

On Fuzzy Pairwise α - Continuous and Fuzzy Pairwise α - open mapping

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Abstract :

We introduce the concept of fuzzy pairwise α -Continuous maps and fuzzy pairwise α - open maps and investigate the relation between these concepts from side and several properties of such maps .

المستخلص :

في هذا البحث ستقدم مفهوم (دوال α المستمرة الضبابية) ودوال α -المفتوحة الضبابية والعلاقة بينهما والخواص المهمة لهذه الدوال .

Introduction

Fuzzy topological spaces were introduced by C.L. Chang [2] who studies a number of basic concepts including fuzzy continuous maps and compactness fuzzy topological spaces are natural generalization of topological spaces .A bitopological space is a collection of two topologies defined on non-empty set X , or if (X, τ_1) and (X, τ_2) are two topologies defined on X , then the bitopology X is denoted by (X, τ_1, τ_2) . In (1965) O.Njastad [6] introduced a concepts of α - open set and α - continuous mapping also in (1983) S .Mashhour, I.A. Hasanein and S.N. EL-Deeb [4] introduced an α -open mapping and they were also studied an α -continuous mapping . In this research we shall study a fuzzy pairwise α -continuous in bitopological spaces and also we shall give anew definitions of a fuzzy concept of an α - open mapping in bitopological spaces .

A mapping $f : (X, \tau) \rightarrow (Y, \rho)$ is called α - continuous (briefly. α -cont.) iff the inverse image of each open set of Y is α -open in X

where X and Y are topological spaces [4] . A mapping $f : (X, \tau) \rightarrow (Y, \rho)$ is called α -open iff $f(A)$ is α -open in Y for each open set in X [4].

1- Some basic definitions

In this section ,we will give the basic definitions .

Definition (1.1) [3]

A fuzzy topology τ on X is a collection of subsets of X such that

1. $0, 1 \in \tau$
2. if $\lambda, \mu \in \tau$ then $\lambda \wedge \mu \in \tau$
3. if $\lambda_i \in \tau$ for all $i \in J$,then $\bigvee \lambda_i \in \tau$.

And (X, τ) is called fuzzy topological space , members of τ are called fuzzy open sets in (X, τ) and its complement is called fuzzy closed set .

Definition (1.2)

A fuzzy bitopological space (briefly , fbts) is a non- empty set with two fuzzy topological spaces τ_1 and τ_2 define on X , in other word is the triple (X, τ_1, τ_2) where (X, τ_1) and (X, τ_2) are two fuzzy topological spaces .

Definition (1.3) [5]

Let $f : X \rightarrow Y$ be a mapping from X into Y , if λ is a fuzzy set in X and μ is a fuzzy set in Y , then we define $f(\lambda)$ and $f^{-1}(\mu)$ as follows

$$f(\lambda)(y) = \begin{cases} \text{Sup}\{\lambda(x)\} & \text{iff } f^{-1}(y) \text{ is non-empty} \\ x \in f^{-1}(y) \\ 0 & \text{Other wise} \end{cases}$$

And $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

Definition (1.4) [4]

For a fuzzy set λ of X , the τ_i - closure and τ_i -interior are defined respectively , as follows

$$\tau_i\text{-cl}(\lambda) = \inf \{ v : v \geq \lambda , 1-v \in \tau_i \}$$

$$\tau_i\text{-int}(\lambda) = \sup \{ v : v \leq \lambda , v \in \tau_i \}$$

2- Fuzzy pairwise α - Continuity

The goal of this section is to study the Fuzzy pairwise α - Continuity.

Definition (2.1)

A subset λ of a fuzzy topological space (X, τ) is called fuzzy α -set if $\lambda \leq \tau_i\text{-int}(\text{cl}(\text{int}(\lambda)))$.

Definition (2.2)

A mapping $f : (X, \tau) \rightarrow (Y, \rho)$ is said to be fuzzy α - continuous (briefly f α - cont.) iff the inverse image of each fuzzy open set of Y is a fuzzy α - set .

Definition (2.3)

A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \rho_1, \rho_2)$ is said to be fuzzy pairwise α - continuous (briefly f P α - cont.) iff the mapping $g : (X, \tau_1) \rightarrow (Y, \rho_1)$ and $h : (X, \tau_2) \rightarrow (Y, \rho_2)$ are fuzzy α - continuous .

Theorem (2.4)

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \rho_1, \rho_2)$ be a mapping from fuzzy bitopological space X into fuzzy bitopological space Y , then the following statements are equivalent :

- i) f is fuzzy pairwise α - continuous .
- ii) For each $x \in X$ and each fuzzy open set v in Y containing $f(x)$, there exists $x \in X$ such that , $w \in X$, $f(w) < v$.
- iii) the inverse image of each fuzzy closed set in Y is fuzzy α - closed .

Before we prove the above theorem ,we have to know that .

Remark (2.5) [4]

Every open set is an α - set .

Remark (2.6) [3]

Every fuzzy open set is a fuzzy α - set .

Theorem (2.7)

Any fuzzy pairwise continuous mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \rho_1, \rho_2)$ is a fuzzy pairwise α - continuous .

Proof :

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \rho_1, \rho_2)$ is fuzzy pairwise continuous mapping , hence $g : (X, \tau_1) \rightarrow (Y, \rho_1)$ and $h : (X, \tau_2) \rightarrow (Y, \rho_2)$ are fuzzy continuous . Now let λ is an open in (Y, ρ_1) and μ is an open in (Y, ρ_2) ,thus $g^{-1}(\lambda)$ and $h^{-1}(\mu)$ are two fuzzy open in (X, τ_1) and (X, τ_2) respectively , and by Remark (2.6) we have $g^{-1}(\lambda)$ and $h^{-1}(\mu)$ are two fuzzy α - set in (X, τ_1) and (X, τ_2) respectively which means that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \rho_1, \rho_2)$ is fuzzy pairwise α - continuous .

Now , we prove theorem (2.4)

i \Leftrightarrow ii , it is easy by theorem (2.7) and definition of fuzzy pairwise continuity .

ii \Leftrightarrow iii , it is easy if we take the complement of theorem (2.7) .

References

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