Anomalous Diffusion in Turbulent plasma

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Abstract:
The semi-empirical equation of Bohm (D_B) doesn't take into the account the fluctuation properties of plasma parameters in the plasma. So, the Bohm formula did not explain the diffusion behavior caused by instability. The 1/16 factor in Bohm formula represent the relation between the diffusion and the instability that causing it. Therefore, the Bohm factor studied as instability factor (C), this factor derived into two turbulence cases (strong and weak turbulence).The results of this factor was tested with experimental results in Q-machine and shown good results. The diffusion coefficients are drived according to this instability factor for strong and weak turbulence cases.
Key Words, Plasma diffusion, Bohm diffusion, Plasma Instability.

الخلاصة :
ان العلاقة الشبه وضعية التي افترضت من قبل Bohm لم تأخذ بنظر الاعتبار خواص الاضطراب لمعلمات البلازما داخل البلازما لذا يمكن القول ان انتشار Bohm لايمكن ان تصف الانتشار الشاذ بسبب اللاستقرارية. ان العامل 1/16 في علاقة Bohm يمثل العلاقة بين الانتشار الشاذ واللاستقرارية المسببة له. لذا فأن العامل 1/16 قد استبدل بالعامل C والذي اطلقنا عليه بعامل اللاستقرارية. في هذا البحث فأن عامل اللاستقرارية C قد اشتق لحالتي الاضطراب القوي والضعيف. وبالنهاية فأن نتائج عامل اللاستقرارية قد قورنت مع النتائج العملية في منظومة Q-machine وقد لوحظ انها تتفق بشكل جيد مع النتائج العملية. ومن ثم تم تحديد علاقة لمعامل الانتشار الشاذ لحالتي الاضطراب القوي والضعيف.
تبرير العمل
هدفنا في هذا البحث هو وضع علاقة يمكن من خلالها وصف الانتشار الشاذ اعتمادا على نوع الاضطراب الذي يحصل بالبلازما. ومن خلال شرط معين يمكن معرفة هل ان البلازما في وضع الاضطراب القوي او الضعيف. وقد يصادف ان يحصل في عمود البلازما كلا الاضطرابين وبالتالي فأن كلتا المعادلتين يمكن ان تستعمل لوصف الانتشار الشاذ.
ان هاتان المعادلتان قد قورنت لمعظم منظومات البلازما والتي تعتبر Q-machine من ضمنها، فقد لوحظ ان هاتان المعادلتان للانتشار تتفق وبشكل جيد جدا مع النتائج العملية المستحصلة من التجربة.

1-Introduction:
In all almost pervious experiments, the diffusion of plasma across magnetic field scaled as B^{-1} rather than B^{-2}, and the decay of plasma with time was found to be exponential rather than reciprocal [1].

Bohm first notes this anomalous diffusion. He found that, the plasma created by electric arc leaked across magnetic field with unexpected fast and large amplitude oscillations of electric field, which observed inside arc. Bohm surmised that, these fluctuated electric fields are caused anomalous diffusion [2].
The semi-empirical formula of Bohm for this diffusion is [1, 3]:

$$D_B = \frac{1}{16}\frac{KT}{eB} \tag{1}$$

where K, T, e, and B is Boltzman constant, plasma temperature, electronic charge, and magnetic field, respectively. The factor 1/16 has no theoretical justification but is an empirical number agreeing with most experiments to within a factor of two or three [1]. This equation is not formally derived.

Historically, the first experiment on anomalous diffusion in positive long tube is performed by Lehnorl et al in Sweden. They found that the diffusion of plasma in helium positive column increases with the increasing of magnetic field above a critical value (B_c). Kadomtsev and Nedosppasov showed that, the instability behavior was established in plasma column at high magnetic field. This instability in the form helical distortion of plasma and was seen directly by Allen et al experiment [1].

In 1960, Spitzer [4] has pointed out, the low-frequency ion wave give arise to anomalous diffusion in slellartor. Spitzer gave relation of Bohm .He suggested an explanation from Bohm's formula that is:

$$D_{\perp} = 2k_1^2 k_2 k_3 \frac{KT}{eB} \tag{2}$$

where k_n is unknown constant at proportionality's. The symbol ⊥ significant across magnetic field. This equation shows the 1/16 factor in formula in equal to $2k_1^2 k_2 k_3$.

According to equation, Sanduk [5] given another modification for Bohm diffusion formula depending on Spitzer parameters which in the form:

$$D_{MB} = C \frac{KT}{eB} \tag{3}$$

where C is the instability factor which equal to[5]:

$$C = \left(\frac{\gamma k_y}{2\omega^2}\right) \left|\frac{k_y \Phi_1}{B}\right|^2 \frac{\Phi_1}{T} \tag{4}$$

where K_y:is the wave number parallel to direction of electric field (perpendicular to direction of magnetic field and density gradient).

ω: is angular frequency of instability.

Φ_1: is the fluctuated potential due to instability.

and γ: is the growth rate of instability.

This equation was tested for many confinement systems [5, 6, 7].

2-Turbulence Diffusion

Plasma turbulence can also be characterized by the ratio of turbulence energy (w) to the plasma particle thermal energy [8]:

$$\eta = \frac{w}{nT_e} \tag{5}$$

where n and T_e are plasma density and electron temperature respectively. The parameter η is frequently used to separate weak and strong turbulence (η<<1 for weak and η>1for strong turbulence). This ratio (η) can be either very small,η<<1, or large, η>1.

To obtain on the relation of diffusion coefficient in turbulence cases, let using the slab ∇n in x-direction (corresponding to r-direction), and E in y-direction (corresponding to θ-direction in cylindrical geometries)).

At the beginning, using the continuity equation and then solving it, the particle flux associated with turbulence can be estimated as:

$$\Gamma = -\frac{\gamma}{2}\left|\frac{k_y \Phi_1}{\omega B}\right|\frac{dn}{dx} \qquad (6)$$

This equation can be studies in two cases:

i-strong turbulence; $\omega \cong \gamma$ **:**

Kadmotsev [9] gave a relation of $e\Phi_1 / KT$ in this case:

$$\frac{e\Phi_1}{KT} = \frac{\omega}{\omega_{De}} \frac{1}{k_\perp \Lambda} \qquad (7)$$

where ω_{De} and Λ are diamagnetic drift frequency of electron ($\omega_{De} = k_y v_{De}$) and density scale length ($\Lambda = -n / \nabla n$), respectively.

By determining the plasma potential from above equation and apply it into equation (6), the particle flux becomes:

$$\Gamma_{\gamma s} = -\frac{\gamma}{k_\perp^2}\frac{dn}{dx} \qquad (8)$$

The diffusion coefficient ($D_{\gamma s}$) can be taken by using the Fick's low on equation (8), one obtain:

$$D_{\gamma s} = \frac{\gamma}{k_\perp^2} \qquad (9)$$

Consequently, according to equation (6), the instability factor ($C_{\gamma s}$) will become equal:

$$C_{\gamma s} = \frac{\gamma}{k_\perp^2}\frac{B[Tesla]}{T[eV]} \qquad (10)$$

where T: temperature

ii-weak turbulence; $\gamma \langle\langle \omega$ **:**

For this problem, Kadmotsev [9] gave another relation of $e\Phi_1 / KT$, which is equal to:

$$\left(\frac{e\Phi_1}{KT}\right)^2 \cong \frac{\gamma}{\omega}\left[\frac{\omega}{\omega_{De}}\frac{1}{k_\perp \Lambda}\right]^2 \qquad (11)$$

Then by using the same technique which used in strong turbulence cases, the particle flux ($\Gamma_{\gamma w}$) becomes:

$$\Gamma_{\gamma w} = -\frac{\gamma^2}{\omega k_\perp^2}\frac{dn}{dx} \qquad (12)$$

where:

$$D_{\gamma w} = \frac{\gamma^2}{\omega k_\perp^2} \tag{13}$$

and

$$C_{\gamma w} = \frac{\gamma^2}{\omega k_\perp^2}\frac{B[Tesla]}{T[eV]} \tag{14}$$

3-Modified Bohm Diffusion

In this attempt, consider Bohm's diffusion in a gradient magnetic field ($\vec{\nabla}$B) for two reasons, (i) to note the variation of local value of D_B,(ii) since instability condition is $\vec{\nabla}B.\vec{\nabla}p<0$, where p is the plasma pressure, a turbulent diffusion does arise, and D_B can be compared with D_γ. In addition, the magnetic gradient normal to B, since the Bohm diffusion occurs in the normal direction to B [5].

4-Diffusion Equation in $\vec{\nabla}$B

Any realistic plasma, the particle fluxes flow from dense region toward reign of low density [10, 11]. Then the Bohm diffusion equation is given by:

$$\vec{\nabla}\cdot\vec{\Gamma} = -\left(\vec{\nabla}n.\vec{\nabla}D_B + D_B\nabla^2 n\right) \tag{15}$$

for equation (1) one obtains:

$$\vec{\nabla}\cdot\vec{\Gamma} = -\frac{1}{16}\left[\vec{\nabla}n\cdot\left(\frac{\vec{\nabla}T}{B}-\frac{T\vec{\nabla}B}{B^2}\right)+\frac{T}{B}\nabla^2 n\right] \tag{16}$$

where $\vec{\nabla}B = \vec{\nabla}|B|$. This equation is called the general form of Bohm diffusion. Where the variation of D_B with distance is:

$$\vec{\nabla}D_B = \frac{1}{16}\left[\frac{\vec{\nabla}T}{B}-\frac{T\vec{\nabla}B}{B^2}\right] \tag{17}$$

The theoretical explanation of Bohm diffusion that have been given by Spitzer[4] and Bernstein [12], assumed a fluctuated electric field (E_1) that may cause to (EXB)particles drift. The fluctuated rang (λ_x) of E_1 is large compared with the larmor radius (r_L) i.e. $\lambda_x > r_L$ and its internal (τ) is long compared to $1/\omega_c$ (where ω_c is cyclotron frequency), $\tau > \omega_c$ [4].The magnetic field is assumed to be in y-direction. Then the general form of D_B will represent to equation (3). According to Spitzer, C may be expressed as:

$$C = 2\frac{\Phi_1}{T}\frac{r_L}{\lambda_x}\frac{\omega_c}{f} = 2\frac{\Phi_1}{T}\frac{V_{th}}{\nu_1} \tag{18}$$

where f= 1/τ, V_{th}= thermal velocity and ν_1= Frequency. It is clear from this equation that, C is B-dependent quantity. According to equation (3), the equation (15) will become:

$$\vec{\nabla}\cdot\vec{\Gamma} = -\left[\vec{\nabla}n\cdot\left(C\frac{\vec{\nabla}T}{B}+\frac{T\vec{\nabla}C}{B}-\frac{CT\vec{\nabla}B}{B^2}\right)+\frac{nT\nabla^2 n}{B}\right] \tag{19}$$

where the variation of general form of D_B is [5]:

$$\vec{\nabla} D_B = C\frac{\vec{\nabla} T}{B} + \frac{T\vec{\nabla} C}{B} - \frac{CT\vec{\nabla} B}{B^2} \tag{20}$$

For isothermal case ($\vec{\nabla}$ T = 0), the equation (20) becomes:

$$\vec{\nabla} D_B = \frac{T\vec{\nabla} C}{B} - \frac{TC\vec{\nabla} B}{B^2} \tag{21}$$

The first term explains the slope due to variations of C. This term modifies the slope of the second one [5]. Because of C is related to the instability and the diffusion flux $\Gamma = \Gamma_o + \Gamma_1$, where $\Gamma_1 = \langle n_1 v_{(EXB)1} \rangle$ and the angular bracket indicates arranging with respect to time, one obtains [5]:

$$D_B = C\frac{T}{B} \equiv \frac{\langle n_1 v_{(EXB)1} \rangle}{\vec{\nabla} n} \tag{22}$$

5-Strong Turbulence

For the slab geometry, we assume B in y-direction, $\vec{\nabla}$ n in x-direction, and E to be in Z-direction. One that finds with aid of equation (9) that:

$$\Gamma_{st} = -\left(\frac{\gamma}{\omega^2}\right)\left|\frac{k_z \Phi_1}{B}\right| \nabla n_0 \tag{23}$$

Thus,

$$CT = \left(\frac{\gamma}{\omega^2 B}\right) |k_z \Phi_1|^2 \tag{24}$$

In term of Φ_1 / T , we get:

$$C_{st} = \left(\frac{\gamma k_z}{\omega^2}\right)\left|\frac{k_z \Phi_1}{B}\right| \frac{\Phi_1}{T} \tag{25}$$

This equation shows that the instability factor depends on:
(i) the of growth rate and the frequency
(ii) the $v_{(EXB)1}$ drift velocity
(iii) the fluctuated level.

Finally, according to equation (3), D_{st} will become:

$$D_{st} = \left(\frac{\gamma k_z}{\omega^2}\right)\left|\frac{k_z \Phi_1}{B}\right| \frac{\Phi_1}{T} \frac{T[eV]}{B[Tesla]} \tag{26}$$

6- Weak Turbulence

With aid of equation (12), the fluctuated particle flux Γ_1 becomes:

$$\Gamma_{wt} = -\left(\frac{\gamma^2}{\omega^3}\right)\left|\frac{k_z\Phi_1}{B}\right|\nabla n_0 \qquad (27)$$

where:

$$C_T = \left(\frac{\gamma^2}{\omega^3 B}\right)\left|k_z\Phi_1\right|^2 \qquad (28)$$

According to above equation and in the term of fluctuation level $(e\Phi_1/KT)$, the instability factor will become:

$$C_{wt} = \left(\frac{\gamma^2}{\omega^2}\right)\left[\frac{k_z\Phi_1}{B}\right]\left(\frac{\Phi_1}{T}\right) \qquad (29)$$

This equation shows that, the C depends on:

(i) the square of the ratio of the growth rate and the frequency.
(ii) the ratio between the $v_{(EXB)}$ and v_{ph}.
(iii) the fluctuated level.

As results of equation (3), the D_{wt} will equal to:

$$D_{wt} = \left(\frac{\gamma^2}{\omega^2}\right)\left(\frac{k_z}{\omega}\right)\left|\frac{k_z\Phi_1}{B}\right|\left(\frac{\Phi_1}{T}\right) \qquad (30)$$

7-Experimental Results and Discussion

The profile of the degree of turbulence, defined as ratio of fluctuation energy and thermal energy [1٣]:

$$\left[\frac{n_1}{n_o}\right]^2 = \frac{n_1}{n_o}\frac{e\Phi}{KT} = \frac{W_{fl}}{W_{th}} \qquad (31)$$

In this section we will apply the results of modified Bohm diffusion for two turbulence cases on Chu et al [1٤, 1٥] experimental results in Q-machine. In figure (1), the results of equation (29) are plotted with plasma radius. This figure shows that, the turbulence is weak in the region near the plasma center (where the plasma density is high in this region) and will becomes strong toward plasma edge.

In consequence to experimental results and equations (25) and (28), the instability factor for two turbulence cases and Bohm diffusion equation was plotted with experimental factor (by apply equation (3)) in figure (2).This figure illustrated, the behavior of C-factor according to equation (25) and (28) has a better agreement with C_{exp} behavior rather than Bohm's factor (1/16) which shows behavior far from C_{exp}.The diffusion coefficients D_{wt}, D_{st}, D_{Bohm}, and D_{exp} are obvious as function of distance in figure (3). From this figure, the behavior of diffusion coefficient have same behavior of instability factor (this accurse in the experiment which have temperature and magnetic field are constant).

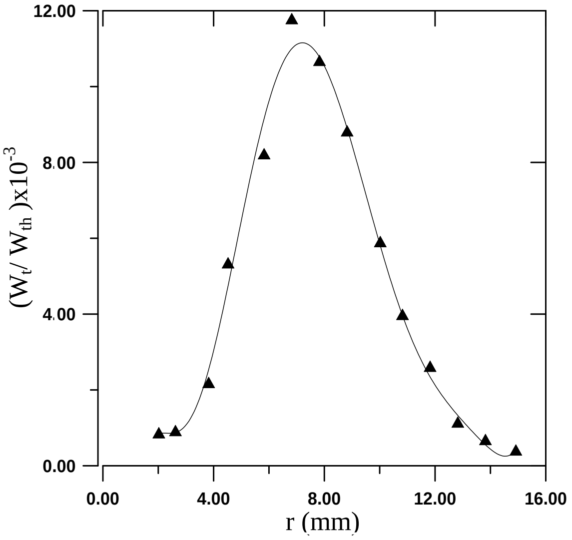

Fig.(1): Variation of W_t/W_{th} as a function of plasma radius.

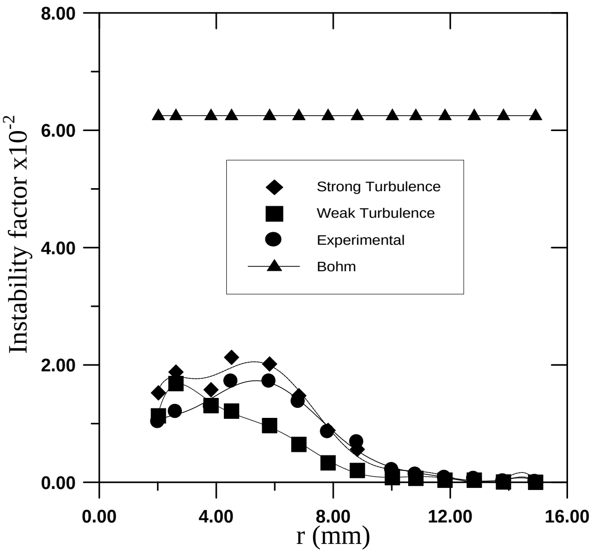

Fig.(2): Computed quantities of C-factor verus plasma radius .

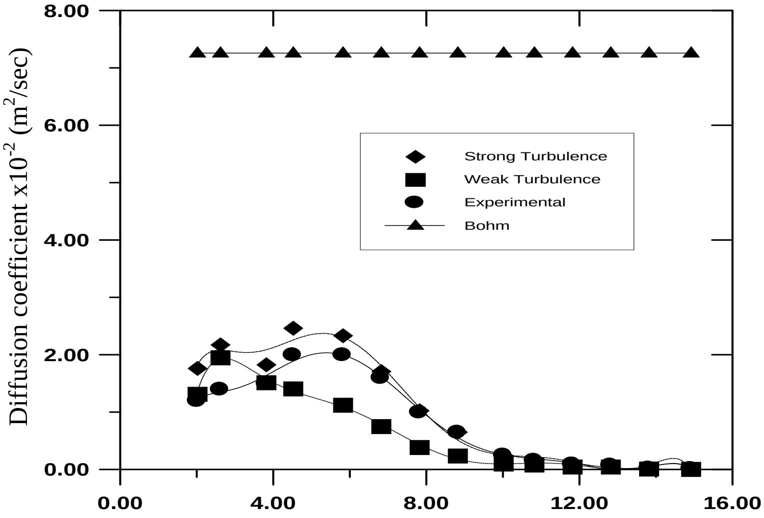

Fig.(3):Variation of diffusion coefficients as a function of plasma radius.

Finally, $D_{\gamma s}$ and $D_{\gamma w}$are plotted with D_{st}, D_{wt}, and D_{exp} in figures (4) and (5). figure (4) shows that, $D_{\gamma s}$ is fitting with D_{exp} because of we assumed $\gamma \cong \omega$(plasma under study is in strong turbulence) and the behavior of D_{st} will clause from D_{exp} in strong turbulence region. So, as a conclusion from this fitting, D_{γ} follows the instability growth rate. While in figure (5), D_{wt} has a good agreement behavior with D_{exp} comparable to $D_{\gamma w}$ near from the center (where the turbulent is weak in this region) but the behavior of D_{wt} and $D_{\gamma w}$ will be agreement and will far from D_{exp} toward strong turbulence region of plasma column.

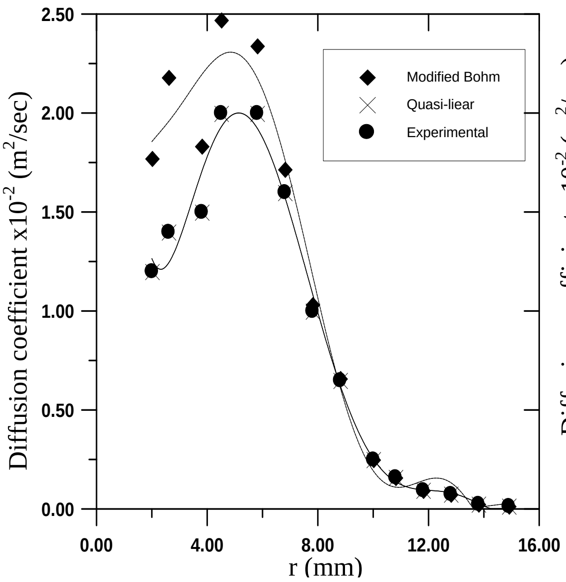

Fig.(4): Calculated diffusion coefficients as a function of plasma radius for Strong turbulent case.

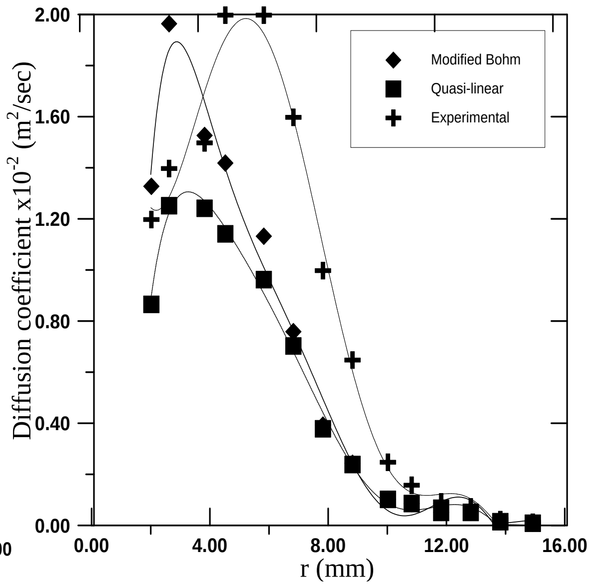

Fig.(5): Calculated diffusion coefficients as a function of plasma radius in weak turbulent case.

8-Conclusion

In consequence of the above discussion, the results of Bohm formula shows a contradiction in the behavior with the prediction of modified Bohm and experimental results. This contraction caused by, the Bohm diffusion did not take into the account the fluctuation properties of plasma parameters in the plasma column. The modified Bohm diffusion theory for two turbulence cases taken these fluctuations properties of plasma parameters which causes to instability.

The comparison between the behavior of instability factor and diffusion coefficient will leads to the fact that, the instability factor behavior is responsible for diffusion coefficient behavior (this across in the experiment which have temperature and magnetic field are constant) (see figures (2) and (3)).

The results, which are shown in figures (4) and (5) were shows, the results modified Bohm diffusion theory which represented by equations (26) and (30) is suitable to study of change of plasma parameters in the plasma column. As well as, this theory is agree with the behavior of quasi-liner theory.

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