Tikrit Journal of Eng. Sciences/Vol.15/No.2/June 2008, (90-111)

NUMERICAL STUDY OF NON-DARCIAN NATURAL CONVECTION HEAT TRANSFER IN A RECTANGULAR ENCLOSURE FILLED WITH POROUS MEDIUM SATURATED WITH VISCOUS FLUID

Mahmood H. Ali Ass. Lecturer

Ahmed Tawfiq Ahmed Ass. Lecturer

Khalaf Ibraheem Hamada Ass. Lecturer

Mechanical Eng. Dept.-University of Tikrit

ABSTRACT

A numerical study of non-Darcian natural convection heat transfer in a rectangular enclosure filled with porous medium saturated with viscous fluid was carried out. The effects of medium Rayleigh number, porosity, particle to fluid thermal conductivity ratio, Darcy number and enclosure aspect ratio on heat transfer were examined to demonstrate the ability of using this construction in thermal insulation of buildings walls.

A modified Brinkman-Forchheimer-extended Darcy flow model was used and no-slip boundary conditions were imposed for velocity at the walls and the governing equations were expressed in dimensionless stream function, vorticity, and temperature formulation. The resulting algebraic equations obtained from finite difference discritization of vorticity and temperature equations are solved using (ADI) method which uses Three Diagonal Matrix Algorithm (TDMA) in each direction, while that of the stream function equation solved using successive iteration method.

The study was done for the range of enclosure aspect ratio ($2 \le H/L \le 30$) which is in the tall layers region at medium Rayleigh number ($5 \le Ra_m \le 2000$), Darcy number (Da=10^{-3}, 10^{-4}, 10^{-5}), porosity (ε=0.35, 0.45, 0.55), particle to fluid thermal conductivity (k_S/k_f=5.77, 38.5, 1385.5).The results showed that the Nusselt number is direct proportional to medium Rayleigh number and porosity and reversely proportional to Darcy number, ratio of particle to fluid thermal conductivity and enclosure aspect ratio. The variables that affect the heat transfer in the above arrangement was correlated in a mathematical equation that account better for their affects on heat transfer which is represented by mean Nusselt number (Nu).

Tikrit Journal of Eng. Sciences/Vol.15/No.2/June 2008, (90-111)

KEY WARDS: Porous medium, Rectangular enclosure, Non-Darcian Natural Convection, Viscose fluid.

NOMENCLATURE

Symbol	Mining
C	Specific heat (J/kg . K)
Da	Darcy number
d_p	Particle diameter(m)
F	Forchheimer coefficient
g	Acceleration of gravity (m/s^2)
H	High of porous bed (m)
h	Convection coefficient (W/m^2 . K)
K	Permeability of porous medium (m^2)
k	Thermal conductivity (W/m . K)
L	Width of porous bed (m)
Nu	Mean Nusselt number
P	Dimensionless pressure
Pr	Prandtl number
p	Pressure (N/m^2)
Ra	Rayleigh number
T	Temperature (K)
T_c,T_h	Cold and hot temperature (K)
t	Time (s)
u,v,w	Velocity components (m/s)
U,V,W	Dimensionless velocity components
x,y,z	Cartesian coordinate (m)
X,Y,Z	Dimensionless Cart. coorinate

Symbol	Mining
α	Thermal diffusivity (m^2/s)
β	Thermal expansion coefficient (K^{-1})
ε	Porosity
θ	Dimensionless temperature
μ	Viscosity ($N.s/m^2$)
ρ	Density (kg/m^3)
τ	Dimensionless time
Ψ	Dimensionless stream function
Ω	Dimensionless vorticity
Subscripts	
d	Dispersive
e	Effective
f	Fluid
m	Medium
s	Solid
st	Stagnant

INTRODUCTION

The study of convection heat transfer in porous media with confined boundary has been a subject of intensive study during the past two decades because of its wide applications including geothermal energy engineering, groundwater pollution transport, nuclear wasted disposal, chemical reactors engineering, insulation of buildings and pipes, and storage of grain and coal and so on. Most of the studies on natural convection in enclosed spaces have been related to rectangular cavities include an air gap or porous medium which is used for insulation[1]. In these cases, it is important to determine the rates of heat transfer across the gap which results from a temperature difference between the opposing walls. Theoretical analysis of the problem usually began with the fundamental differential equations of conservation of mass, momentum and thermal energy together with appropriate equation of state. These equations may be solved numerically, under desired boundary conditions to yield useful results and criteria for design[1].

For the natural convection in the confined porous medium Chan, et al.[1] used a numerical methods to solve field equations for heat transfer in a porous medium filled with a gas and bounded by plane rectangular surfaces at different temperature. They obtained a heat transfer rate as a function of dimensionless parameters (Da , Ra_f) and geometric aspect ratio (H/L) and they proposed a correlation equation between these parameters for product of ($Da.Ra_f$) greater than 40. Kladius and Prasad[2] have studied the influence of Darcy and Prandtl numbers for fluid flow in a porous medium in a rectangular cavity. Jue[3] considered enclosures of some aspect ratios filled with a porous medium and examined the effect of Brinkman resistance in detail for the aspect ratio 1, 2 and 4. The problem of natural convection flow of a fluid in a square cavity filled with porous medium subject to internal heat generation and non-uniformly heated sidewalls has been investigated by Hossain and Wilson[4]. Hossain and Rees[5], considered the flow, which is induced by differential heating on the boundary of a porous cavity heated from below. Attention is focused on how the flow and heat transfer is affected by variation in the cavity aspect ratio, the Grashof number and Darcy number. Deshpande and Srinidhi[6] studied numerically natural convection in a three-dimensional cavity heated from below. Results for the case of a long cavity of span-wise aspect ratio three are interpreted in the light of those obtained results for a cubical cavity where two distinct solutions have been detected. Recognition of these multiple solutions has been helpful in understanding more complex flows in the long cavity.

Phanikumar and Mahaja[7] present numerical and experimental results for buoyancy-induced flows in high porosity metal foams ($0.89 \le \varepsilon \le 0.97$) heated from below. A Brinkman-Forchheimer-extended Darcy flow model was used and effects of thermal dispersion and Darcy number on heat transfer are reported.

As mentioned by Younsi, et,al[8] the no-slip boundary condition is found to modify the velocity profile near the wall which in turn affects the rate of heat transfer from the wall and not considering such effect will result in errors in heat transfer calculations. Also, when the Reynolds number is of an order greater than unity, the inertial force is no longer negligible compared with the viscous force and it must be included.

The aim of the present work is to present numerical results for buoyancy-induced flow in rectangular cavity filled with porous medium saturated with viscose fluid (air) in the tall layers reign including effects of no-slip boundary condition and inertial force because of their necessity to obtain reliable results as mentioned by Younsi, et,al [8].

The non-Darcian model proposed by Brinkman–Forchheimer was used in contrary to the previous studies of natural convection in rectangular cavity filled with porous medium that uses Darcy's model which is applicable in the low Rayleigh number, in addition to small aspect ratio that considered in these studies. The effects of porosity, solid particles to fluid thermal conductivity ratio and high medium Rayleigh numbers on heat transfer for high aspect ratio up to 30 was examined to demonstrate the ability of using this construction in thermal insulation of buildings walls. To the best of our knowledge, no work has been reported for high-aspect ratio geometry using present model as noted from available references.

PROBLEM FORMULATION AND GOVERNING EQUATIONS

To analyze the problem and modeling natural convection flow in rectangular cavity a fundamental configuration shown in Fig.(1) is selected. The problem considered is a two-dimensional natural convection flow of viscous incompressible fluid in an isotropic, homogeneous and rigid porous medium contained in a rectangular cavity consists of two vertical side walls at x=0 and x=L and two horizontal plates at y=0 and y=h. The fluid taken to have initially uniform temperature, T_c . The temperature of the right side wall is maintain at uniform temperature T_c , while the temperature of the left side wall is raised to T_h ($T_h>T_c$) and thereafter maintained uniform. The top and bottom horizontal plates insulated perfectly, i.e $\partial T/\partial y = 0$. The no-slip boundary conditions are imposed along the solid surfaces. For the mathematical formulation of the problem, it is assumed that: (a) the porous medium is saturated with the fluid and is considered as continuum, (b) the fluid flow and temperature distribution are two-dimensional, (c) the fluid and the solid particles are in local thermal equilibrium, and (d) the Boussinseq approximation is applicable. With these assumptions, the transient macroscopic conservation equations in vector forms are [9,10,11]:

Continuity equation:

$$\nabla.\overline{V} = 0 \qquad \text{..................(1)}$$

Momentum equation:

$$\rho_f\left[\frac{\partial \overline{V}}{\partial t} + \frac{\overline{V}}{\varepsilon}.\nabla\overline{V}\right] = -\nabla P + \rho_f \beta_f (T - T_\infty) g\varepsilon + \mu_f \nabla^2 \overline{V} - \frac{\mu_f \varepsilon \overline{V}}{K} - \rho_f \frac{F}{\sqrt{K}}\left|\overline{V}\right|\overline{V}\varepsilon \qquad .(2)$$

Energy equation:

$$\frac{\partial}{\partial t}\left[(\rho C)_f\,\varepsilon + (\rho C)_s (1-\varepsilon)\right]T + (\rho C)_f \overline{V}.(\nabla T) = \nabla\left[k_e.\nabla T\right] \qquad \text{..........(3)}$$

In equation (2), the last three terms are the Brinkman (or friction) term, the Darcy term and Forchheimer (or inertia) term. For a packed-sphere bed, the permeability (K) and the Forchheimer coefficient (F) are related to the porosity by [9,11]:

$$K = \frac{d_p^2 \varepsilon^3}{150(1-\varepsilon)^2}\ , \ F = \frac{1.75}{\sqrt{1.75\varepsilon^{3/2}}} \qquad \text{...(4)}$$

The value of the stagnant thermal conductivity (**k_{st}**) of the saturated porous medium can be computed according to the following expression given by Zehner and Schluender's [11,12]:

$$\frac{k_{st}}{k_f}=1-\sqrt{(1-\varepsilon)}+\frac{2\sqrt{(1-\varepsilon)}}{1-\lambda B}$$
$$\times\left[\frac{(1-\lambda)B}{(1-\lambda B)^2}\ln\left(\frac{1}{\lambda B}\right)-\frac{B+1}{2}-\frac{B-1}{1-\lambda B}\right] \quad \text{.........(5)}$$

where: $B=1.25[(1-\varepsilon)/\varepsilon]^{10/9}$ and $\lambda=k_f/k_s$

The effective thermal conductivity (k_e) is superposition of the stagnant thermal conductivity(k_{st}) and dispersive conductivity (k_d)[11,12], i.e. $k_e=k_{st}+k_d$. The dispersive conductivity is depends on the velocity which is small in the natural convection thus the dispersive conductivity is small and can be neglected and the effective conductivity $k_e= k_{st}$. Using the following dimensionless variables:

$X=x/L$, $Y=y/L$, $U=uL/\alpha_f$, $V=vL/\alpha_f$, $P=pL^2/\rho_f\alpha^2_f$, $\tau=t\alpha_f/L^2$, $\theta=\dfrac{T-T_c}{T_h-T_c}$ ..(6)

The governing equations (1)-(3) in term of the dimensionless variables and constant property become:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad \text{..................(7)}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\varepsilon\partial X}+V\frac{\partial U}{\varepsilon\partial Y}=-\frac{\partial P}{\partial X}+\Pr_f\nabla^2U$$
$$-\left(\frac{\Pr_f}{Da}+\frac{F}{\sqrt{Da}}|W|\right)\varepsilon U \quad \text{..................(8)}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\varepsilon\partial X}+V\frac{\partial V}{\varepsilon\partial Y}=-\frac{\partial P}{\partial Y}+\Pr_f\nabla^2V$$
$$-\left(\frac{\Pr_f}{Da}+\frac{F}{\sqrt{Da}}|W|\right)\varepsilon V+Ra_f\Pr_f\theta \quad \text{...(9)}$$

$$\sigma\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=-\frac{\alpha_e}{\alpha_f}$$
$$\times\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \quad \text{......................(10)}$$

Where σ, α_e, Da, $\Pr_f$, and Ra_f are the heat capacity ratio of the saturated porous medium to that of the fluid, the effective thermal diffusivity of the saturated porous medium, the Darcy number, the fluid Prandtl number, and the fluid Rayleigh number respectively, which are defined as:

$$\sigma=\frac{(\rho C)_f\varepsilon+(\rho C)_s(1-\varepsilon)}{(\rho C)_f},\quad \alpha_e=\frac{k_e}{(\rho C)_f},$$

$$Da=\frac{K}{L^2},\quad \Pr_f=\frac{\upsilon_f}{\alpha_f},\quad Ra_f=\frac{g\beta_f(T_h-T_c)L^3}{\alpha_f\upsilon_f}$$

While $|W|=(U^2+V^2)^{1/2}$ is the absolute value of the local velocity. The fluid Rayleigh number related to the medium Rayleigh number (Ra_m) by:

$$Ra_f=Ra_m/Da(\alpha_e/\alpha_f)$$

and medium Rayleigh number is:

$$Ra_m = \frac{g\beta_f (T_h - T_c) KL}{\alpha_e \upsilon_f}$$

Eliminating the pressure terms in equations (8) and (9) by cross differentiation (differentiate equation(8) with respect to Y and equation(9) with respect to X). We can express the resulting equations in term of dimensionless stream function and vorticity as:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\Omega \qquad \ldots\ldots\ldots\ldots(11)$$

$$\frac{\partial \Omega}{\partial \tau} + \frac{\partial \Psi}{\partial Y}\frac{\partial \Omega}{\varepsilon \partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \Omega}{\varepsilon \partial Y} = \Pr_f \nabla^2 \Omega$$
$$-\left(\frac{\Pr_f}{Da} + \frac{F}{\sqrt{Da}}|W|\right)\varepsilon\Omega + Ra_f \Pr_f \theta \; .....(12)$$

$$\sigma \frac{\partial \theta}{\partial \tau} + \frac{\partial \Psi}{\partial Y}\frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} = -\frac{\alpha_e}{\alpha_f}$$
$$\times\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \; ....(13)$$

Where the dimensionless stream function and vorticity related to the velocity as:

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X}, \quad \Omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}$$

The local Nusselt number is defined as the ratio between the heat transfer by convection to that of conduction or:

$$Nu_y = \frac{q_{conv}}{q_{cond}} = \frac{hy}{k_e} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Substituting for (h) and (k_e) in term of temperature difference, the local Nusselt number can be expressed in term of dimensionless parameters as:

$$Nu_Y = -\left.\frac{\partial \theta}{\partial X}\right|_{X=0} \qquad \text{for left side wall}$$

$$Nu_Y = \left.\frac{\partial \theta}{\partial X}\right|_{X=1} \qquad \text{for right side wall}$$

Using four point forward deference the temperature gradient expressed as[13]:

$$\frac{\partial \theta}{\partial X} = \frac{-11\phi_{(i,j)} - 18\phi_{(i,j+1)} + 9\phi_{(i,j+2)} - 2\phi_{(i,j+3)}}{6\Delta X}$$
.......................(15)

And the mean Nusselt number can be calculated by integrating its local value along the wall:

$$Nu = \frac{1}{H/L}\int_0^{H/L} Nu_Y . dY \qquad \ldots\ldots\ldots\ldots\ldots(16)$$

The above equation was integrated numerically using the Simpson's one-third formula as follows[13]:

$$Nu = \frac{\Delta Y}{3H/L}\left[Nu_1 + Nu_n + 2(Nu_3 + Nu_5 + ...) + 4(Nu_2 + Nu_4 + ...)\right] \quad ....(17)$$

At the steady state the mean Nusselt number at right and left walls must be equal as the upper and lower plates are insulated since the energy in to system equal to the energy out from it according to energy saving law.

Initial and Boundary Conditions

The problem of the present work is steady two-dimensional natural convection but, the term of time derivative in the governing equations is kept in order to use time marching technique starting with initial state continuing until reaching steady state solution[14], thus initial conditions are required to start solving the problem. So, as mentioned in the problem formulation the porous medium are initially at a uniform temperature T_c . Thus, the initial conditions of the problem in the dimensionless form are:

$\Psi=\Omega=\theta=0$ at **τ=0**

The boundary conditions in the dimensionless form are:

$$\Psi = 0, \quad \theta = 1, \quad \Omega = -\frac{\partial^2 \Psi}{\partial X^2}$$ at **X=0**

$$\Psi = 0, \quad \theta = 0, \quad \Omega = -\frac{\partial^2 \Psi}{\partial X^2}$$ at **X=1**

$$\Psi = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad \Omega = -\frac{\partial^2 \Psi}{\partial Y^2}$$

at **Y=0** and **Y=H/L**

NUMERICAL PROCEDURE

The coupled transient equations are solved to obtain a steady state solution using time marching technique and when a convergent result is approached, the transient terms vanish and the steady-state equations are solved. This approach allows to eliminate instabilities. The differential equations (11)-(13) are discretized based on the finite difference method. A first-order forward difference approximation is used for the time derivative and a second-order central difference approximation is used for space derivative.

The algebraic equations obtained from finite deference for the stream function were solved by successive iteration method, while those for vorticity-stream function and temperature were solved by the alternating direction implicit (ADI) method in two time step each of ($\Delta\tau/2$) for each direction. The solution of resulting equations was obtained using line-by-line method, which results a three diagonal matrix (TDM) solved numerically using (TDMA) algorithm[15]. Starting with equation (13) all quantities are known at previous time step and the new values of the temperatures at time step ($\tau+\Delta\tau$) are obtained, with this values known, equation (12) was solved to obtain the vorticity values (Ω) which depend on the value of (θ) as seen in the last term in this equation. Lastly the stream function equation equation (11) was solved iteratively to get the new values of stream function (Ψ) at present time step ($\tau+\Delta\tau$), and the process is continued until the steady state is reached. The solution is assumed to be reached during the time marching process, when the maximum percentage relative difference between the temperatures (θ) in two successive time step is smaller than 10^{-4} i.e. $\left|\frac{\theta^{i}-\theta^{i+1}}{\theta^{i+1}}\right| \le 10^{-4}$ where **(i)** denotes the number of time increments. To assure steady state the computations continue until the deference between the Nusselt number at the left and right walls in the same time step is not more than (1%) and decreases with a further increase in the time domain. The specified accuracy to stop the iterations of the stream function equation within each time step was (10^{-4}) i.e. $\left|\frac{\Psi^{j}-\psi^{j+1}}{\psi^{j+1}}\right| \le 10^{-4}$

where **(j)** denotes iteration number in successive iteration procedure.

The time step ($\Delta\tau$) used was (10^{-4}) reduced gradually to (10^{-5}) in some cases eliminating the instability in the stream function equation specially at high Rayleigh numbers. In this paper the computations was cared out using proper grid number according to the each aspect ratio as following:

H/L	**2**	**3**	**5**	**8**
m x n	21x41	21x51	21x81	21x121
H/L	**12**	**18**	**24**	**30**
m x n	21x181	21x251	21x331	21x421

Where (**m**) and (**n**) is number of grids in (**X**) and (**Y**) direction respectively.

Test Validation

To validate our numerical model and program code, the accuracy of the present study has been checked with the results predicted by Wilkes and Churchill[16] for natural convection of pure fluid in a rectangular enclosure.

A test study for enclosure with aspect ratio (H/L=3) at Rightly number (Ra_f=14660) was carried out, which is attend in our model by eliminating second term in the right hand side of equation (12) ant putting σ=1.0, ε=1.0 and $\alpha_e=\alpha_f$. We obtained an excellent agreement for the streamlines distribution and temperature profile consequently mean Nusselt number as showed in Fig.(2) and Fig.(3) .

RESULTS AND DISCUSSION

The porous medium used in the calculations is assumed to consists of solid spherical particle saturated with air which its thermal conductivity is (0.026 W/m.k) and density (1.2 kg/m^3). We considered three different materials for the solid particles with wide rang of thermal conductivity as follows:

Material	Alumina	Glass	Asbestos
k_s/k_f	1538.5	38.5	5.77

Three porosity(ε) values was considered 0.35, 0.45 and 0.55 . The aspect ratio (H/L) was varied from 2 to 30 and medium Rayleigh number (Ra_m) varied from (5 to 2000) while Darcy numbers (Da) considered are (10^{-3}, 10^{-4}, 10^{-5}).

To illustrate the effect of the parameters (Ra_m, ε, k_s/k_f , and Da), on streamlines and temperature profile, due to large number of cases studied which is reached to (460) case and for brevity, only a selected results are presented as a typical example and showed in figures (4-7).

The effect of medium Rayleigh number is shown in Fig.(4) for aspect ratio of (H/L=5), at the first Rayleigh number the isothermal lines seem to be a vertical parallel lines which is very close to pure conduction, as Rayleigh number increase a gradual increasing development of convective motion is occur accompanied by distortion of the temperature field when comparing two consecutive state. Also the streamlines at the first value of the Rayleigh number approximately symmetric about a vertical line through the med span, while Rayleigh number increase the local values of stream function are increase and a gradual boundary layers development occur downward on the cold wall and upward on the hot wall. These predictions are consistent with the known phenomenon of natural convection in enclosure.

Fig.(5) shows the effect of porosity on streamlines and temperature profile. It is clear from this figure that increase of porosity led to farther distortion of isothermal lines which mean increase of convection heat transfer due to increase of the amount of fluid in motion. The increase of porosity led to decrease in fluid velocity because the passages between the solid particles become larger as void ratio is increase. This can be noted from streamlines patterns which decrease in strength with porosity increase.

The effect of thermal conductivity ratio (k_s/k_f) on stream lines and temperature profile is presented in Fig.(6), clearly the increase in thermal conductivity ratio causes decrease distortion of isotherms caused by increase of conduction participation in heat transfer with respect to convection between two vertical walls of the enclosure, which results in decrease of temperature difference between them. In the same time the stream lines increase in strength because of increase of locally difference in the temperature between solid particles and adjacent fluid layers, which results in decrease of fluid density and increase of its velocity inside the enclosure.

Fig.(7) shows the effect of Darcy number on streamlines and isotherms. It is clear from this figure that Darcy number increase lead to decrease distortion of isotherms and weaken convection cell as noted from streamlines.

Fig.(8) shows results of mean Nusselt number as a function of aspect ratio (H/L) at some Rayleigh numbers. As noted from this figure, Nusselt number decrease with increase of aspect ratio at all Rayleigh numbers. This is consistent with known phenomenon of natural convection in enclosed cavities, the only difference noted is that the curves become more flatter as Rayleigh number increase.

The effect of porosity on heat transfer for each aspect ratio at different Rayleigh number is presented in Fig.(9), from this figure we can note that the Nusselt number increase with porosity increase at any Rayleigh number and aspect ratio this resulting from increase of void ratio as porosity increase, which reduce the hindering to fluid motion allowing increase in convection cells strength thus increasing heat transfer rate.

Fig.(10) shows effect of thermal conductivity ratio (k_s/k_f) on heat transfer rate. This figure illustrates that the Nusselt number is decrease with increase of thermal conductivity ratio, this observation is due to increase of heat transfer by conduction with respect to convection which lead to reduce temperature difference between two vertical walls of the enclosure. This is also consistence with definition of Nusselt number (eq. 14) which is decreased as thermal conductivity is increased despite the increase in the convection.

Darcy's number effect on heat transfer is presented in Fig.(11). The heat transfer rate which is represented by Nusselt number is increased as Darcy number is decreased.

Figures (8-11) showed that the Nusselt number directly proportional to Rayleigh number, regardless of other parameters because increase of Rayleigh number mean increase of the bouncy force which lead to increase convection heat transfer hence Nusselt number.

To provide imperical correlation equation between the parameters studied the statistical computer program (SPSS) was used for this purpose. To get an equation of greater accuracy the data sets of each aspect ratio was analyzed individually, because a large deviation is noted from obtained equation if the data of all aspect ratios are analyzed together. The mean Nusselt number was expressed as a function of the dimensionless parameters that affect the heat transfer rate and a mathematical equation of the following form are suggested for each aspect ratio:

$$\mathbf{Nu = a\,(Ra_m)^b\,(\varepsilon)^c\,(k_s/k_f)^d\,(Da)^e} \quad \ldots(18)$$

The enclosure aspect ratio is implicitly incorporated into the above equation by expressing each of the constants (a, b, c, d, e) as a function of the aspect ratio (H/L) of the power type obtained from the results summarized in Table (1) . The final form of the correlated equation is:

$$Nu = \frac{Ra^{\frac{0.43078}{0.15773(H/L)}}\;\varepsilon^{0.1887(H/L)^{0.23534}}}{(k_S/k_f)^{0.02634(H/L)^{0.21643}}\;Da^{0.0572(H/L)^{0.17416}}} \quad \ldots\ldots\ldots(19)$$

SUMMARY AND CONCLUSIONS

A numerical analysis of natural convection heat transfer in porous medium saturated with viscous fluid confined by rectangular enclosure is presented. A non-Darcian model proposed by Brinkman-Forchheimer was used. The effects of important parameters on heat transfer is illustrated including Darcy number, porosity, solid to fluid thermal conductivity ratio, enclosure aspect ratio in addition to medium Rayleigh number. The main conclusions are summarized as follows:

1- The present results have shown the importance of the effects of above parameters on heat transfer and disability of omitting effect of any of them to get reliable results.

2- With the increase of aspect ratio the heat transfer rate (Nu) is decreased so, the effect of convection appears when Rayleigh number becomes larger.

3- The role of porous media is to reduce significantly the velocities and hence the convection hear transfer.

4- A correlation equation was obtained and could be used to estimate the heat transfer rate for specified aspect ratio, or estimate a suitable insulation thickness (aspect ratio) when it used as insulated partitions.

REFERENCES

1- Chan, B. K., Ivey, C. M., and Barry, J. M., "Natural Convection in Enclosed Porous Media with Rectangular Boundaries", J. Heat Transfer, Pp. 21-27, Feb. 1970.

2- Kladias, N., and Prashad, V., "Natural Convection in a Horizontal Porous Layer: Effects of Darcy and Prandtl Number", J. Heat Transfer, No. 111, Pp. 929–935, 1989 (Internet).

3- Jue, T. C., "Analysis of Be´Nard Convection in Rectangular Cavities Filled with Porous Medium", Acta Mechanica, No. 146, Pp. 21–29, 2001 (Internet).

4- Hossain, M. A., and Wilson, M., "Effect of Heat Generation on Natural Convection Flow in a Fluid-Saturated Porous Medium Enclosed by Non-Isothermal Walls", Int. J. Thermal Sci., 2001 (Internet).

5- Hossain, M. A., and Rees, D. A. S., "Natural Convection Flow of a Viscous Incompressible Fluid in a Rectangular Porous Cavity Heated from Below with Cold Sidewalls", J. Heat and Mass Transfer, Vol. 39, Pp. 657–663, 2003.

6- Deshpande, M. D., and Srinidhi, B. G., "Multiple Solutions in a Differentially Heated Cavity", The Tenth Asian Congress of Fluid Mechanics, 17–21 May 2004 (Internet).

7- Phanikumar, M. S., and Mahajan, R. L.,"Non-DarcyNatural Convection in High Porosity Metal Foams", Int J.

Heat and Mass Transfer, Vol. 45, Pp. 3781-3793, 2002.

8- Younsi, R., Harkati, A., and Kalache, D., "Numerical Simulation of Double-Diffusive Natural Convection in a Porous Cavity: Opposing Flow", The Arabian Journal for Science and Engineering, Vol. 27, Number 2C, 2002.

9- Alazmi, B., and Vafai, K., "Analysis of Variants within The Porous Media Transport Models", J. Heat Transfer, Vol. 122, No. 5, Pp. 303-326, 2000.

10- Hslao, S. W., Cheng, P., and Chen, C. K., "Non-Uniform Porpsity and Thermal Dispersion Effects on Natural Convection about a Heated Horizontal Cylinder in an Enclosed Porous Medium", Int. J. Heat and Mass Transfer, Vol. 35, Pp. 3407-3417, 1992.

11- Kaviany, M., "Principle of Heat Transfer in Porous Media", 2nd Edition, Mechanical Engineering Series, Springer-Verlag, 1999.

12- Bejan, A., "Convective Heat Transfer in Porous Media", John Wiley and Sons, New York, 1987.

13- Al-Khafaji, A. W., and Tooley, J. R., "Numerical Methods in Engineering Practice", 1st Edition, CBS Publishing, Japan, 1986.

14- Robert, W. Hornbeck, "Numerical Marching Techniques for Fluid Flows with Heat Transfer", NASA, 1973.

15- Fletcher, C. A. J., "Computational Techniques for Fluid Dynamics 2", Springer Series in Computational Physics, Berlin, Springer-Verlag, 1991.

16- Wilkes, J. O., and Churchill, S. W., "The Finite Difference Computation of Natural Convection in a Rectangular Enclosure", A.I.Ch.E. Journal, Vol. 12, No. 1, Pp. 161-166, 1966.

Table (1) Variation of correlating constants (a, b, c, d, e) with (H/L)

H/L	a	b	c	d	e
2	0.28956	0.24682	0.38212	-0.031014	-0.06900
3	0.29156	0.23690	0.35711	-0.033860	-0.06900
8	0.25296	0.25618	0.32302	-0.037350	-0.06990
12	0.22003	0.33795	0.29881	-0.048600	-0.09497
18	0.25568	0.38785	0.26144	-0.048800	-0.09063
24	0.20907	0.41682	0.26874	-0.052967	-0.09871
30	0.20379	0.43134	0.24511	-0.054855	-0.11181

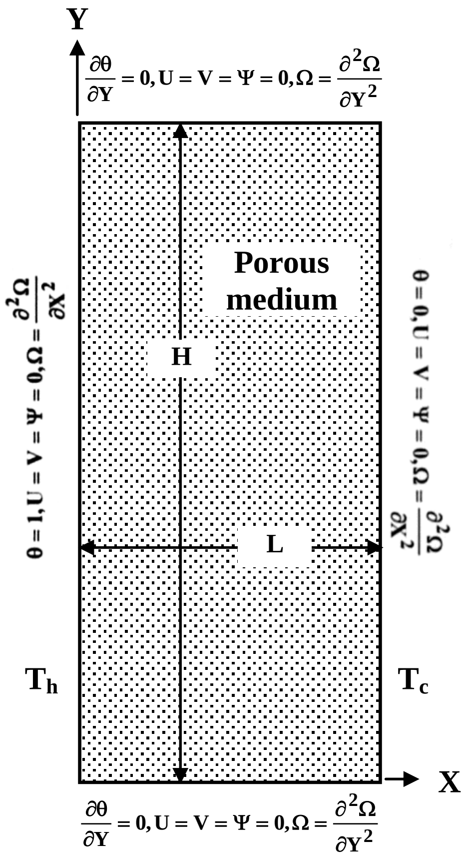


Fig.(1) Schematic diagram of the problem and the corresponding coordinate system

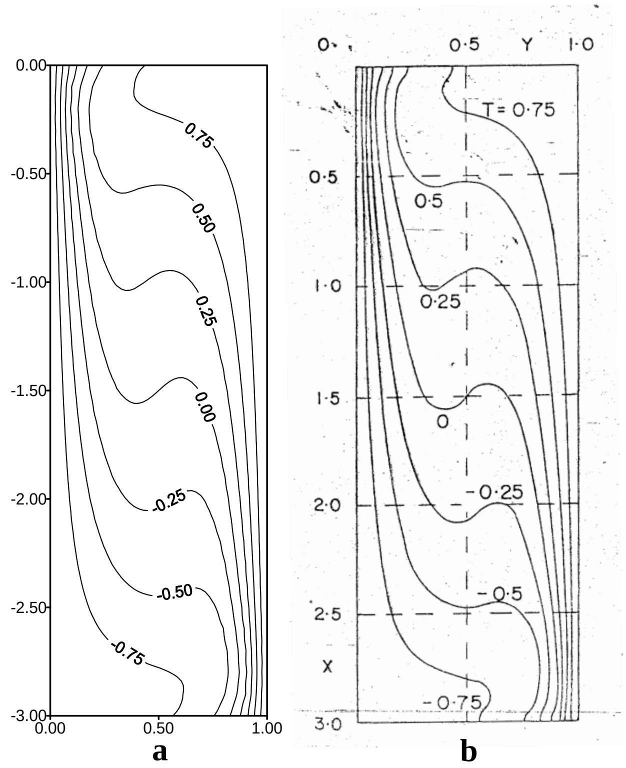


Fig.(2) Isotherms for H/L=3 and Ra_f=14660 . (a) Present code, (b) Reference [16]

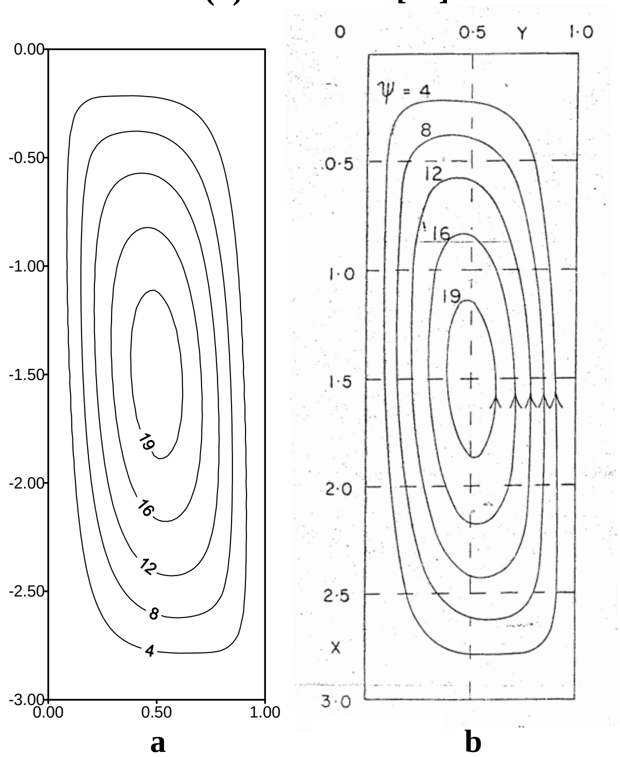
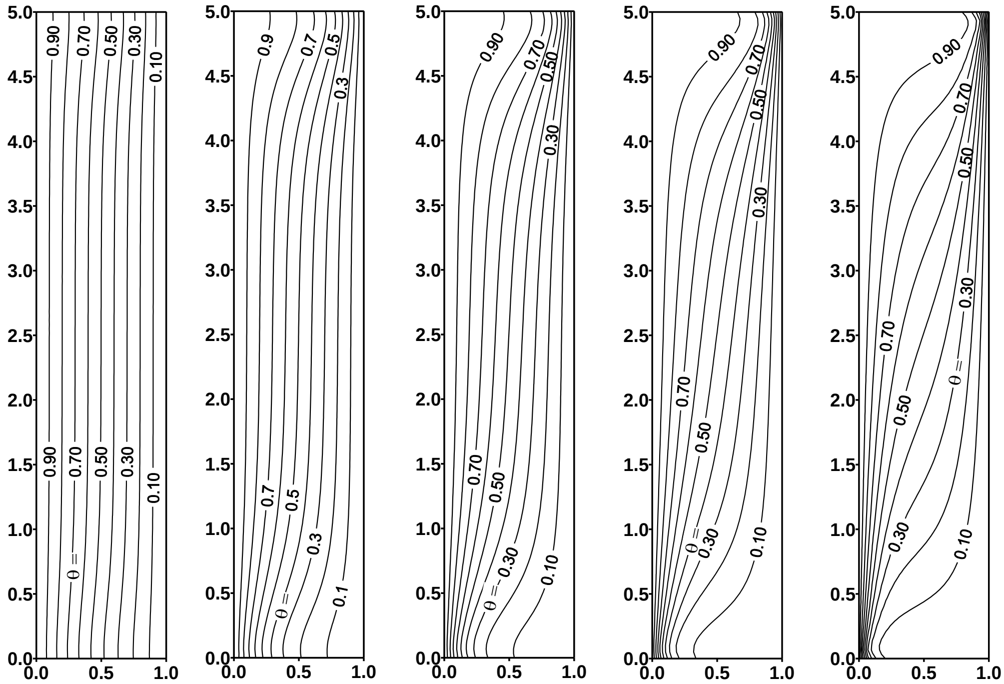
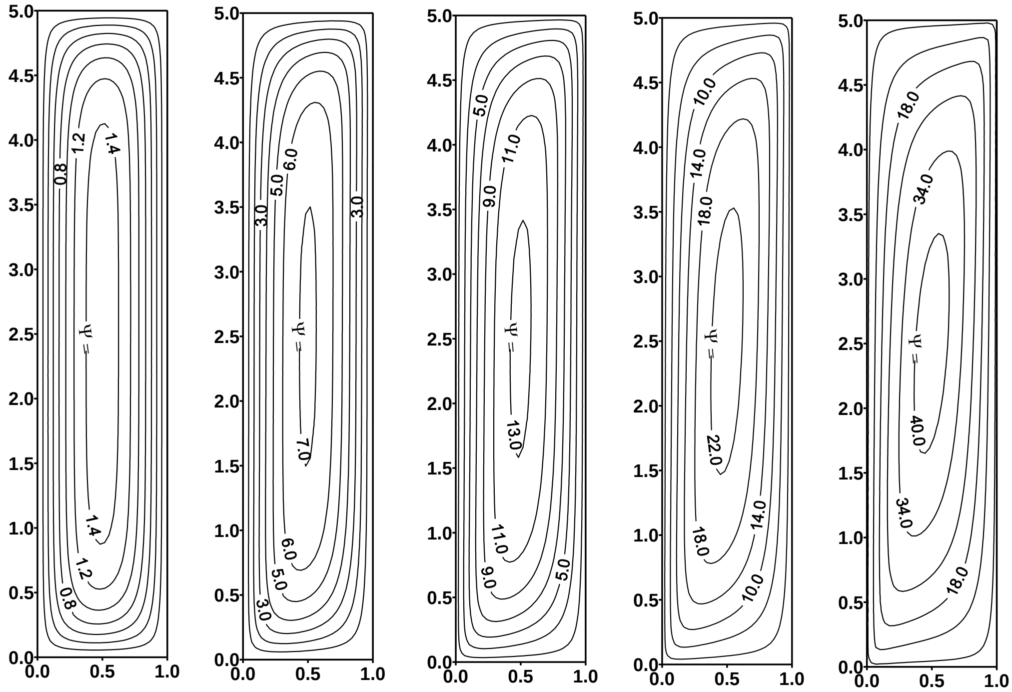


Fig.(3) Streamlines for H/L=3 and Ra_f=14660 . (a) Present code, (b) Reference [16]



Ra_m=5 Ra_m=25 Ra_m=50 Ra_m=100 Ra_m=250

Temperature profile (θ)



Ra_m=5 Ra_m=25 Ra_m=50 Ra_m=100 Ra_m=250

Streamlines (Ψ)

Fig.(4) Effect of medium Rayleigh number (Ra_m) on Temperature profile and Streamlines for H/L=5, Da=10^{-4}, ε=0.45, k_s/k_f =5.77

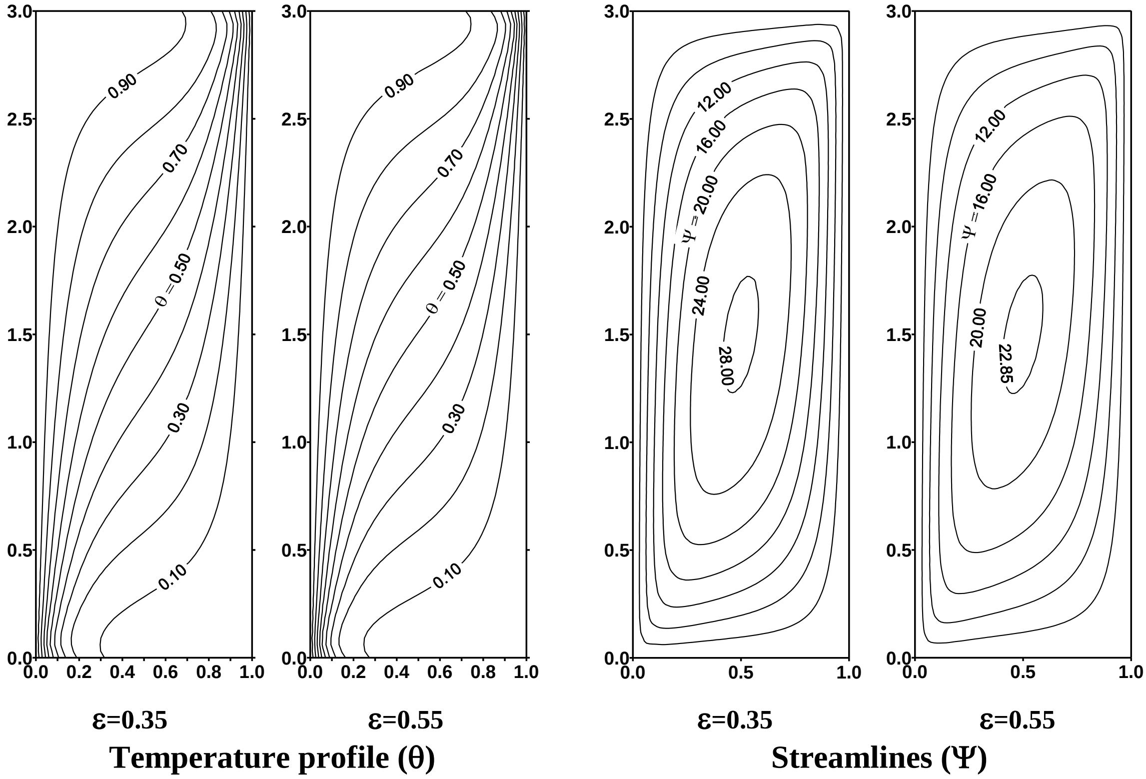


Fig.(5) Effect of porosity (ε) on Temperature profile and Streamlines for H/L=3, Ra_m=150, Da=10^{-4}, k_s/k_f =5.77

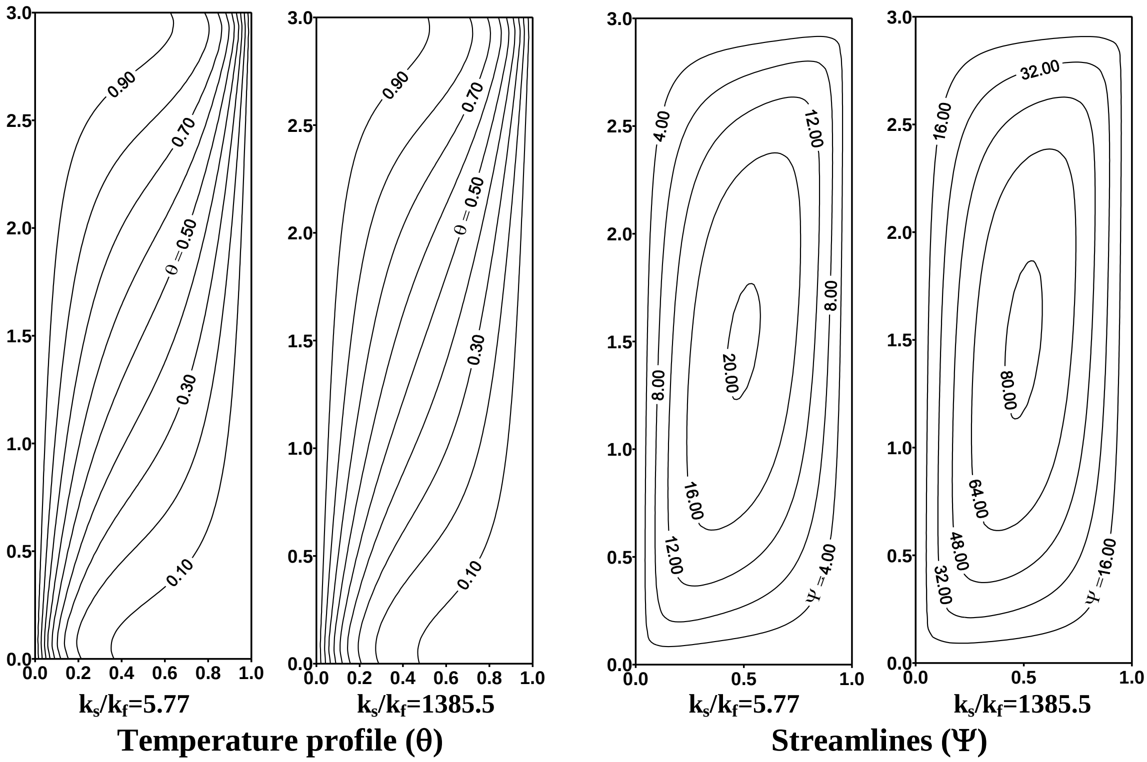


Fig.(6) Effect of thermal conductivity ratio (k_s/k_f) on Temperature profile and Streamlines for H/L=3, Ra_m=150, ε=0.45, Da=10^{-4}

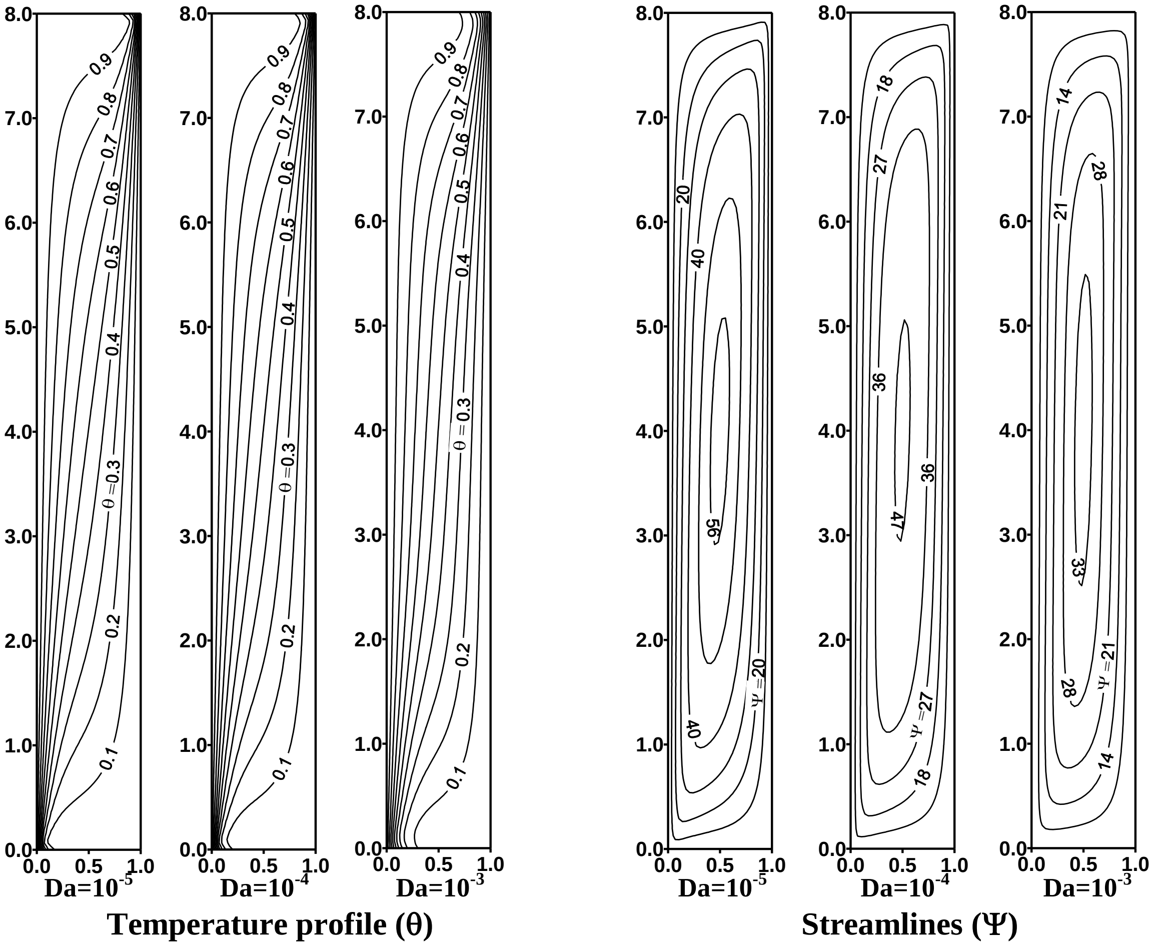


Fig.(7) Effect of Darcy number (Da) on Temperature profile and Streamlines for H/L=8, Ra_m=250, ε=0.45, k_s/k_f =5.77

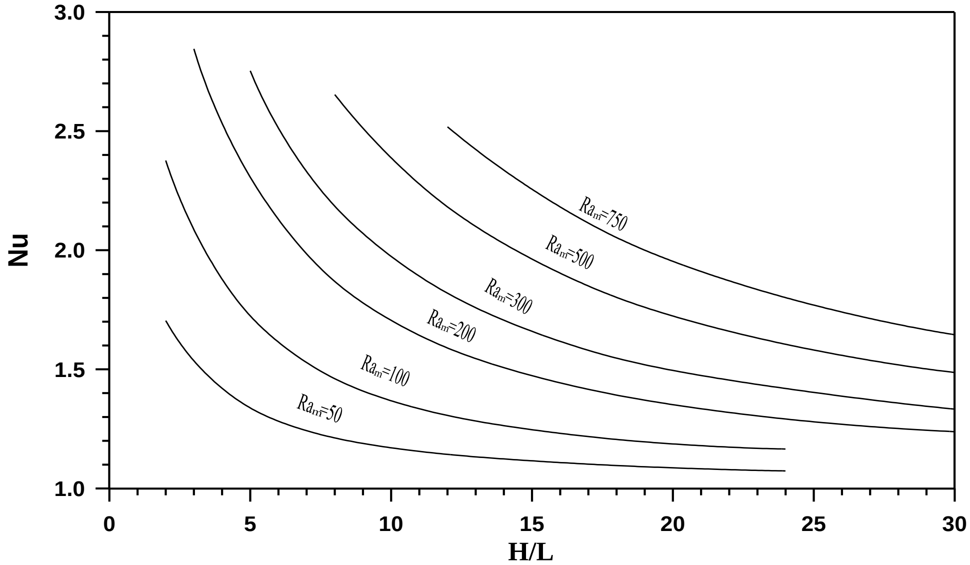


Fig.(8) Variation of mean Nusselt number (Nu) with aspect ratio (H/L) at various medium Rayleigh number (Ra_m) for Da=10^{-4}, ε=0.45, k_s/k_f =5.77

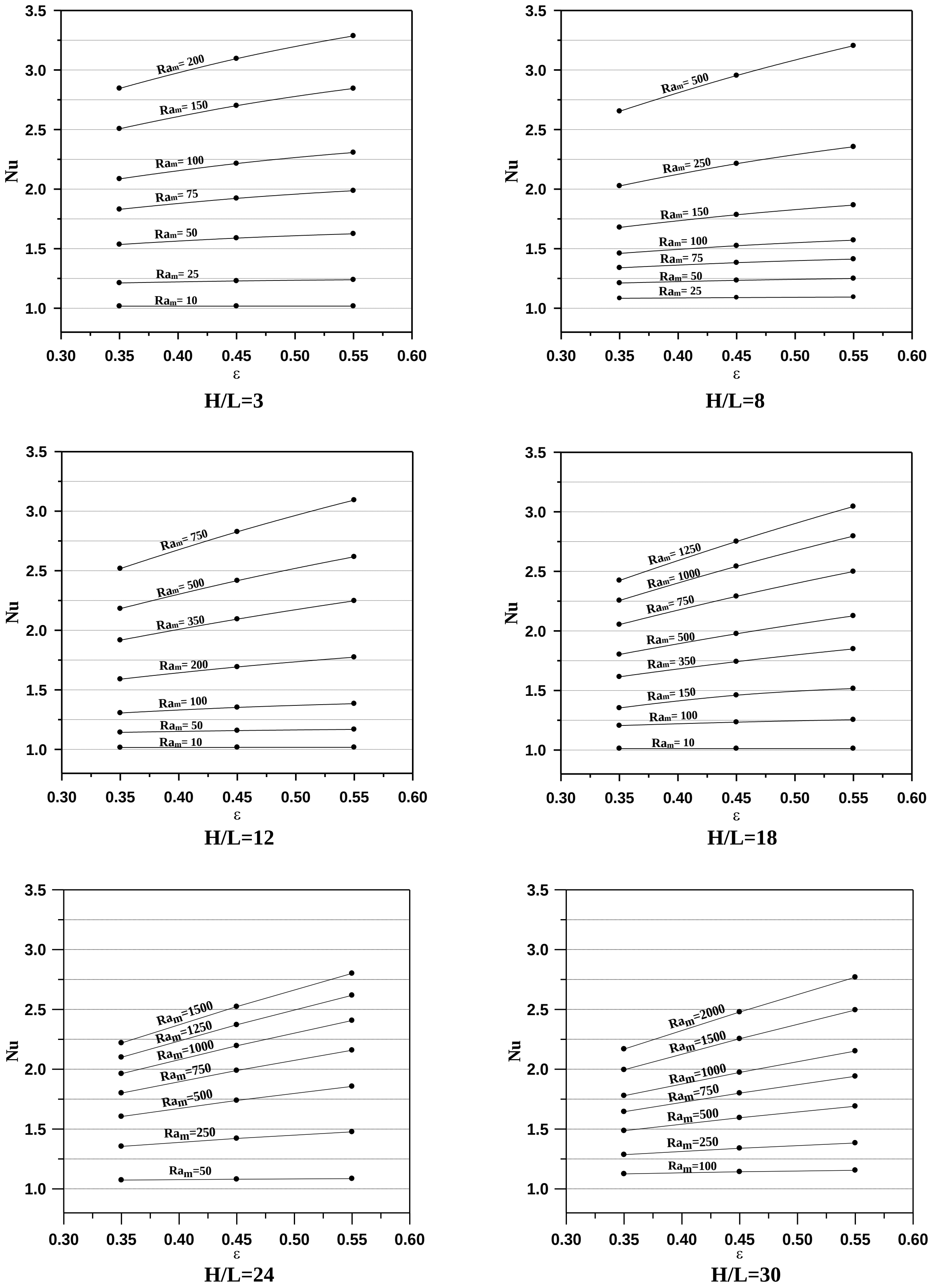


Fig.(9) Variation of mean Nusselt number (Nu) with porosity (ε) at different medium Rayleigh number and aspect ratio for $Da=10^{-4}$, $k_s/k_f=5.77$

Tikrit Journal of Eng. Sciences/Vol.15/No.2/June 2008, (90-111)

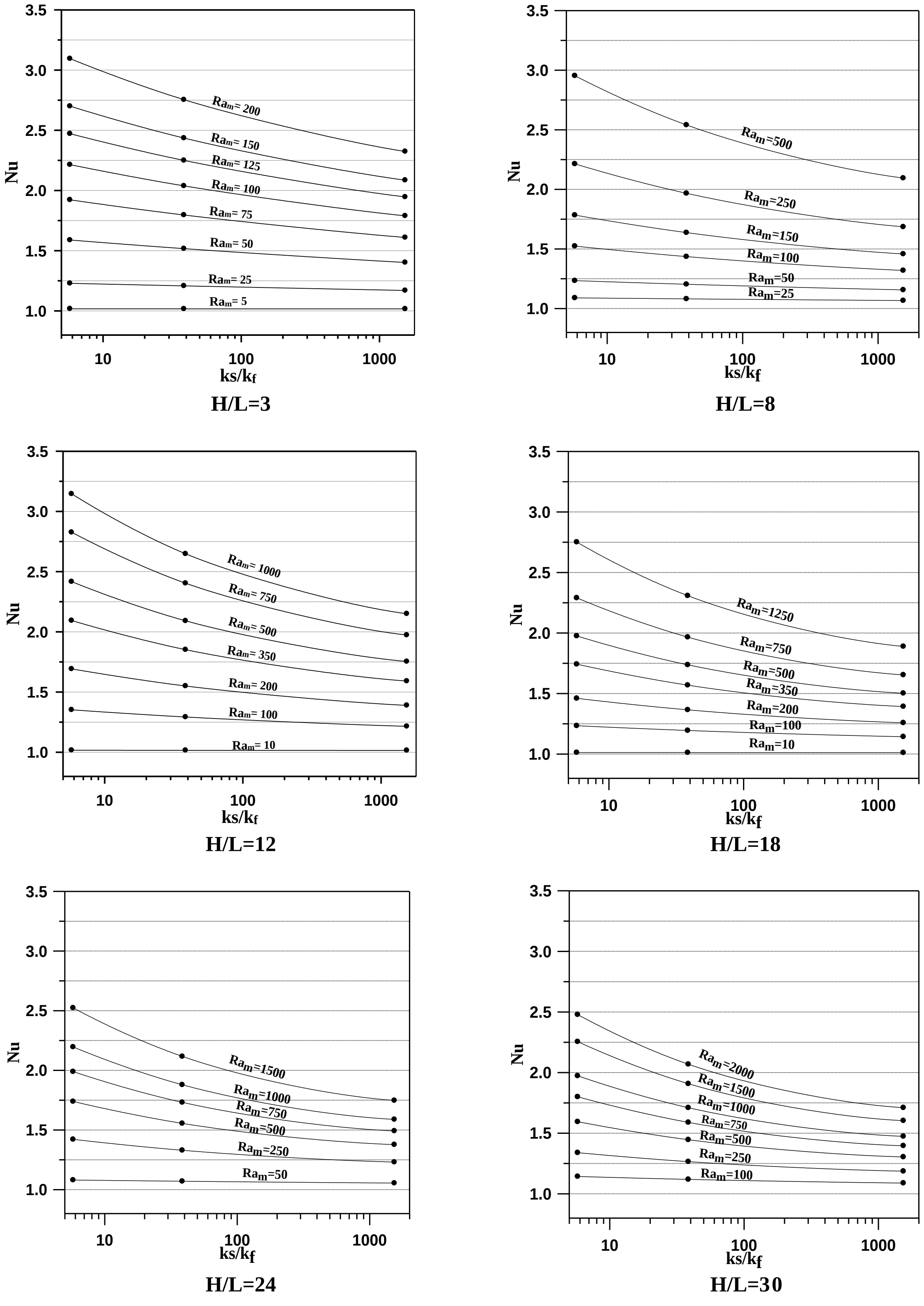


Fig.(10) Variation of mean Nusselt number (Nu) with thermal conductivity ratio (k_s/k_f) at different medium Rayleigh number and aspect ratio for $Da=10^{-4}$, $\varepsilon=0.45$

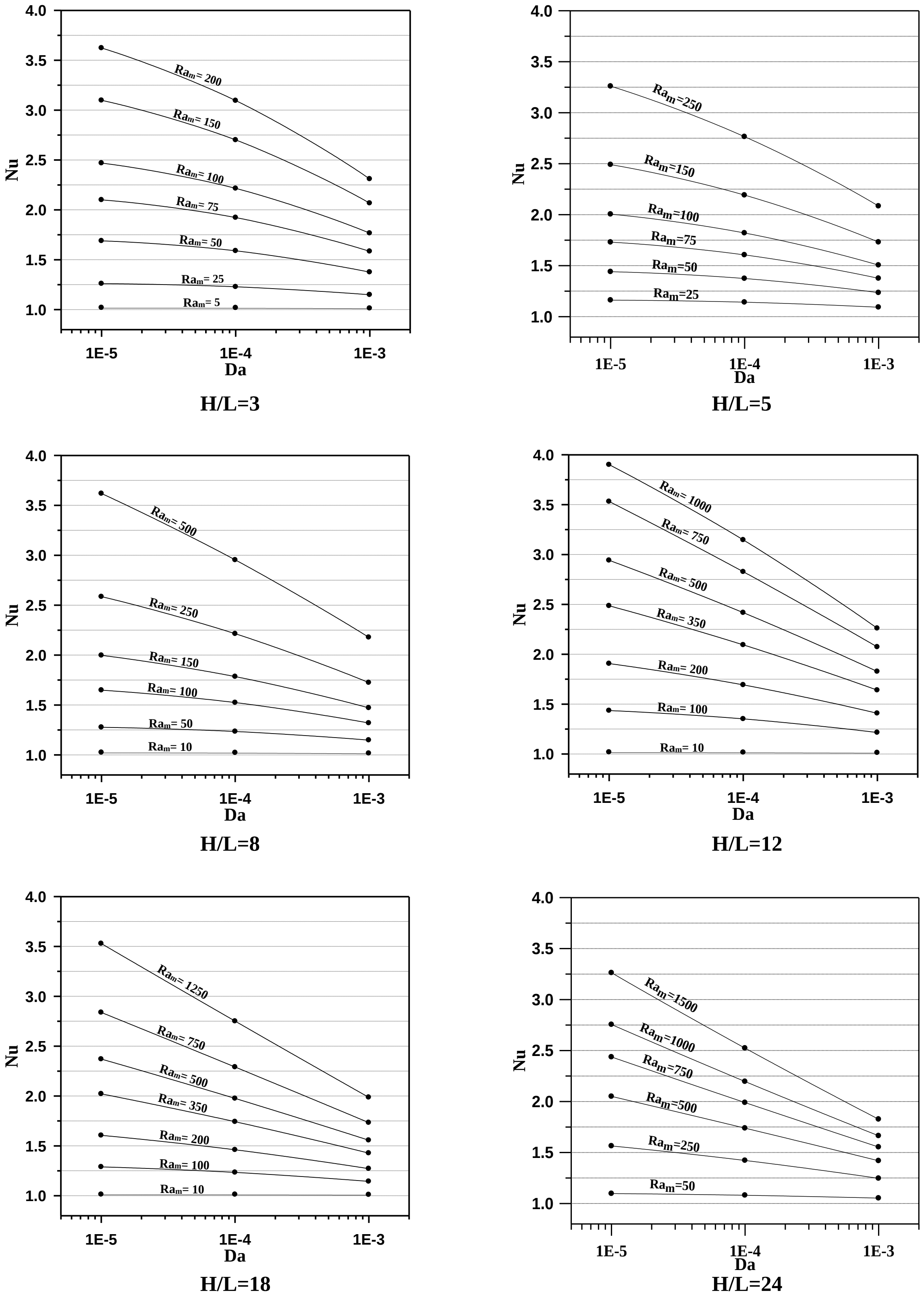


Fig.(11) Variation of mean Nusselt number (Nu) with Darcy number (Da) at different medium Rayleigh number and aspect ratio for ε=0.45, k_s/k_f=5.77

دراسة عددية لانتقال الحرارة بالحمل الحر اللا دارسيني داخل حيز مستطيل مملوء بوسط مسامي مشبع بمائع لزج

محمود حسين علي	أحمد توفيق أحمد	خلف إبراهيم حمادة
مدرس مساعد	مدرس مساعد	مدرس مساعد

قسم الهندسة الميكانيكية – جامعة تكريت

الخلاصة

أُجريت دراسة عددية لانتقال الحرارة بالحمل الحر اللا دارسيني داخل حيز مستطيل مملوء بوسط مسامي مشبع بمائع لزج. شملت الدراسة بيان تأثير كل من عدد رالي للوسط والمسامية ونسبة الموصلية الحرارية لكرات الوسط إلى المائع وعدد دارسي والنسبة الباعية للحيز (H/L) على انتقال الحرارة لبيان إمكانية استخدام هذا التركيب في العزل الحراري لجدران الأبنية.

أُستخدم إنموذج جريان دارسي–فورشماير–بركمان المطور مع شرط عدم الانزلاق للسرعة على الجدران، وقد تم التعبير عن المعادلات الحاكمة باستخدام دالة الجريان والدوامية ودرجة الحرارة اللا بعدية. حُلت المعادلات الجبرية الناتجة من تمثيل معادلتي الدوامية ودرجة الحرارة بطريقة الفروقات المحددة باستخدام طريقة الاتجاه المتناوب الضمني (ADI) والتي تستخدم خوارزمية (TDMA) في كل اتجاه، أما المعادلات الجبرية الناتجة من تمثيل معادلة دالة الانسياب فتم حلها باستخدام طريقة التكرار المتعاقب.

أُنجزت الدراسة ضمن مدى من النسب الباعية (H/L) تراوحت بين ($2 \le H/L \le 30$) ضمن منطقة الطبقات الطولية عند أعداد رالي للوسط تراوحت بين ($5 \le Ra_m \le 2000$)، فضلاً عن أعداد دارسي عند ($Da=10^{-3}, 10^{-4}, 10^{-5}$) والمسامية عند ($\varepsilon=0.35, 0.45, 0.55$) ونسبة الموصلية الحرارية لكرات الوسط إلى المائع عند ($k_s/k_f=5.77, 38.5, 1385.5$). أظهرت النتائج أن عدد نسلت يتناسب طردياً مع كل من عدد رالي للوسط والمسامية وعكسياً مع كل من عدد دارسي ونسبة الموصلية الحرارية والنسبة الباعية للحيز، وأُستنتجت علاقة إرتباطية ممثلة بأفضل معادلة رياضية ممكنة تصف تأثير تلك المتغيرات على انتقال الحرارة والممثلة بمتوسط عدد نسلت (Nu) في التركيب أعلاه.

الكلمات الدالة: وسط مسامي، حيز مستطيل، حمل حر لا دارسيني، مائع لزج.