# ESTIMATION OF HYPERBOLIC STRESS-STRAIN PARAMETERS FOR GYPSEOUS SOILS

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**ABSTRACT:**

The hyperbolic model is a simple stress-strain relationship based on the concept of incrementally nonlinear elastic behavior. The hyperbolic stress-strain relationship was developed for use in finite element analysis of stresses and movements in earth masses. To estimate hyperbolic parameter values required for nonlinear finite element analysis, data used from the triaxial compression tests for the gypseous soils exposed to the effect of drying and wetting cycles carried out by (Mohammed, 1993). From these data, the parameters (C, $\phi$, $K$, $n$, $R_f$), which are required by Duncan-Chang model, 1970 can obtained for analyses of dams, excavations and various types of soil-structure interaction problems.

In addition, it can be found that the primary loading modulus, $K$, the exponent number, $n$, and the failure ratio, $R_f$, have random values during rewetting cycles for CU and UU triaxial compression tests.

**الخلاصة:**

ان النموذج ذو المقطع الزائد هو عبارة عن علاقة بسيطة بين الاجهاد والانفعال تعتمد على مفهوم التصرف التزايدي اللاخطي المرن. هذه العلاقة وظفت كي تستخدم في طريقة تحليل العناصر المحددة للاجهادات وحركات الكتل الترابية. لتخمين قيم متغيرات المقطع الزائد المطلوبة للتحليل اللاخطي للعناصر المحددة اخذت معلومات من فحوصات الانضغاط ثلاثي المحاور للترب الجبسية المعرضة لتأثير دورات التجفيف والترطيب والمنجزة من قبل (Mohammed, 1993). ومن هذه المعلومات ، تم الحصول على المتغيرات (C, $\phi$, $K$, $n$, $R_f$) المطلوبة لنموذج Duncan-Chang model, 1970 لتحليل السدود والحفريات والعديد من أنواع مشاكل التداخل بين التربة والمنشأ.

بالإضافة الى ذلك وجد ان معامل التحميل الابتدائي $(K)$، العدد التكاملي$(n)$، ونسبة الفشل $(R_f)$ لها قيم عشوائية خلال دورات اعادة الترطيب ضمن فحوصات الانضغاط ثلاثي المحاور الغير مبزول بنوعية غير المنضم (UU) والمنضم (CU).

**Introduction:**

The hyperbolic stress-strain relationship was developed for use in finite element analyses of stresses and movements in earth masses. In the ten years since its development, the model has been used in analyses of a large number of dams, braced and open excavations, and a variety of types of soil-structure interaction problems.

In its original form, as described by Duncan and Chang (1970)(1), the hyperbolic model employed tangent values of Young's modulus ($E_t$), which varied with the magnitudes of the stresses, and constant values of Poisson's ratio. The Young's modulus relationships remain the same as described by Duncan and Chang (1970)(1).

The principal advantage of the hyperbolic model is its generality. It can be used to represent the stress-strain behavior of soil ranging from clays and silts through sands, gravels and rockfills. It can be used for partly saturated or fully saturated soils, and for either drained or undrained loading conditions in compacted earth material or naturally- occurring soils. Experience with treating these various types of problems, and the accumulated background of stress-strain parameter values for a wide variety of soils, provide a useful base for further applications. To estimate hyperbolic stress-strain parameters required for finite element analysis, the data collected into triaxial compression tests of gypseous soils carried out by (Mohammed, 1993)(2).

**Stress-Strain Relationships:**

The hyperbolic stress-strain relationships are developed for incremental analyses of soil deformations where nonlinear behavior is modeled by a series of linear increments. The relationship between stress and strain is assumed to be governed by the generalized Hook's Law of elastic deformations. For plane strain conditions this relationship may be expressed as follows (3):

$$\begin{Bmatrix}\Delta\sigma_x\\ \Delta\sigma_y\\ \Delta\tau_{xy}\end{Bmatrix}=\frac{E_t}{(1+\nu_t)(1-\nu_t)}\begin{bmatrix}(1-\nu_t) & \nu_t & 0\\ \nu_t & (1-\nu_t) & 0\\ 0 & 0 & (1-2\nu_t)/2\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_x\\ \Delta\varepsilon_y\\ \Delta\gamma_{xy}\end{Bmatrix} \quad \text{….(1)}$$

where:

$\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\tau_{xy}$ = are the increments of stress during a step of analysis.

$\Delta\varepsilon_x$, $\Delta\varepsilon_y$ and $\Delta\gamma_{xy}$ = are the corresponding increments of strain.

$E_t$ = is the tangent value of Young's modulus.

$\upsilon_t$ = is the tangent value of Poisson's ratio.

The value of both $E_t$ and $\upsilon_t$ in each element change during each increment of loading in accordance with the calculated stresses in that element, in order to account for three important characteristics of the stress-strain behavior of soil, namely non linearity, stress-dependency, and inelasticity. The procedures used to account for these characteristics are described in the following paragraphs.

**Nonlinear Stress-Strain Curves Represented By Hyperbolas:**

Kondner (1963)(4) showed that the stress-strain curves for many of soils, both clay and sand, can be approximated reasonably accurate by hyperbolas like the one shown in figure (1-A). This hyperbola can be represented by an equation of the form:

$$(\sigma_1-\sigma_3)=\frac{\varepsilon}{\dfrac{1}{E_i}+\dfrac{\varepsilon}{(\sigma_1-\sigma_3)_{ult}}} \quad \text{……………(2)}$$

These hyperbolas have two characteristics which make their use convenient:

1. The parameters which appear in the hyperbolic equation have physical significance. $E_i$ is the initial tangent modulus or initial slope of the stress-strain curve, and $(\sigma_1-\sigma_3)_{ult}$ is the asymptotic value of stress difference which is related closely to the strength of the soil. The value of $(\sigma_1-\sigma_3)_{ult}$ is always greater than the compressive strength of the soils.
2. The values of $E_i$ and $(\sigma_1-\sigma_3)_{ult}$ for a given stress-strain curve can be determined easily. If the hyperbolic equation is transformed as shown in figure (1-B), it represents a linear relationship between $[\varepsilon/(\sigma_1-\sigma_3)]$ and $\varepsilon$.

Thus, to determine the best-fit hyperbola for the stress-strain curve, values of [$\varepsilon/(\sigma_1-\sigma_3)$] calculated from the test data are plotted against $\varepsilon$. The best-fit straight line on this transformed plot corresponds to the best-fit hyperbola on the stress-strain plot.

This research, data from the triaxial compression tests of gypseous soils exposed to rewetting cycles presented by researcher (Mohammed, 1993)(2) is used to re-plot the stress-strain relations to calculate the required nonlinearity parameters.

Figures (2) to (7) show stress-strain relationships for triaxial compression tests of gypseous soils exposed to rewetting cycles carried out by the researcher (Mohammed, 1993)(2).

The stress-strain relations obtained in figures (2) to (7) have been processed as shown in figures (8) to (13). For each stress-strain curve, the values of [$\varepsilon/(\sigma_1-\sigma_3)$] are calculated for each single curve, and then, [$\varepsilon/(\sigma_1-\sigma_3)$] are plotted against $\varepsilon$. For each of these plotted curves the values of (a) and (b) are obtained, where (a) is the value of the (y-axis) intercept and (b) is the slope of the curve as explained in figure (1-B).

The value of the initial modulus of elasticity ($E_i$) is obtained as:

$$E_i = \frac{1}{a} \qquad \text{..............(3)}$$

and the value of asymptotic ultimate deviator stress $(\sigma_1 - \sigma_3)_{ult}$ is obtained as:-

$$(\sigma_1 - \sigma_3)_{ult} = \frac{1}{b} \qquad \text{.................(4)}$$

The failure ratio ($R_f$) is evaluated as:

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} \qquad \text{..................(5)}$$

The variation of $(\sigma_1-\sigma_3)_f$ with $\sigma_3$ is represented by the familiar More-Coulomb strength relationship, which can be expressed as follows:

Mohr-Coulomb failure criterion, then:

$$(\sigma_1 - \sigma_3)_f = \frac{2c\cos\phi + 2\sigma_3 \sin\phi}{1-\sin\phi} \qquad \text{................(6)}$$

In which c and $\phi$ are the cohesion intercept and the friction angle.

**Stress-Dependent Stress-Strain Behavior:**

For all soils, except fully saturated soils tested under unconsolidated-undrained conditions, an increase in confining pressure will result in a steeper stress-strain curve and a higher strength and the values of $E_i$ and $(\sigma_1-\sigma_3)_{ult}$ therefore increase with increasing confining pressure. This stress-dependency is taken into account by using empirical equations to represent the variations of $E_i$ and $(\sigma_1-\sigma_3)_{ult}$ with confining pressure (1),( 3).

The variation of $E_i$ with $\sigma_3$ is represented by an equation suggested to be Janbu (1963)(5) of the form:-

$$E_i = K\,P_a\left(\frac{\sigma_3}{P_a}\right)^n \qquad \text{...............(7)}$$

The variation of $E_i$ with $\sigma_3$ corresponding to this equation is shown in figure (14). The parameter K in equation (7) is the modulus number, and n is the modulus exponent. Both factors are dimensionless numbers. $P_a$ is atmospheric pressure, introduced into the equation to make conversion from one system of units to another more convenient system. The values of K and n are the same for any system of units, and the units of $E_i$ are the same as the units of $P_a$.

Data of figures (8) to (13) are used to plot the relations between the values of $E_i$ and $\sigma_3$ for each series of tests as shown in figures (15) to (20).

The value of K is obtained by taking the value of initial modulus corresponding to one unit of confining pressure, while n is evaluated to be the slope of the ( $E_i$ - $\sigma_3$ ) relation, which is a straight line on a log-log scale.

**Relationship Between $E_t$ and The Stresses:**

The instantaneous slope of the stress-strain curve is the tangent modulus, $E_t$. By differentiating equation (2) with respect to $\varepsilon$ and substituting the expressions of equations (5), (6), (7) into the resulting expression for $E_t$, the following equation can be derived:

$$E_t = \left[1-\frac{R_f(1-\sin\phi)(\sigma_1-\sigma)}{2c\cos\phi+2\sigma_3\sin\phi}\right]KP_a\left(\frac{\sigma_3}{P_a}\right)^n \qquad \text{................(8)}$$

This equation can be used to calculate the appropriate value of tangent modulus for any stress conditions [$\sigma_3$ and ($\sigma_1$- $\sigma_3$)] if the values of the parameters K, n, c, $\phi$, and $R_f$ are known.

**Results And Discussion of Hyperbolic Stress-Strain Parameters:**

This section includes the results of hyperbolic stress-strain parameters extracted from the triaxial compression tests of different gypseous soils exposed to the effect of wetting and drying cycles and summarized in the tables (1) and (2).

The variation of the Duncan-Chang model parameters with wetting and drying cycles is presented in figures (21) to (23).

From figures (21) to (23), it can be observed that the primary loading modulus (K), the exponent number (n) and failure ratio ($R_f$) have random values during rewetting cycles for both tests.

**Conclusions:**

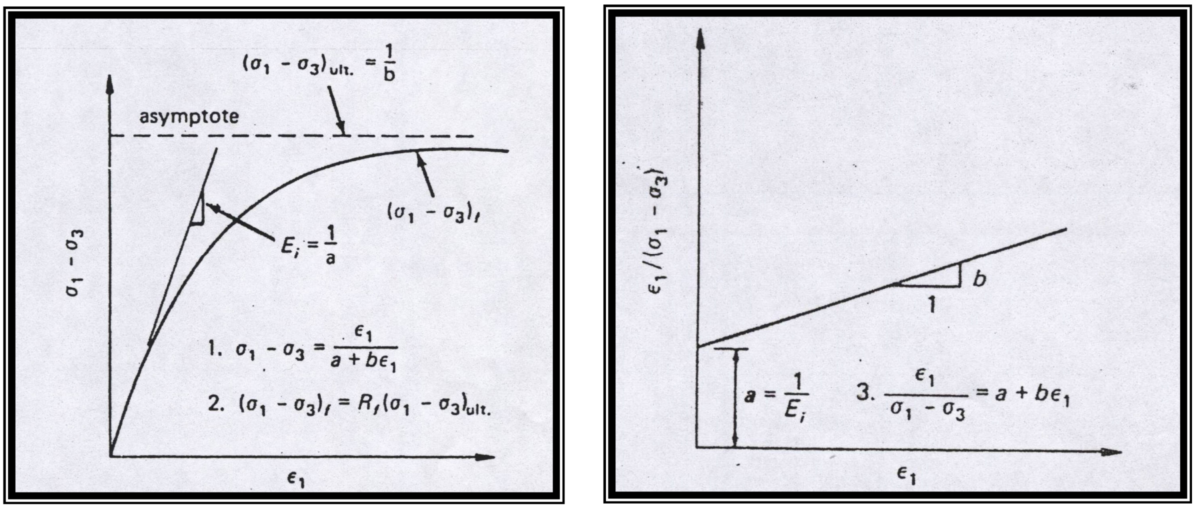
The hyperbolic model is a simple stress-strain relationship based on the concept of incrementally nonlinear elastic behavior. It is applicable to virtually any type of soil and to drained or undrained conditions. Experience in applying the hyperbolic model to analyses of dams, excavations and various types of soil-structure interaction problems has shown that it is useful for calculation movements in stable earth masses, and is not suitable for predicting instability or collapse loads. Like any theory hypothesis of soil behavior, its successful application requires the exercise of engineering judgment. In addition, it can be concluded that the primary loading modulus, K, the exponent number, n, and the failure ratio, $R_f$, have random values during rewetting cycles for both tests.

**Table (1): Hyperbolic model parameters for gypseous soil samples (the present study).**

| Parameters | Disturbed Gypseous Soil | | | | |
|---|---|---|---|---|---|
| | Cycle (1) | Cycle (15) | Cycle (30) | Cycle (45) | Cycle (60) |
| K | 270 | 100 | 200 | 145 | 275 |
| n | 0.84 | 2.32 | 0.10 | 0.56 | 0.21 |
| $R_f$ | 0.85 | 0.65 | 0.78 | 0.82 | 0.70 |
| $K_{ur}$ | - | - | - | - | - |
| **c (kPa)** | 59.0 | 98.3 | 92.2 | 56.0 | 59.7 |
| **$\phi$ (degree)** | 36.4 | 23.8 | 22.7 | 31.5 | 25.1 |
| **$\gamma$ kN/m$^3$** | 17.08 | 17.41 | 16.78 | 17.89 | 17.61 |
| **Soil Location** | Al-Sherqat area | | | | |
| **Classified Soil** | Clayey sandy silt | | | | |
| **Test Type** | UU-triaxial test | | | | |
| **Reference** | Mohammed, 1993 | | | | |

**Table (2): Hyperbolic model parameters for gypseous soil samples (the present study).**

| **Parameters** | **Disturbed Gypseous Soil** | | | | |
|---|---|---|---|---|---|
| | **Cycle (1)** | **Cycle (15)** | **Cycle (30)** | **Cycle (45)** | **Cycle (60)** |
| **K** | 800 | 800 | 990 | 750 | 1785 |
| **n** | 0.74 | 0.22 | 0.03 | 0.41 | 1.24 |
| **$R_f$** | 0.73 | 0.97 | 0.95 | 0.75 | 0.95 |
| **$K_{ur}$** | - | - | - | - | - |
| **c (kPa)** | 104.1 | 131.4 | 99.1 | 79.1 | 61.8 |
| **$\phi$ (degree)** | 35.5 | 34.3 | 36.0 | 36.3 | 35.8 |
| **$\gamma$ kN/m$^3$** | 17.64 | 17.3 | 17.07 | 17.29 | 16.98 |
| **Soil Location** | Al-Sherqat area | | | | |
| **Classified Soil** | Clayey sandy silt | | | | |
| **Test Type** | CU-triaxial test | | | | |
| **Reference** | Mohammed, 1993 | | | | |




A- Real B-Transformed

Figure (1): Hyperbolic representation of a stress-strain curve (after Kondner, 1963).

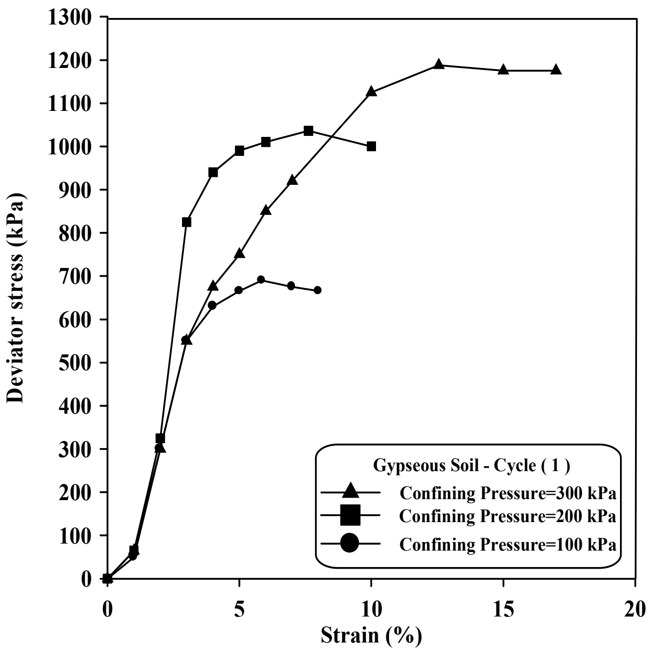



Figure (2): Stress-strain relationship for CU- triaxial testing (Mohammed, 1993).

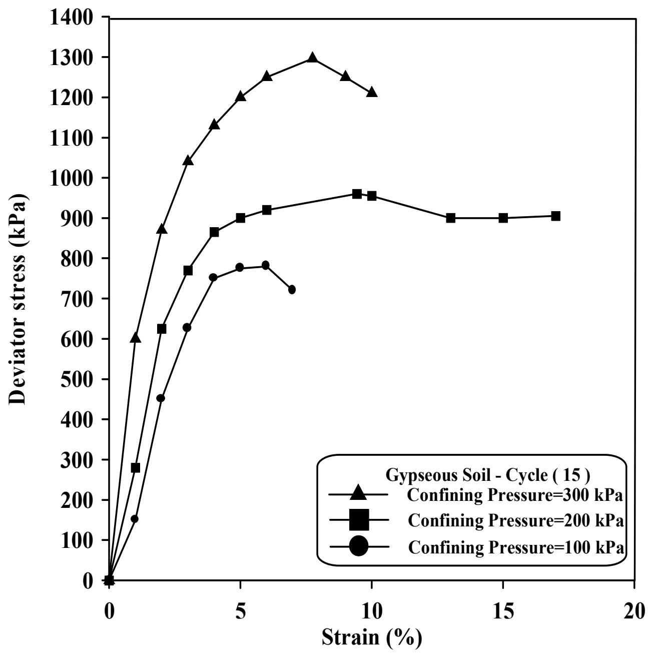



Figure (3): Stress-strain relationship for CU- triaxial testing (Mohammed, 1993).

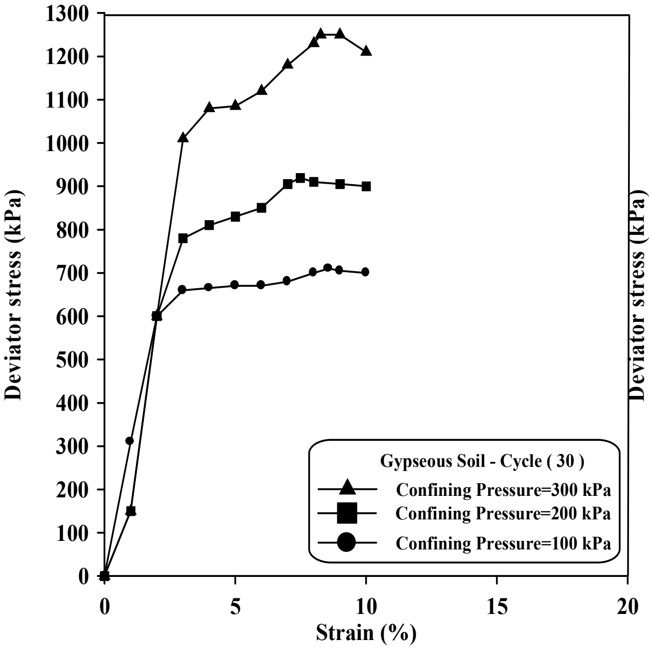



Figure (4): Stress-strain relationship for CU- triaxial testing (Mohammed, 1993).

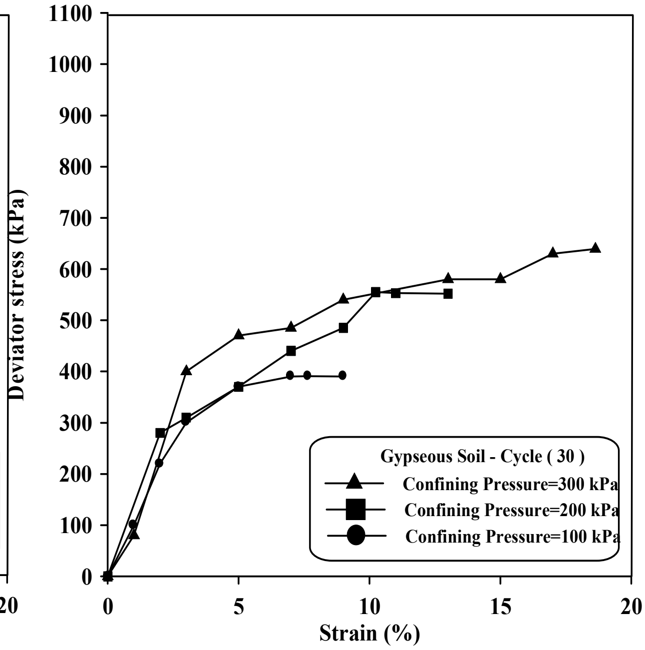



Figure (5): Stress-strain relationship for UU- triaxial testing (Mohammed, 1993).

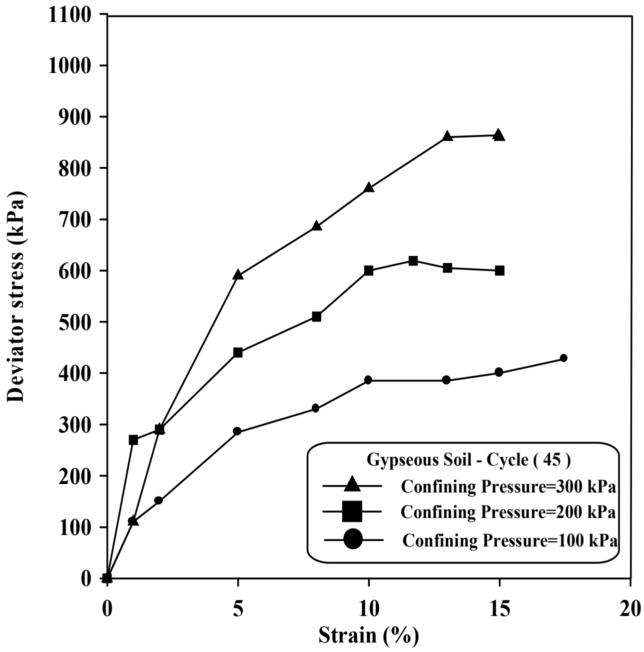



Figure (6): Stress-strain relationship for UU- triaxial testing (Mohammed, 1993).

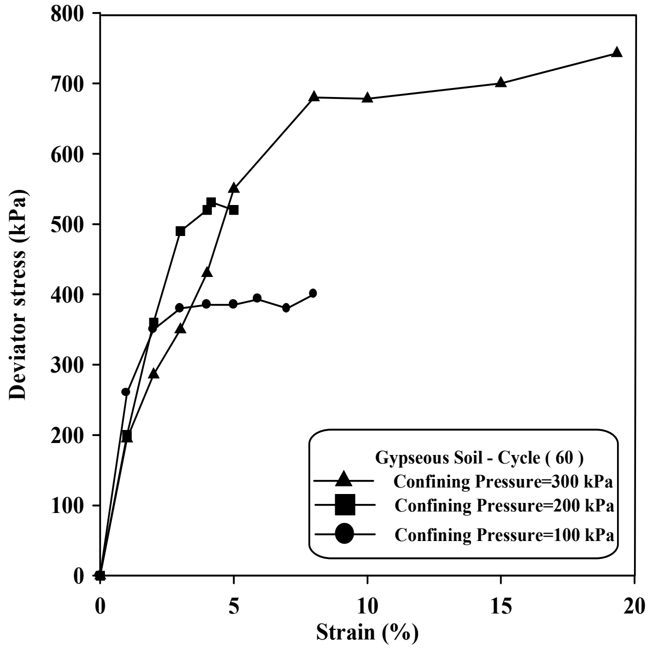



Figure (7): Stress-strain relationship for UU- triaxial testing (Mohammed, 1993).

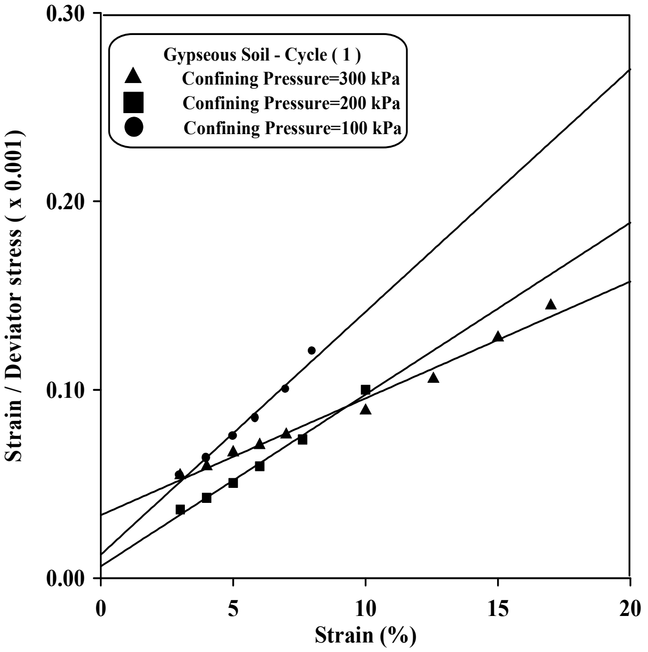



Figure (8): Strain / Stress ratio vs. strain for CU- triaxial testing (Mohammed, 1993).

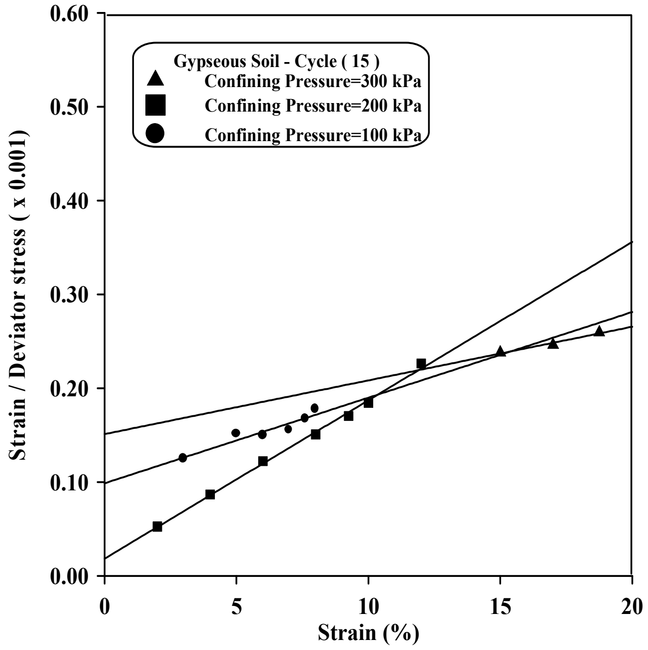



Figure (9): Strain / Stress ratio vs. strain for CU- triaxial testing (Mohammed, 1993).

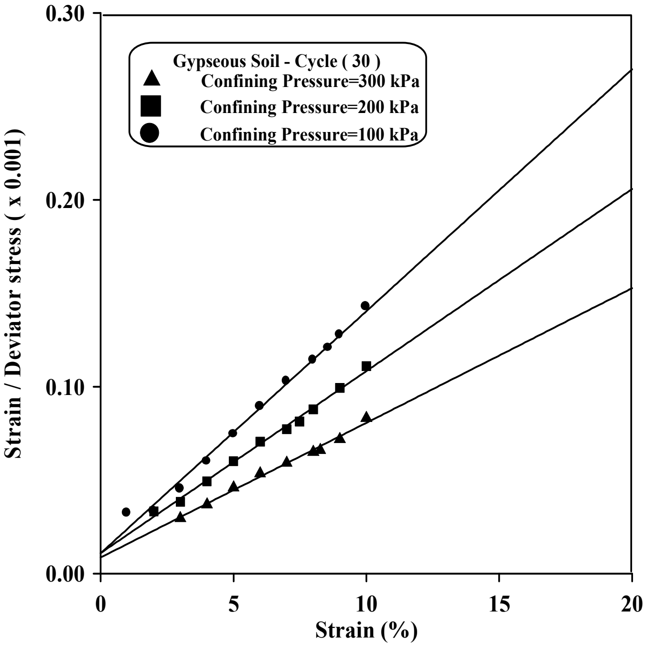



Figure (10): Strain / Stress ratio vs. strain for CU- triaxial testing (Mohammed, 1993).

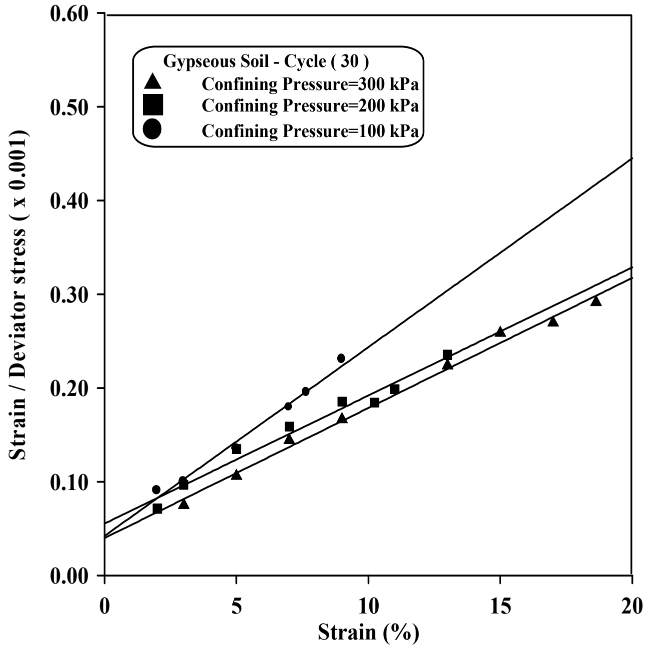



Figure (11): Strain / Stress ratio vs. strain for UU- triaxial testing (Mohammed, 1993).

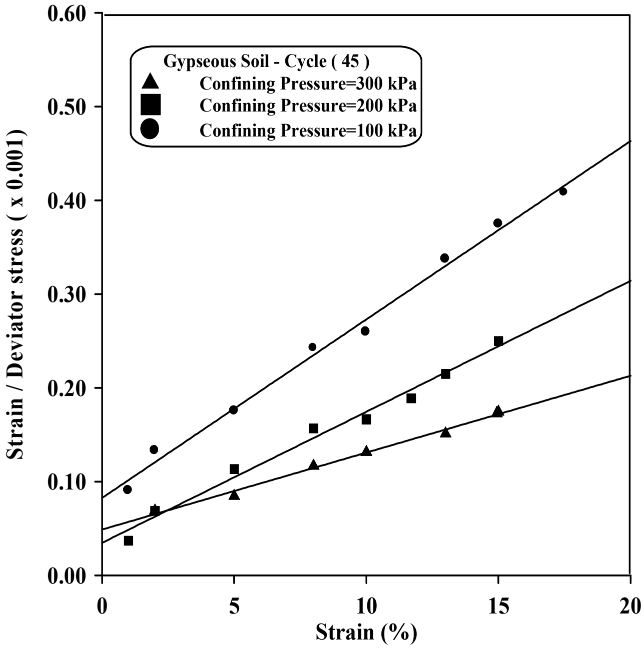



Figure (12): Strain / Stress ratio vs. strain for UU- triaxial testing (Mohammed, 1993).

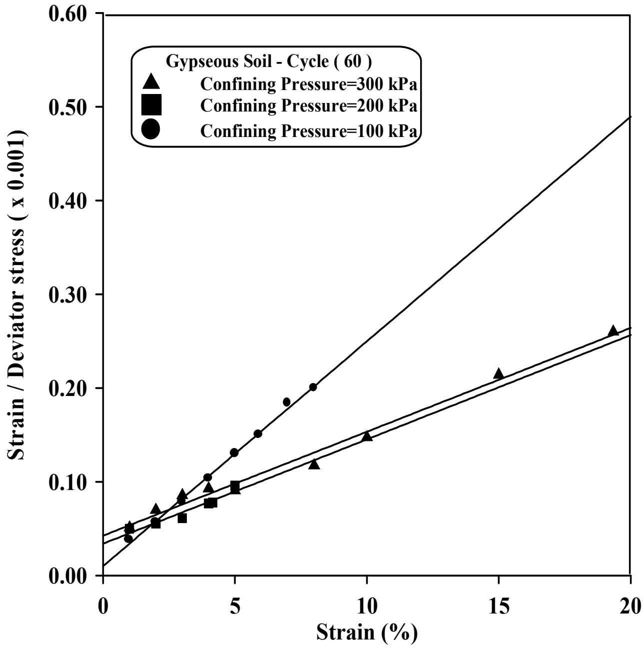



Figure (13): Strain / Stress ratio vs. strain for UU- triaxial testing (Mohammed, 1993).

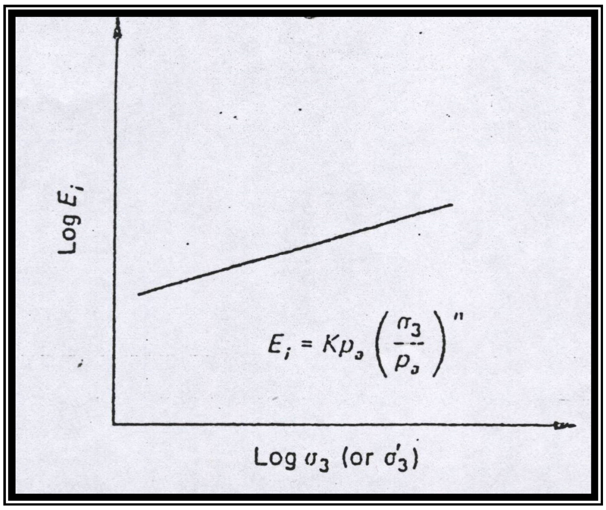



Figure (14): Variation of initial Tangent modulus with confining Pressure (after Duncan, 1981).

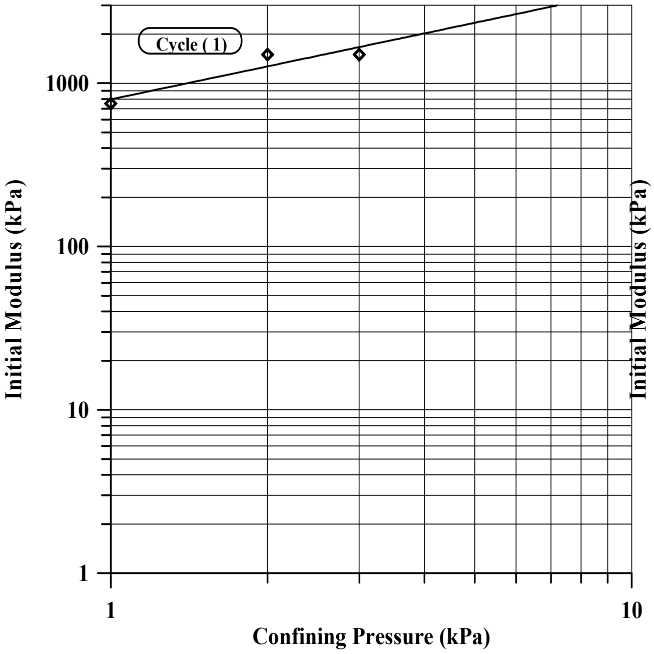



Figure (15): Initial modulus vs. confining pressure for CU-test (Mohammed, 1993).

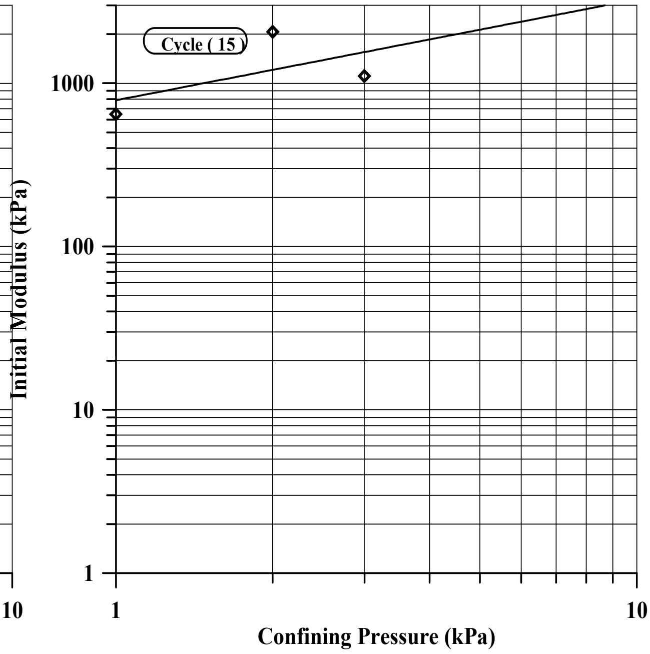



Figure (16): Initial modulus vs. confining pressure for CU-test (Mohammed, 1993).

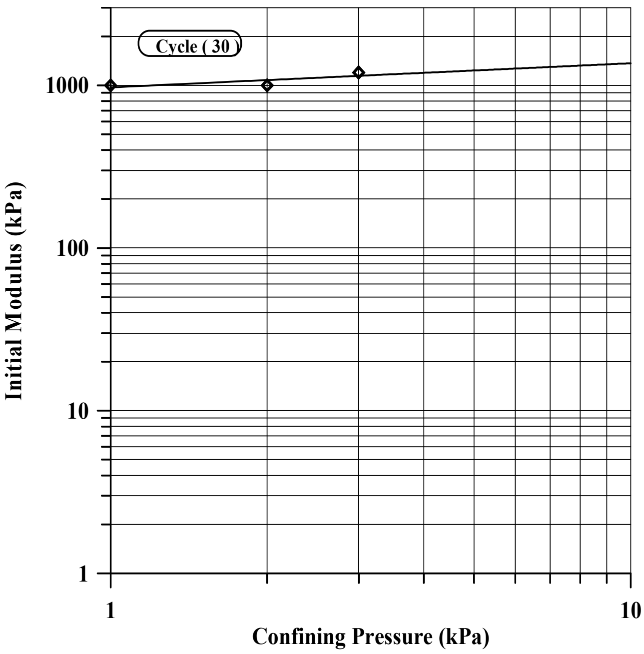



Figure (17): Initial modulus vs. confining pressure for CU-test (Mohammed, 1993).

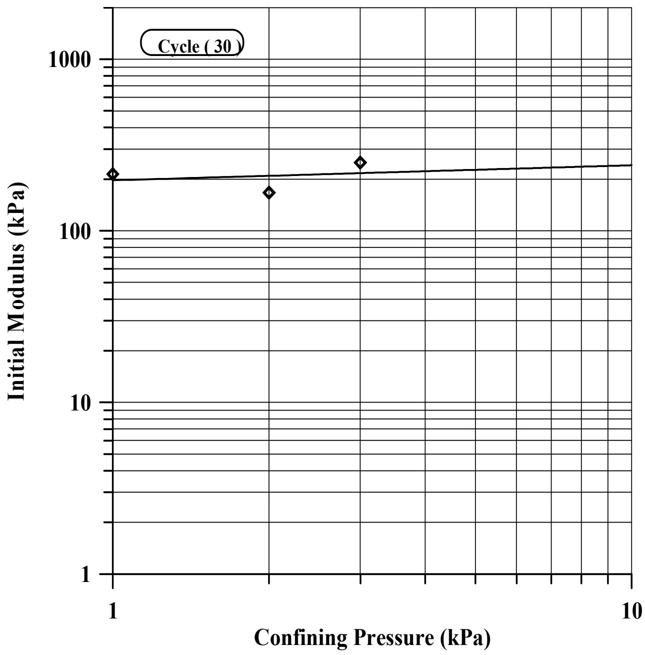



Figure (18): Initial modulus vs. confining pressure for UU-test (Mohammed, 1993).

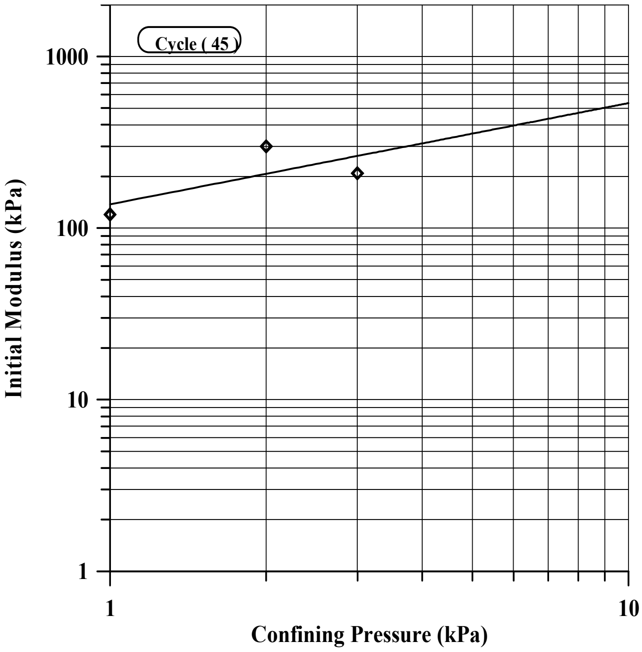



Figure (19): Initial modulus vs. confining pressure for UU-test (Mohammed, 1993).

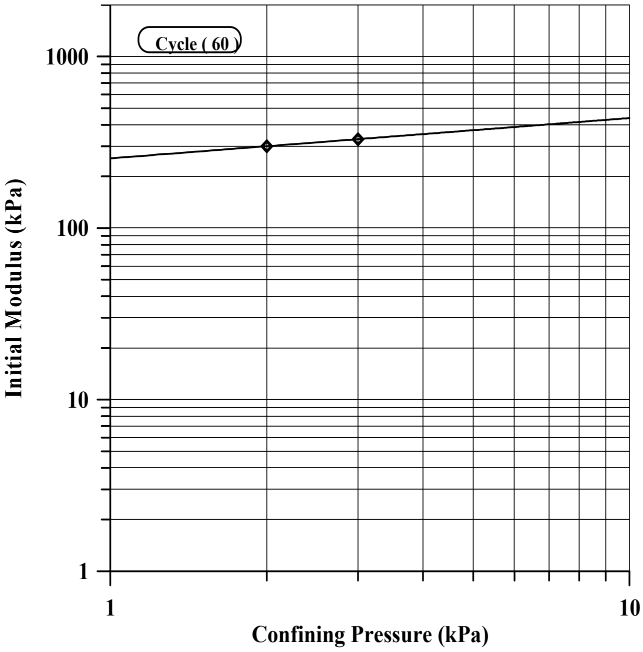



Figure (20): Initial modulus vs. confining pressure for UU-test (Mohammed, 1993).

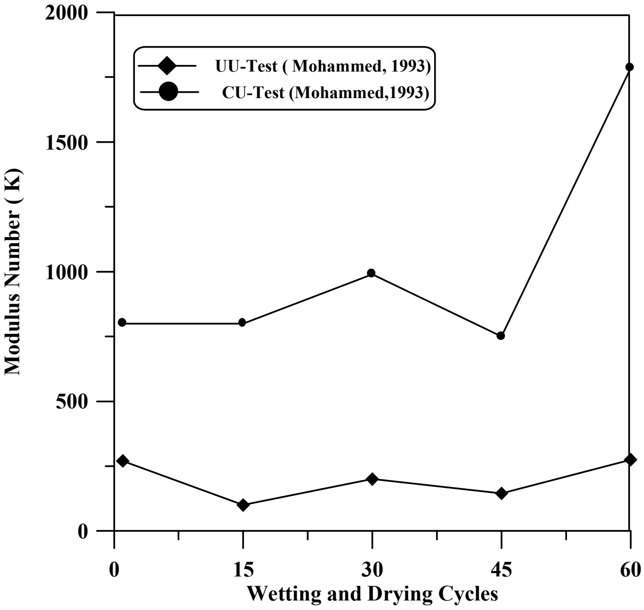



Figure (21): Variation of primary loading modulus number with rewetting cycles.

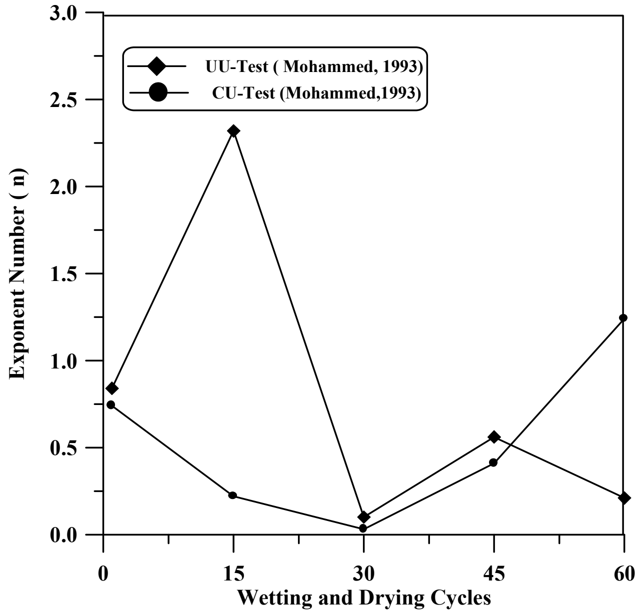



Figure (22): Variation of exponent number with rewetting cycles.

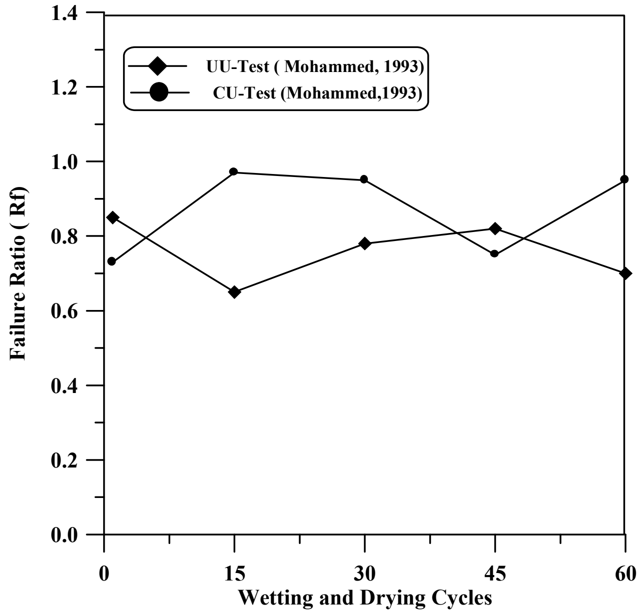



Figure (23): Variation of failure ratio with rewetting cycles.

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**Notations:**

| | |
|---|---|
| A | Hyperbolic constant for stress-strain relationship. |
| B | Hyperbolic constant for stress-strain relationship. |
| C | Cohesion. |
| UU | Unconsolidated undrained (triaxial test). |
| CU | Consolidated undrained (triaxial test) |
| $E_i$ | Initial tangent modulus. |
| $E_t$ | Tangential modulus. |
| $K$ | Modulus number. |
| $n$ | Exponent determining rate of variation of $E_i$ with $\sigma_3$. |
| $R_f$ | Failure ratio. |
| $P_a$ | atmospheric pressure. |
| $\varepsilon$ | Strain. |
| $\gamma$ | Unit weight. |
| $\sigma_1$, $\sigma_3$ | Major and minor principal stresses |
| $\phi$ | Angle of shear resistance (internal friction |
| $\sigma_1$-$\sigma_3$ | Deviator stress |
| $(\sigma_1-\sigma_3)_f$ | Deviator stress at failure |
| $(\sigma_1-\sigma_3)_{ult}$ | Asymptotic value of deviator stress |