ON self –C-injective modules

حول المقاسات من النمط المغلق الشبه الاغماري

الاعداد من قبل
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Abstract :
Let R be a commutative ring with unity . In this paper we study R-modules which satisfy self-C - injective modules.

المستخلص
لتكن R زمرة ابدالية ذات محايد . في هذا البحث ندرس المقاسات التي تحقق المقاس من النمط المغلق الشبه اغماري

Introduction :
Let R be a commutative ring with unity and M be an R- module. An R-module M is said to be self – C-injective module , if for each N closed sub module of M , and for each R-homomorphism f : N→M can be extended to g:M→M {[6],[7]}
We study the relation between self –C- injective and other well – known concepts as semi-injective module, ∏- injective , continuous module and quasi – injective module.
First , we give the following proposition .
Proposition 1: Let M be a self –C- injective R-module such that for any A,B closed in M ,A+B is closed in MR , then $ann_s(A\cap B)=ann_s(A)+ann_s(B)$,where $ann_s(X)=\{s \in s: Sx=o\}$ and $S=End_R(M)$.

Proof:-
Let $f \in ann_s(A\cap B)$,A,B are closed sub module thus f€ s and $f(A\cap B)=O$
Consider the diagram A+B→M
£ F μ
M
Since A,B are closed , so A+B is closed ,
hence $£_f$ can be extended to μ :M→M such that $\mu\, oi=£_f$.
Since M is self – C-injective by assumption so $£_f(a+b)= f(a)$,
for any a € A and b € B ,$\mu\, oi(a+b)=£_f(a+b)$
μ(a+b)=f(a)
μ(a)+μ (b)=f(a)
(f-μ) (a)+μ (b)=o
But f-μ € anns(A), since f(a)- μ (a)=o
for any a € A and μ € anns(B), since μ (b)=μ (o+b)
$=£_f(o+b)$
=f(o)
There fore f=(f-μ)+μ) € $ann_s(a)+ann_s(B)$*

Recall that , M satisfies C1 ,if every sub module is essential in a direct summand.
<u>**Proposition (2)**</u> **Every module that satisfies C1-condition is self – C-injective**

Proof:-
Assume M satisfies C1,then every closed sub module of M is a direct sum and by [(5),prop2.4, p.20] .

Thus for every closed sub- module N of M,there exists a sub- module W of M such that M =N+W.
Let f :N→M be any R-homomorphism.

Define g: M→-M by g(x) ={ F(x) , x € N; O , other wise.

It is easy to check that g is an R-homomorphism and goi=f .
Thus M is self –C- injective R-module *

Example (3) [2,lemma(101), p.61]

Let M be an R- module whose lattice of sub- modules is:

N1

(o) N1+N2→M

N2

where N1 is not iso- morphic to N2.
N1 and N2 are closed sub- module of M ,to showthat N1has no proper essential extenstion in M
N1 N1 +N2 , but N2 N1 +N2 and N1∩N2=(O)
SO N1 ≤ N1+N2
Similarly , N2 ≤ N1 +N2 , but N1 and N2 are not direct sum and of M, hence M does not satisfy C1 , and N1 +N2 M , thus N1 +N2 is not closed .

Remark (4):

M in example (3) is not self –C- injective module.Recall that , a right R- module M is called an Ikeda – Nakayama module (briefly M_R is an IN-module) If Ls(A^B)=Ls(A)+Ls(B)for any sub-modules A and B of MR, where S=End(M) and Ls(X)={s S :Sx=o} for any sub- module X of MR . [9]

Proposition (5)

Every IN –module is self –C- injective module .

Proof:-Since every IN –module is ∏ injective module (C1+C3) ,so by proposition (2),M is self –C-injective*

An R- module M is called continuous if it satisfies C1 and C2
where M is said to be satisfy C1, if every sub- module is essential in a direct summand and M satisfies C2 if every sub- module isomorphic to a direct summand is itself a direct summand of M. And M is called quasi –continuous (∏- injective)if C1 holds and if for any two direct summand M1, M2 of M with M 1 ∩M2 =(O) , M1+ M2 is also a direct summand of M C3 [5 , DEF(2.3) , P.18]

proposition (6):-

Every continuous module is self- C- injective module *

Proof:-

Since M is continuous (C1+C2) , so by prop (2)
M is self –C- injective module *

Proposition(7):-

Every ∏– injective (quasi – continuous) module is self –C- injective module.

Proof: Since M is ∏- injective (C1+C3) ,so by prop (2)
M is self –C- injective module *

Recall that , an R –module M is said to be semi –injective, if for each sub module N of M , every R – homomorphism F:N→N can be extended to an R- homomorphism g:M→M [8]

Proposition (8).

Every semi –injective module is self –C- injective module . Proof. Since by [1 , car(9.3), p.85] every semi – injective is ∏- injective (C1+C3) , thus by prop .(2) M is self –C- injective * Recall that , an R-module M is uniform if M≠o and every non Zero sub module of M is essential in M [3 , p.85] . Thus every uniform module satisfies C1- condition , and by prop .(2), M is self –C-injective . Now, we have the following An R-module M is said to be semi simple module, if every sub- module is a direct summand (**4)**

Proposition (9).

Every semi simple module is self –C- injective module .

Proof.Since every semi simple module is quasi-injective module and every quasi –injective module is ∏-injective(C1+C3), thus by prop (2), M is self –C-injective*

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