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Notes On The Weak Denseness In Topological Spaces

حول المجموعات الضعيفة الكثيفة في الفضاءات التبلوجية

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Abstract :

In this paper we introduced the notion "weak dense, weak dense - in-itself, and nowhere weak dense sets" and proved some of their related theorems by using the concept of weak open set.

الخلاصة :

في هذا البحث قدمنا ملاحظات حول الجموعات الضعيفة الكثيفة والضعيفة الكثيفة بنفسها والمجموعات الضعيفة الغير كثيفة واثبتنا بعض النظريات المتعلقة بها باستخدام مفهوم المجموعات الضعيفة المفتوحة.

1-INTRODUCTION AND PRELIMINARIES

Before we present the weak dense, weak dense - in-itself, and no where weak dense sets, we give some definitions and remarks

Definition(1-1): A subset A of a space X is Called a **weak open**, if there is an open set U such that cl(A)=cl(u).[5]

Remark(1-2): All semi open sets are weakly open set. [4]

Example (1-3):let X={a,b,c,d} and $\tau =\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$
Then the set $\{X, \phi,\{a\},\{b,c\}\}$ is a weak open set .

Definition(1-4): Let (X, τ) be a topological space and $A \subseteq X$.
Then A is called a **weak neighborhood** of a point x in X, if There exists a weak open set U of x such that $x \in U \subset A$.

Definition(1-5):The union of all weak open sets contained in a set A is called a **weak interior** of A and denoted by **wkint(A).**

Definition(1-6): let $A \subset X$, then
i)$x\in X$ is called **weak limit point** (**wk limit point**) of A, if each open neighborhood N_x of X, $(cl(N_x)-\{x\}) \cap A \neq \emptyset$.The set of all weak limit point of A, denoted by **A″** and is called weak derive set of A, we denoted by **wd(A)**.
ii) A is said to be **weakly closed set** if $wd(A) \subset A$.[2]

Remark(1-7): weak closedness implies closedness .[1]

Definition(1-8):The **weak closure** denoted by **A˜** of A is the set $A\cup$ wd (A), the same as , if we say $A^{\sim} = \cap\{F{:}F$ is weak closed $\supset A\}$

2.weak dense, weak dense-in-itself and no where weak dense

Definition (2-1):Let A be a subsets of the topological spaces (X, τ).Then A is said to be **weak dense** in X if $A^{\sim}=X$.[2]

Theorem (2-2) : Every weak dense subset of a topological space(X, τ) is dense subset of X .

Proof:let A be dense subset of X, then $A^{\sim}=X$ since $A^{\sim} = \cap \{F{:}F$ is weak close set $\supset A\}$and since weak closed implies closed set and that's implies $A^{\sim} = \cap\{F{:}F$ is close set $\supset A\}=cl(A)$ hence A is dense set .

Theorem (2-3): Let A be a subset of the topological spaces (X, τ), A is weak dense in X if and only if $A^{\sim}$ is weak dense in X.

Theorm(2-4):Let A,B,C be a subsets of the topological spaces (X, τ), if A is weak dense in B and B is a weak dense in C then A is weak dense in C.

Theorem (2-5):A is weak dense in X if and only if every weak open set in X contains a point of A.

Definition (2-6) : A subset A of a topological space (X,τ) is called **weak Dense- in-itself**, if $A \subseteq wd(A)$ that is every points of A is wklimit point of A

Example (2-7): let X={a,b,c,d} and τ ={ϕ,X,{a},{b,c},{a,b,c}}
Let A={b,c} then A″={b,c} is the set of weak drived set
Hence A is weak dense –in-itself.

Theorem (2-8): Every weak dense -in-itself set is dense - in – itself.

Proof: let A be a weak dense-in-itself set, that is (every point in A is wk limit point of A) ,since every wk limit point is a limit point , then each point of A is a limit point , then A is dense-in-itself. •

Theorem (2-9): Let A be a subset of the topological spaces (X, τ),if A is a weak dense-in-itself and $A\subset B\subset A^{\sim}$ then B is a weak dense –in-itself..

Theorem (2-10): $A^{\sim}$ is weak dense-in-itself of X, if A is a weak dense –in-itself subset of X.

Theorem (2-11): The union of any family of weak dense-in-itself subsets of X is weak α-dense -in-itself.

Proof: let $\{A_i\}$, $i\in I$, be a family of weak dense-in-itself. So $A_i \subseteq wd(A_i)$,$\forall\ i\in I$. Let $p\in\cup A_i$, then $p\in A_i$ .for some $i\in I$ Hence for each weak open set U with $p\in U$, $A_i\cap(U-\{p\}) \neq \emptyset$.Thus $(\cup A_i) \cap(U-\{p\}) \neq\emptyset$, hence $p \in (wd (UA_i))$.
Therefore $\cup A_i \subseteq (wd (\cup A_i))$; hence $\cup A_i$ is weak denes-in-itsef. •

Definition(2-12): A subset A of a topological space (X,τ) is **nowhere weak dense set**, if wkint ($A^{\sim}$) = $\varnothing$,that is the weak interior of the weak closure of A is empty.

Theorem(2-13) :Let A be a subset of a topological space (X,τ). Then the following statements are equivalent

i) A is nowhere weak dense in X.

ii) $A^{\sim}$ contains no weak nhd.

Proof : (i) $\Leftrightarrow$ (ii) we have A is no where weak dense

$\Leftrightarrow$ No point of X is a wkint point of $A^{\sim}$

$\Leftrightarrow$ $A^{\sim}$ has not a weak nhd of any of its Points

$\Leftrightarrow$ $A^{\sim}$contains no weak nhd •

Theorem(2-14):if A is nowhere weak dense subset of X and B $\subseteq$ X then B is nowhere weak dense subset of X.

Theorem(2-15):if A is nowhere weak dense subset of X then x-A is weak dense subset of X.

Theorem(2-16): Let A be a subset of topological spaces (X, τ) if A is no where weak dense, then $A^{\sim}$is not the entire space X.

Proof: Since X is weak closed, then $X=X^{\sim}$. Again since X is weak open, we have wkint($X^{\sim}$)=wkint(X)=X. Since A is nowhere weak dense in X,
wkint ($A^{\sim}$) = $\varnothing$. Thus wkint ($X^{\sim}$)=X ,and wkint ($A^{\sim}$)= $\varnothing$. It follows $A \neq X$ •

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