

## Evaluation of Low Density Parity Check Codes Over Various Channel Types

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### Abstract

Low density parity check (LDPC) codes are one of the best error correcting codes in today's coding world and are known to approach the Shannon limit. As with all other channel coding schemes, LDPC codes add redundancy to the uncoded input data to make it more immune to channel impairments. In this paper, the impact of low-Density Parity-Check code (LDPC) on the performance of system under Binary Phase Shift keying (BPSK) over an Additive White Gaussian Noise (AWGN) and other fading (Rayleigh and Rician) channels is investigated. Obtained results show that LDPC can improve transceiver system for various channel types. At Bit Error Rate (BER) of  $10^{-4}$  such code with code rate of  $\frac{1}{2}$  reduces the Signal to Noise Ratio (SNR) by range of 6.5 to 9 dB for fading channels in contrast to uncoded system. By studying modern research it has been found that turbo code can achieved same manner but LDPC decoder faster than turbo decoder and can be implemented in parallel.

**Keywords:** LDPC, BPSK, Rician, Rayleigh.

### تقييم شيفرة المتكافئ المنخفض الكثافة على قنوات مختلفة للاتصالات اللاسلكية

#### الخلاصة

شيفرة التفرقة المتكافئ المنخفض الكثافة (LDPC) واحدة من افضل المشفرات في الوقت الراهن وباستخدامها يمكن الوصول الى الهدف المحدد من قبل العالم شانون. وكما هو الحال لاي مشفر فان (LDPC) يضيف بيانات اضافية الى البيانات الاصلية ليتمكنها من تقليل تشويش القنوات. في هذا البحث تم التحقق من تاثير المشفر (LDPC) على نظام تراسل يستخدم مضمن التزحيف الطوري (BPSK) لقنوات ذات اضمحلال متعدد. النتائج بينت ان (LDPC) يمكنه تحسين اداء هذا النظام لمختلف انواع القنوات. عند معدل خطأ مقداره  $10^{-4}$  هذ المشفر وبمعدل تشفير يساوي  $\frac{1}{2}$  يمكنه تقليل نسبة الاشارة الى الضوضاء بمدى 6.5 الى 9 ديسيبل في قنوات الاضمحلال مقارنة مع نظام بدون مشفر. بمراجعة مصادر حديثة تبين بان المشفر النفاذ يمكنه تحقيق ذات النتائج لكن مفك شفرة (LDPC) اسرع من مفك شيفرة النفاذ وكذلك يمكن تنفيذه على التوازي.

## 1. Introduction

LDPC codes were developed by Robert Gallager in his PhD thesis at MIT in 1962 [1]. These codes were ignored for about 30 years and rediscovered in the late 1990s by D. J. C. MacKay and R. M. Neal [2]. LDPC codes have certain advantages over other codes, e.g. turbo codes. They not only have a simple description of their code structure but can also have a fully parallelizable decoding implementation [3].

Because of their excellent forward error correction properties, LDPC codes are set to be used as a standard in Digital Video Broadcasting (DVB-S2) and 4G mobile communication. Another advantage of LDPC codes is that they are highly parallelizable in hardware. Also, their minimum distance ( $d_{\min}$ ) increases proportionally with an increase in the block length [4].

Low-Density Parity-Check (LDPC) codes have attracted a lot of attention in recent years. The codes have several properties, which make them favorable choices for real-time and high-throughput communications. First, the codes are capacity approaching. Second, the codes can be efficiently decoded by parallel iterative decoding algorithms with low latency [5]. LDPC was and still Get the concerning of researchers to evaluated and develop its performances for many applications. [6] Evaluates that the Concatenation of LDPC and Reed-Solomon (RS)

Codes in Magnetic Recording. They conclude that using outer RS codes does not always improve the overall sector error rates for a fixed user bit density and a fixed SNR. In [7] researchers present an importance sampling method for the evaluation of the low frame error rate (FER) performance of LDPC codes under iterative decoding. They obtain good agreement with the experimental results obtained from a fast hardware emulator of the decoder.

For mobile phones, LDPC codes may prove a better choice, since they can employ a fully parallelizable decoder. They have even been observed to outperform turbo codes on the Rayleigh fading channel [8]. LDPC codes have been applied to magnetic recording, and have once again been shown to outperform turbo codes [9]. It knows that in magnetic recording errors tend to occur in bursts also. Despite this, the LDPC codes exhibited good performance.

The high-speed decoder hardware implementation is obviously one of the most crucial issues determining the extent of LDPC applications in the real world [10].

In this paper, the LDPC has been applied with assistance of BPSK modem scheme for transmission over AWGN, Rician and Rayleigh fading channels, which requires no bandwidth expansion. The evaluation of Bit Error Rate (BER) performance of the LDPC-BPSK is achieved over three types of channel.

This paper is organized as follows. Section 2 briefly reviews the basics of

LDPC codes and their decoding in section 3. Section 4 gives an overview of system model. In Section 5, the results of LDPC codes on the channels and discussion have been presented. Lastly, conclusions are displayed in Section 6.

**2. Code Structure**

A low-density parity-check (LDPC) code is defined by a parity-check matrix that is sparse. A regular  $(j, L)$  LDPC code is defined by an  $(n-k) \times n$  parity-check matrix with  $n$  block length of the code and  $k$  information bits generated by the binary source. Such matrix having exactly  $j$  ones in each column and exactly  $L$  ones in each row, where  $j < L$  and both are small compared to  $n$ . An irregular LDPC matrix is also sparse, but not all rows and columns contain the same number of ones. Figure 1 shows the parity-check matrix of a (3, 6) LDPC code. [11].

By the definition of regular LDPC codes, every parity-check equation involves exactly  $L$  bits, and every bit is involved in exactly  $j$  parity-check equations. Observe that the fraction of ones in a regular  $(j, L)$  LDPC matrix is  $L/n$ . The “low density” terminology derives from the fact that this fraction approaches zero as  $n \rightarrow \infty$  [12]. In contrast, the average fraction of ones in a purely random binary matrix (with independent components equally likely to be zero or one) is 1/2.

Any parity-check code (including an LDPC code) may be specified by a Tanner graph, which is essentially a visual representation of the parity

check matrix  $H$  [13]. Recall that an  $(n-k) \times n$  parity-check matrix  $H$  defines a code in which then bits of each codeword satisfy a set of  $(n-k)$  parity-check equations. The Tanner graph contains  $n$  “variable” nodes, one for each codeword bit, and  $(n-k)$  “check” nodes, one for each of the parity-check equations [14]. Figure 2 shows the Tanner graph corresponding to the  $H$  matrix.

The generator matrix for a code with parity-check matrix  $H$  can be found by performing Gauss-Jordan elimination on  $H$  to obtain it in the form  $H = [A, I_{n-k}]$  ..... (1)

Where  $A$  is a  $(n - k) \times k$  binary matrix and  $I_{n-k}$  is the size  $n-k$  identity matrix. The generator matrix is then

$$G = [I_k, A^T] \dots\dots\dots(2)$$

Where  $[ ]^T$  is the matrix Transposition. The row space of  $G$  is orthogonal to  $H$ . Thus if  $G$  is the generator matrix for a code with parity-check matrix  $H$  then

$$GH^T = 0 \dots\dots\dots(3)$$

An LDPC code parity-check matrix is called  $(w_c, w_r)$ -regular if each code bit is contained in a fixed number,  $w_c$ , of parity checks and each parity-check equation contains a fixed number,  $w_r$ , of code bits. A regular LDPC code will have:

$$m \cdot w_r = n \dots\dots\dots(4)$$

Where  $w_c$  and  $w_r$  are number of ones in each column and row for regular parity check matrix of LDPC code respectively,  $m$  vertices for the parity-check equations (called check nodes see Figure 2).

For an irregular parity-check matrix it must designate the fraction of columns of weight  $i$  by  $v_i$  and the fraction of rows of weight  $i$  by  $h_i$ . Collectively the set  $v$  and  $h$  is called the degree distribution of the code ones in its parity-check matrix. Similarly, for an irregular code [15]:

$$m \left( \sum_i h_i x^i \right) = n \left( \sum_i v_i x^i \right)$$

**3. Decoding**

One of the most widely used decoding methods for LDPC codes is based on belief propagation. It is performed by applying the maximum a posteriori (MAP) algorithm. This algorithm aims at minimizing the bit error rate of the decoded sequence and iteratively calculates the a posteriori probabilities [16].

The MAP algorithm computes the a posteriori probability of each state transition given the noisy observation at the receiver. There is a one to one correspondence between a state transition and its corresponding code symbol. The states connected by the MAP-found state transition need not form a continuous path. The algorithm computes the a posteriori probabilities (APP) of each possible state transition and chooses the one which is more likely (highest probability). For more details see [17].

Consider a regular  $(j, k)$  LDPC code with  $v$  as the Log-Likelihood Ratio (LLR) message passed from a variable node of degree  $j$  to a check node of degree  $k$ , given as [18],

$$v = v_0 + \sum_{i=1}^{j-1} r_i \dots \dots \dots (5)$$

In (5),  $v_0$  is intrinsic information conditioned on the channel output, and  $r_i$  for all  $i = 1, \dots, j - 1$ , is the extrinsic information. Extrinsic information is part of the overall LLR stemming from the observation of the received samples. The check nodes update rule is obtained by noticing the duality between variable and check nodes. It is based upon the well known tanh rule and it is given as [19]

$$\frac{\tanh r}{2} = \prod_{i=1}^{k-1} \frac{\tanh v_i}{2} \dots \dots \dots (6)$$

Where  $v_i$ , for all  $i = 1, \dots, k - 1$ , are the incoming LLRs from the neighboring edges.

**4. System Model**

The system model used is shown in Figure 3:

- **Data source:** The data to be transmitted over the channel was randomly generated by the binary source. The binary source is assumed to be memoryless, which is often the result of source coding (data compression), and therefore all information sequences are equally probable.
- **LDPC Encoder:** A binary  $[k, n]$  generator matrix  $G$  may be used by the channel encoder to map the information bits  $u$  to a codeword  $c$ , where the mapping from information bits to codeword is done through the matrix multiplication  $c = uG$ .

- **Modulator:** Before a codeword  $\mathbf{c}$  is transmitted over the channel, it is mapped to a modulated signal  $x(i)$  where  $i=1, \dots, n$ .
- **Channel:** The received signal vector  $\mathbf{y}$  is given by  $\mathbf{y}=\mathbf{x}+\mathbf{N}$  where  $\mathbf{N}$  is the noise vector. The noise is assumed to be additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2$ , in addition to multipath fading channel for such system, in this paper Rician and Rayleigh fading model are employed.
- **Receiver:** An estimate  $\hat{\mathbf{c}}$  of the transmitted codeword  $\mathbf{c}$  is derived from the received vector  $\mathbf{y}$  at the channel decoder. Since the generator matrix is systematic, the estimated information bits are simply  $\hat{\mathbf{u}}=(\hat{c}_1, \dots, \hat{c}_k)$ . Finally, the information bits are passed to the sink.

## 5. Simulation and Results

In this paper, the performance of LDPC codes are evaluated over three types of channel (AWGN, Rayleigh, and Rician), with three levels of code rate ( $R$ ) = 1/2, 3/5, and 8/9 for each channel. Jakes Doppler filter impulse response of fading channels is employed for all simulations. A (7200, 64800) regular LDPC coded bit stream was used. The number of iterations is taken to be only 10 to avoid delay time. The simulations are applied on system model shown in Figure (3), using the MATLAB software package. The results are

displayed as graphs in which the (BER) is plotted versus (SNR), measured in decibel (dB).

### 5-1 AWGN Channel

The first experiment is to evaluate LDPC with the system shown in Figure 3 over AWGN channel to measure the amount of improvement resulting by using this code. Figure 4 illustrates the performance of such code. It is clear that the LDPC codes with 8/9 code rate can achieve 4.25 dB gain over uncoded message at  $10^{-4}$  BER. It also profits 6 and 9 dB for 3/5 and 1/2 code rate respectively. The code gains achieved for each code rate are summarized in Table 1.

### 5-2 Rician Channel

In this subsection we first evaluate the performance of LDPC over static Rician channel. The path gain is  $1.1375 + 0.1956i$  and  $K=2$ , (The **K-factor** parameter, which is part of the statistical description of the Rician distribution, represents the ratio between the power in the line-of-sight component and the power in the diffuse component). Figure 5 illustrates this performance. Table 2 summarizes the results at BER level of  $10^{-4}$ . It is shown that this code can outperform uncoded system by 2.5, 5, and 6.5dB for code rate 8/9, 3/5, and 1/2 respectively at BER level of  $10^{-4}$ . The second evaluation is to use the Rician flat fading instead of static type. Here we consider 100Hz Doppler shift for mobility, input sample period is  $416.7\mu\text{s}$ , path gain is  $1.0085 - 0.2519i$  and similar other parameters of previous case.

Because of fading environments the officiating becomes worse than precedent evaluation but the gain is resemblance, as shown in Figure 6. The results for both parts are listed in Table 2.

### 5-3. Rayleigh Channel

Same parameters of Rician fading channel are used in this subsection but the path gain is  $0.5647 + 0.1127i$ . Figure 7 clear out the benefits of using LDPC codes with Rayleigh fading channel. Such code obtains significant gain like-that more than 8 dB is achieved with  $1/2$  code rate at  $10^{-4}$  BER. But the range of SNR becomes higher than precedent cases because of worse channel situation. The code gain for various code rate are listed in table 3.

### 5.4 Discussion

From the results of simulation it is clear that this code achieved significant improvement for SNR at low BER. For wireless communication (Rayleigh and Rician channels) 6.5 to 9 dB code gain can be achieved for  $1/2$  code rate with low range of SNR at BER of  $10^{-4}$ . For  $3/5$  code rate achieved 5 to 6.2 dB for various types of channel. While  $8/9$  code rate can reduce the SNR by the range of 2.5 to 4 dB. Note that the latest code rate added one bit to every 8 bits that means it maintains spectrum efficiency with suitable gain. Generally the lower the code rate, the higher the coding gains. In other word, better codes provide better coding gains and higher complexity.

In the light of the results found in [20] and [21] it is seem that no

significant difference between LDPC and Turbo codes. Turbo codes have a fixed number of iterations in the decoder. This implies that the time spent in the decoding and the bit rate out of the decoder, are constant entities. In contrast, the LDPC decoder stops when a legal code word is found, implying that there is potential for significantly reducing the amount of work to be done relative to Turbo codes. This also implies that the bit rate out of the decoder will vary, and a buffer system must be designed in order to make the bit rate constant. The LDPC decoder will become faster the higher the SNR. An advantage with LDPC codes is that the decoders may be implemented in parallel. This has significant advantages when considering long codes.

### 6. Conclusions

Low-density-parity-check codes have been studied a lot in the last years and huge progresses have been made in the understanding and ability to design iterative coding systems. The iterative decoding approach is already used in turbo codes but the structure of LDPC codes give even better results. In many cases they allow a higher code rate and also a lower error floor rate. In other achieve good coding gain performance, good LDPC code design is essential.

It has been observed that the system with LDPC codes has good performance in Rician and Rayleigh fading channel. Obtained results show that better performance in terms of BER can be achieved with such codes by decreasing code rate. More than 8

dB code gain over uncoded system can be achieved with 1/2 code rate of LDPC codes over Rician and Rayleigh channel. Also the results confirm that such code with high code rate (8/9) can achieve beneficial gain while maintaining spectrum efficiency because of no more redundant information added to the message.

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**Table 1: AWGN results**

Code rate	1/2	3/5	8/9
Code gain (dB)	9	5.8	4

**Table 2: Rician results**

Channel	Code rate	1/2	3/5	8/9
Static Rician	Code gain (dB)	6.5	5	2.5
Fading Rician	Code gain (dB)	8.2	6.2	2.8

**Table 3: Rayleigh results**

Code rate	1/2	3/5	8/9
Code gain (dB)	8.6	6.2	4

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \textcircled{1} & 1 & 1 & \textcircled{1} & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & \textcircled{1} & 0 & 0 & \textcircled{1} & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Figure 1: A regular (3, 6) parity-check matrix H, the circled 1s show a 4-loop

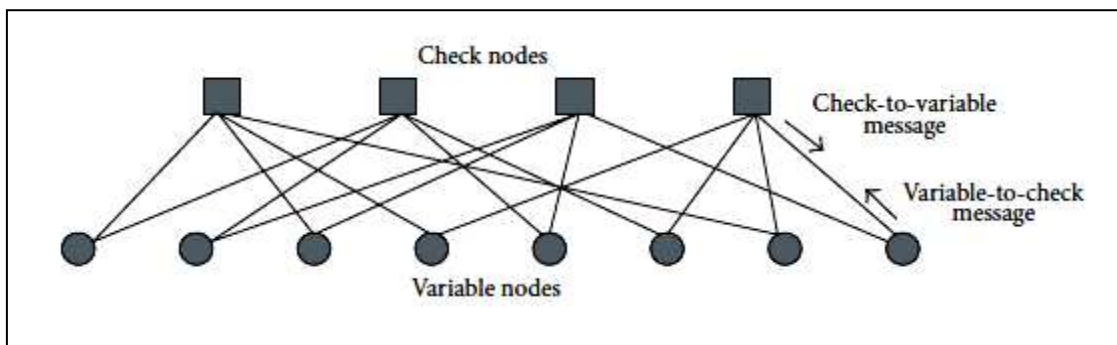


Figure 2: Tanner graph representation of an LDPC code [10]

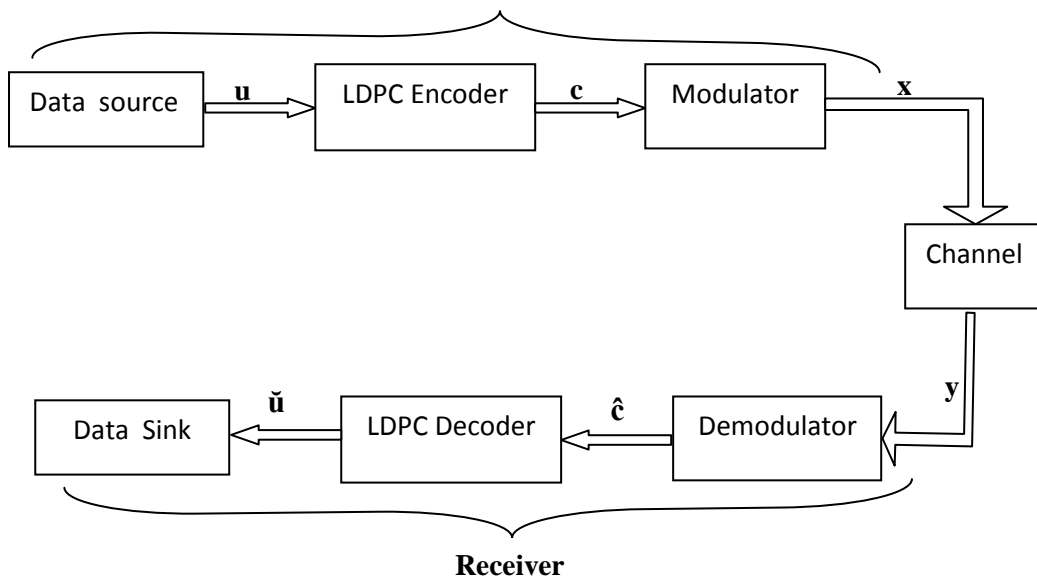


Figure 3: System Model

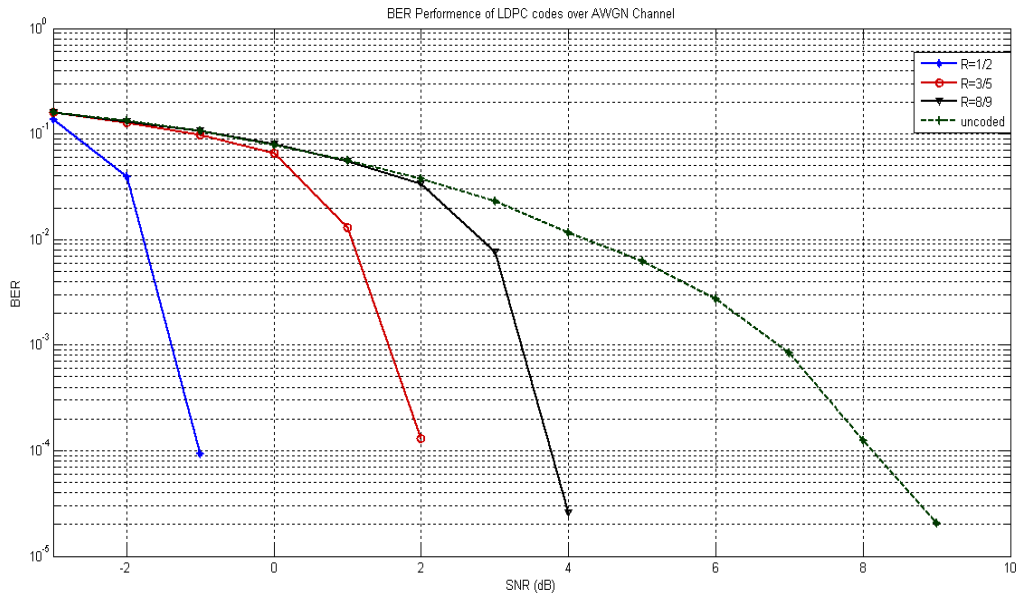


Figure 4: The Performance of LDPC codes over AWGN channel

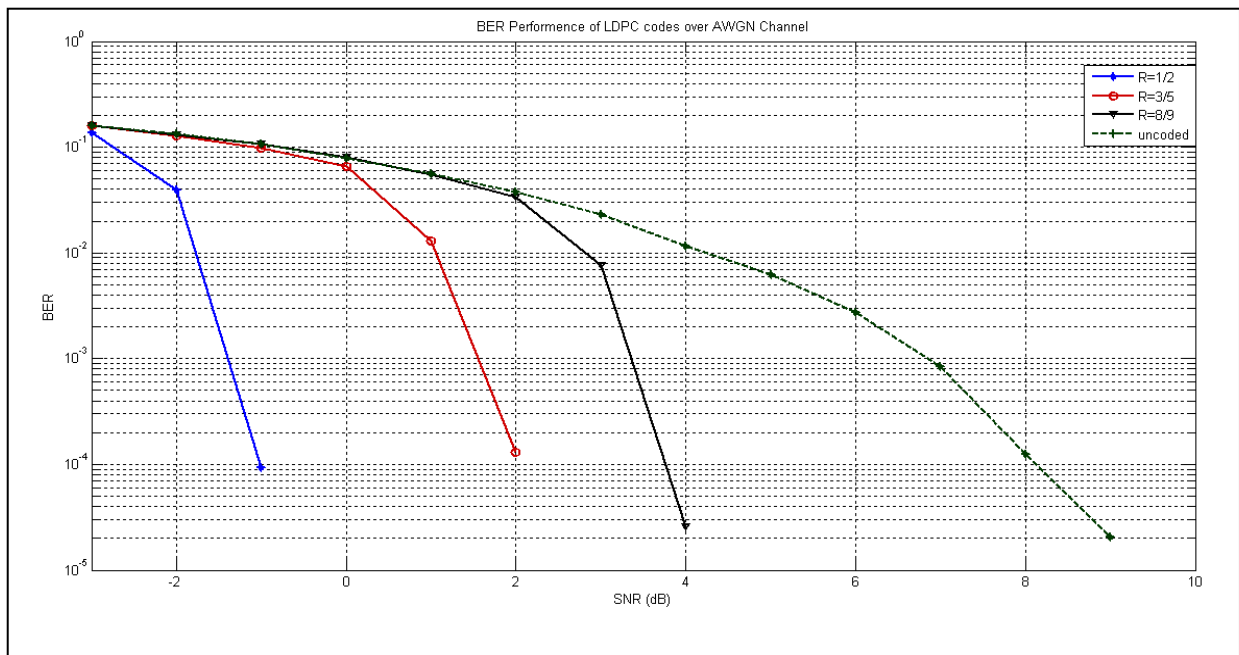


Figure 5: The performance of LDPC codes over static Rician channel

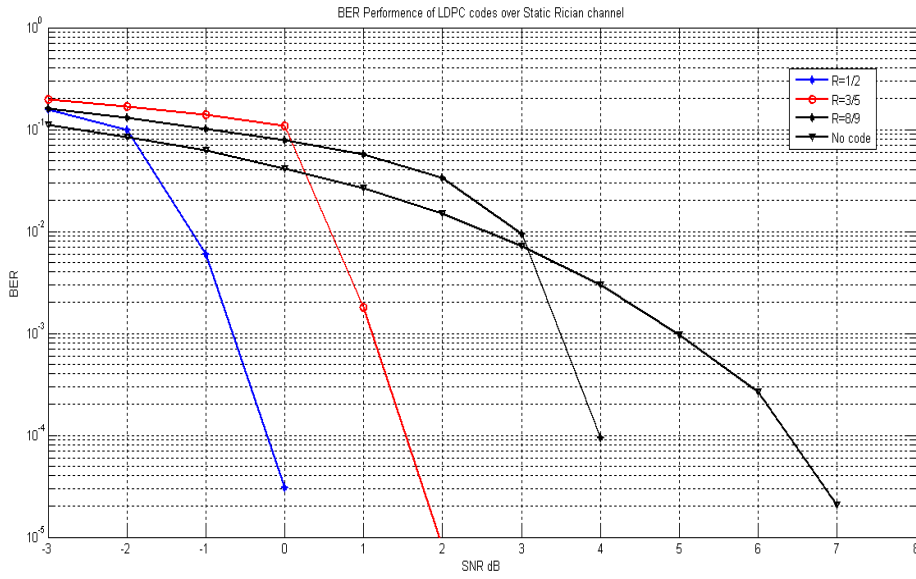


Figure 6: The performance of LDPC codes over fading Rician channel

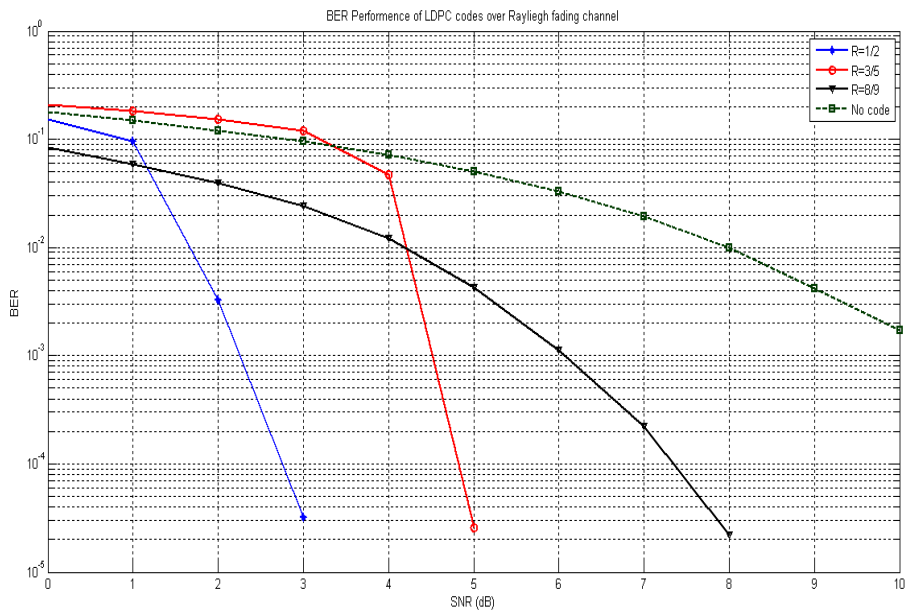


Figure 7: The performance of LDPC codes over Rayleigh fading channel