interaction energy. To explore how the strength of interactions impacts the critical behavior of the 2D Ising model, we adjusted the coupling constant J from its typical value of $J = 1$ to values of $J = 0.8, 0.85, 0.90, 0.95, 1.05, 1.1, 1.15$, and 1. 2. For each value of J, we determined the critical temperature T_C and derived the critical exponents through finite-size scaling analysis. This systematic adjustment enables us to assess how variations in the reference interaction strength affect the phase transition temperature and investigate the stability of the universality class of the 2D Ising model.

In Figure 4, we illustrate the temperature dependence of the principal thermodynamic quantities of the two-dimensional square-lattice Ising model for different J values ($0.80 \leq J \leq 1.20$). The magnetization curves in Figure 4a show the expected transition from ferromagnetic to paramagnetic states, and that the critical temperature increases gradually with J. This behavior is consistent with Onsager's exact result for T_C, as given in Eq. 2, which indicates a linear dependence of T_C on the coupling strength. The susceptibility (Figure 4b), the specific heat (Figure 4c) exhibit peaks near the transition points as predicted, related to enhanced fluctuations in the critical region that move to higher temperatures as J is increased. Our present results do not yet support the theoretical expectation that the specific heat should exhibit a logarithmic divergence in the framework of the two-dimensional Ising universality class. It is also uncertain whether the critical exponents stay the same or vary as a result of changes in the strength of the interaction, even though the shift in T_C with J is widely documented. The next analysis will explore these unresolved issues by doing a finite-size scaling analysis of specific heat, magnetization, and susceptibility.

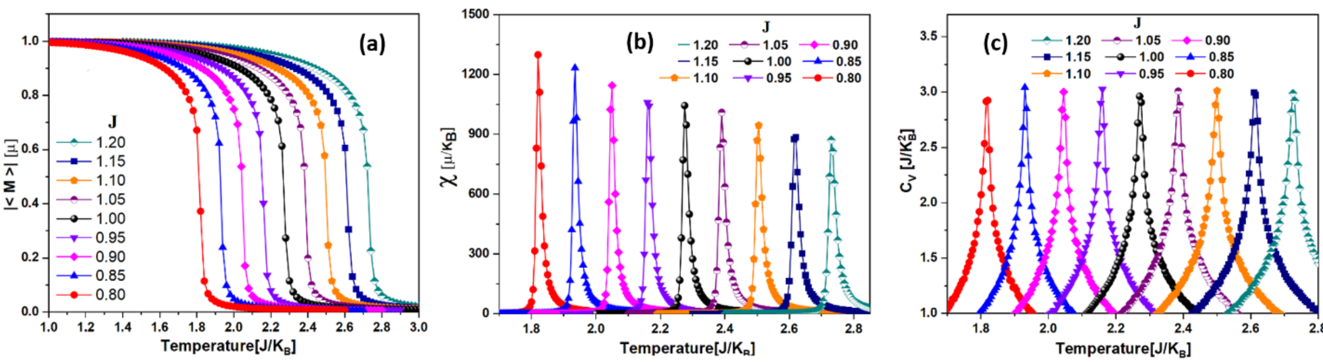

Fig. 4 The plot of thermodynamic properties for values of $0.80 \le J \le 1.20$ with a system size of $L = 300$ includes: (a) magnetization M, (b) magnetic susceptibility χ, and (c) specific heat C_v as functions of temperature. An increase in J raises the critical temperature to higher levels.

We now focus on the scenario $J = 0.80$ and use finite-size scaling to analyze its critical behavior in order to better understand how changing the interaction strength affects the model's universality. These values provide a thorough evaluation of the stability of the universality class since they show the most departures from the reference case $J = 1$. We can determine the corresponding critical exponents and directly compare them with the established results at $J = 1$ by examining the scaling of magnetization, susceptibility, and specific heat across various system sizes. Through this comparison, we are able to determine whether changes in the interaction strength alter the basic scaling behavior or just the critical temperature.

First, we identify the critical temperature T_C by examining the peak of the magnetic susceptibility χ as it varies with temperature across different system sizes L. Since finite-size effects can cause shifts in the peak locations, we carry out a finite-size extrapolation to the thermodynamic limit ($L \to \infty$) to achieve a more precise estimate of T_C. This extrapolated critical temperature is then utilized as the baseline for determining the critical exponents of the system using established finite-size scaling relations. As shown in Figure 5, the critical temperature obtained from finite-size extrapolation of the susceptibility-peak locations is $T_C = 1.8153(3)$, which is in excellent agreement with the Onsager value that is obtained from Eq. 2. We will now examine the universality class of the Ising model with $J = 0.\ 8$. According to finite-size scaling theory, the distinct behavior of thermodynamic properties close to the critical point is determined by power-law relationships with the system size L. At the critical temperature Tc, the scaling forms can be found from Eqs. 8 - 11. From Figure 6a, we find the correlation length exponent $\nu = 1.02(2)$; the magnetization scaling yields $\beta = 0.123(3)$ (Figure 6b); the susceptibility χ gives us $\gamma = 1.748(2)$ (Figure 6c); and the specific heat exhibits a logarithmic divergence, indicating $\alpha = 0$ (Figure 6d). These measurements align remarkably well with the established exact exponents for the two-dimensional Ising universality class and fall within the error margins of those reported for the standard case of $J = 1$. This suggests that altering the coupling strength J merely changes the critical temperature while leaving the universal critical behavior unchanged [33, 34].

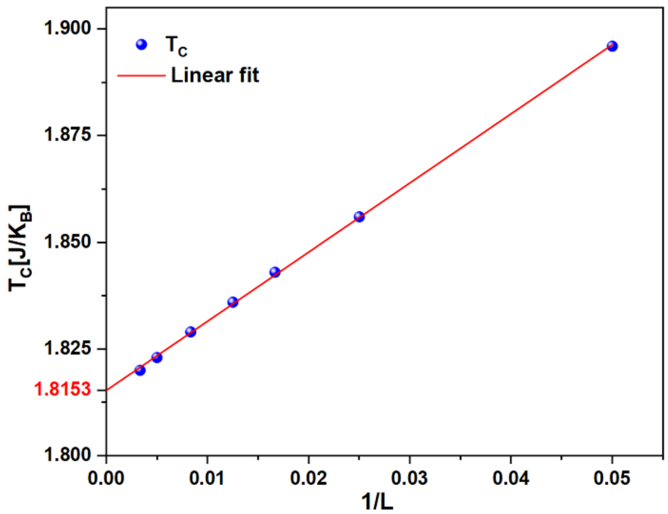

Fig. 5 Finite-size extrapolation of the pseudo-critical temperatures $T_C(L)$, derived from the peaks of susceptibility, is illustrated as a function of $1/L$. When extrapolated to the thermodynamic limit, a consistent value of $T_C = 1.8153(3)$ is obtained.

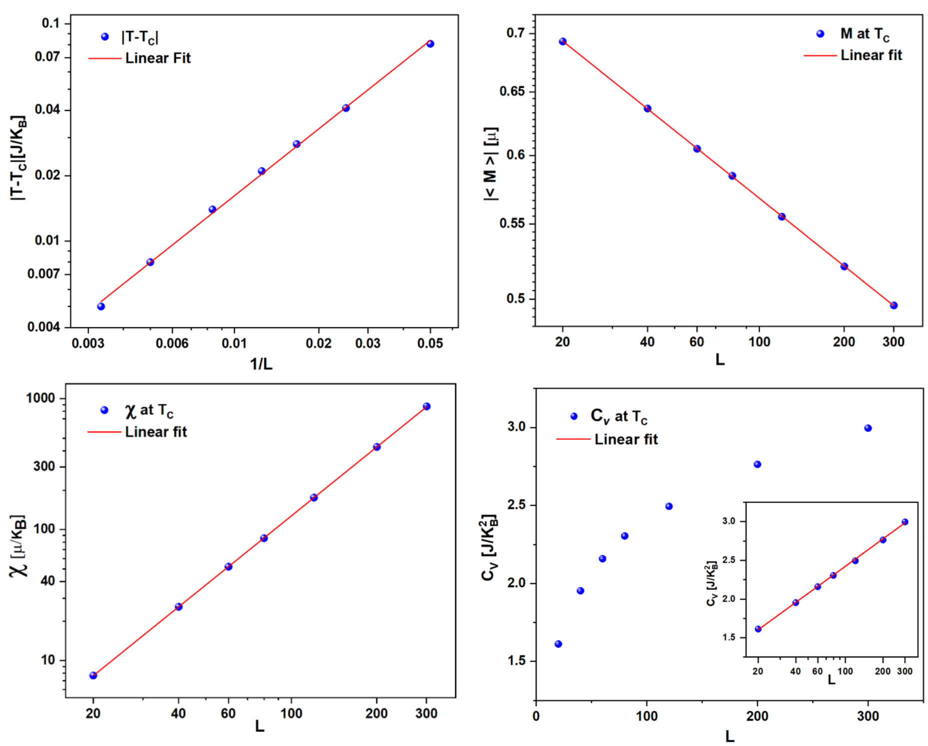

Fig. 6 Finite-size scaling investigations of the critical exponents: (a) correlation-length exponent ν, (b) magnetization exponent β, (c) susceptibility exponent γ, and (d) power-law scaling of the specific-heat exponent α in the main panel and semi-logarithmic scaling in the inset.

4 CONCLUSION

We have investigated the influence of varying interaction strength Jon the two-dimensional square-lattice Ising model using Monte Carlo simulations and finite-size scaling analysis. Across the range $J = 0.80$ to 1.20 , the estimated critical temperatures obtained from the extrapolated susceptibility peaks show excellent agreement with Onsager's exact analytical predictions, confirming the reliability of our numerical approach. The extracted critical exponents for magnetization, susceptibility, and specific heat consistently match the exact Ising values, demonstrating that the universal scaling laws remain preserved. Together, these results confirm that modifying Jshifts the critical temperature linearly, as expected from the energy scaling of the Hamiltonian, while leaving the universality class and critical behavior unchanged. This robustness highlights the fundamental stability of the 2D Ising model's critical properties under variations in microscopic coupling strength.

Author contributions

Both authors designed the study, conducted simulations, analyzed the data, and wrote the original manuscript.

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Data availability

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DECLARATIONS

Conflict of interest

The authors declare no competing interests

Consent to publish

Both authors consent to the publication of this work. Written informed consent for publication was obtained from the participants.

Ethical approval

N/A

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