# Differential Equation For Higher Frequency Periodic Direction Function

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## Abstract :

In this research, we will study an ordinary differential equation (n degree) for higher Frecuency periodic direction function $(f_0, f_1)$.

**ملخص البحث:**

في بحثنا هذا تم دراسة دوال ( $f_0$ and $f1$ ) الدورية الاتجاهية في معادلة تفاضلية اعتيادية من الدرجة (n) (وفي هذا الحالة تكون عالية التردد ) حيث وضعنا في بحثنا أدناه شروط معية وخاصة , حيث تميزنا بدراسة معادلة تفاضلية من الدرجة (n) ولأي دالتين دوريتين متجهتين وبهذا المعنى فان هذه الدراسة يمكن اعتبارها دراسة موسعة ومهمة.

## Introduction :

In this research , $f_0$ *and* $f_1$ Vector Periodic in normal high order differential equations of order (8) with high frequency that a specific conditions are put in this research is differ from (Bateman H.1985 , Boyce W.E. , Di Prima R.C. 1977) . whom study a normal differential equation of order one for trigonometric function . While ( Greenberg M.D.1994 and Struble R.A) deal with study of differential equation of order two for vector periodic function during which they put a specific conditions . While in this research we work studying a differential of order (n) for any two vector periodic function so by this work we consider this study is an important and expand study.

## 1- Basic Concept

Let m and p-natural number , and also n-even , $n = 2p$ , and $G_i$ , $i = 0,1,2,3,....., p$ organic domain in space $R^m$ .We have to study problem $2\pi\omega^{-1}$periodic solution for differential equations to be formal n .

$$\frac{d^n u}{dt^n} = f_0(u, \frac{du}{dt}, ....., \frac{d^p u}{dt^p}, \omega t) + \omega^p f_1(u, \omega t) \quad ..........................(1)$$

Where $\omega$-big parameter .We will be presupposed the following :

1.Vector functions $f_0(e,\tau)$ defined in the set $\Omega_0 = \{e, \tau, e \in G_0 \times G_1 \times .....\times G_p, \tau \in R\}$ u vector functions $f_1(u,\tau)$ defined in the set $\Omega_1 = \{u, \tau, u \in G_0, \tau \in R\}$, have meaning in $R^m$.

2. Vector functions $f_0(e,\tau)$ and $f_1(u,\tau)$ have continuously differentiable for any order with respect to e and u respectively .

**2-** Asymptotic expansion solution equation (1) will be sought in the form

$$u_\omega(t)=\sum_{j=0}^{\infty}\omega^{-j}u_j+\sum_{j=p}^{\infty}\omega^{-j}v_j(\omega t)\ldots\ldots\ldots(2)$$

where $v_j(\tau)$ - $2\pi$ periodic vector functions have meaning in $R^m$. $u_j$-vector in $R^m$ and

$$<v_j>=\frac{1}{2\pi}\int_0^{2\pi}v(\tau)d\tau=0$$

We substitute equation (1) in place of $u,\frac{du}{dt},\ldots,\frac{d^pu}{dt^p}$ expression (2) and we develop nonlinear $f_0$ and $f_1$ in Taylor series , as a result we have the following equation :

$$\sum_{j=p}^{\infty}\omega^{-j+n}\frac{\partial^n v_j}{\partial\tau^n}=f_0(u_0,0,\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)+\frac{\partial f_0(u_0,0,\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)}{\partial e_0}\left[\sum_{j=1}^{\infty}\omega^{-j}u_j+\sum_{j=p}^{\infty}\omega^{-j}v_j\right]+$$

$$\frac{\partial f_0(u_0,0,\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)}{\partial e_1}\sum_{j=p}^{\infty}\omega^{-j+1}\frac{\partial^n v_j}{\partial\tau}+\ldots+f_0(u_0,0,\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)\sum_{j=p}^{\infty}\omega^{-j+p}\frac{\partial^p v_j}{\partial\tau^p}+\ldots+$$

$$\omega^p\{f_1(u_0,\tau)+$$

$$\frac{\partial f_1(u,\tau)}{\partial u}[\sum_{j=1}^{\infty}\omega^{-j}u_j+\sum_{j=1}^{\infty}\omega^{-j}v_j]+\frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}[\sum_{j=1}^{\infty}\omega^{-j}u_j+\sum_{j=p}^{\infty}\omega^{-j}v_j]^2+\ldots\}\ldots\ldots$$

$$\ldots\ldots(3)$$

Where

$$\frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}[\sum_{j=1}^{\infty}\omega^{-j}u_j+\sum_{j=p}^{\infty}\omega^{-j}v_j]^2=$$

$$\frac{1}{2!}\sum_{k,s=1}^{n}[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_k\partial u_s}[\sum_{j=1}^{\infty}\omega^{-j}u_{j_k}+\sum_{j=p}^{\infty}\omega^{-j}v_{j_k}][\sum_{j=1}^{\infty}\omega^{-j}u_{j_s}+\sum_{j=p}^{\infty}\omega^{-j}v_{j_s}]]$$

Equations coefficient with positive degree keep in mind:

$$\omega^p:\frac{\partial^n v_p}{\partial\tau^n}=f_1(u_0,\tau),\ldots\ldots\ldots(4)$$

$$\omega^{p-1}:\frac{\partial^n v_{p+1}}{\partial\tau^n}=f_1(u_0,\tau)u_1,\ldots\ldots\ldots(5)$$

$$\omega^{p-2}: \frac{\partial^n v_{p+2}}{\partial \tau^n} = \frac{\partial f_1(u_0,\tau)}{\partial u} u_2 + \frac{1}{2!} \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} u_1{}^2, \ldots\ldots\ldots\ldots(6)$$

$$\omega^1: \frac{\partial^n v_{n-1}}{\partial \tau^n} = \frac{\partial f_1(u_0,\tau)}{\partial u} u_{p-1} + \frac{1}{2!} \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} \sum_{\substack{i+j=p-1 \\ i,j\geq 1}} (u_i u_j) + \ldots + \frac{1}{(p-1)!} \frac{\partial^{p-1} f_1(u_0,\tau)}{\partial u^{p-1}} u_1{}^{p-1},$$

$$\ldots\ldots(7)$$

Where

$$\frac{1}{2!} \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} \sum_{\substack{i+j=p-1 \\ i,j\geq 1}} (u_i u_j) = \frac{1}{2!} \sum_{s,\ell=1}^{n} [\frac{\partial^2 f_1(u_0,\tau)}{\partial u_s \partial u_\ell} \sum_{\substack{i+j=p-1 \\ i,j\geq 1}} (u_{i_s} u_{j_\ell})]$$

And

$$\frac{1}{(p-1)!} \frac{\partial^{p-1} f_1(u_0,\tau)}{\partial u^{p-1}} u_1{}^{p-1} = \sum_{i_1,i_2,\ldots,i_{p-1}=1}^{n} \frac{\partial^{p-1} f_1(u_0,\tau)}{\partial u_{i_1} \partial u_{i_2} \ldots \partial u_{i_{p-1}}} (u_{1j_1} u_{1j_2} \ldots u_{1j_{p-1}}).$$

The equation (4) , where $u_0$ be considered parameter , it is well known have unique satisfying condition $< v_p(\tau) >= \frac{1}{2\pi} \int_0^{2\pi} v(s) ds = 0$. We produce this solution in the from

$$v_p(\tau) = \varphi_p(u_0,\tau) \ldots\ldots\ldots\ldots(8)$$

Analogous we have the solution equations (5)- (7) with zero mean :

$$v_s(\tau) = \frac{\partial \varphi_p(u_0,\tau)}{\partial u} u_{s-p} + F_s \quad s=p+1, \ldots, n+1 \ldots\ldots\ldots\ldots(9)$$

Where $F_s$ -expression depending on $u_i, 1 \leq i \leq s-p-1$ .Now we equate in (3) coefficient expansion with $\omega^0$ :

$$\frac{\partial^n v_j}{\partial \tau^n} = f_0(u_0, 0, \ldots.., 0, \frac{\partial^p v_p}{\partial \tau^p}, \tau) + \frac{\partial f_1(u_0,\tau)}{\partial u} (u_p + v_p) \ldots\ldots\ldots\ldots(10)$$

If we substitute expression $v_p$ in equation (10) and from average , we have the equation:

$$\Phi(u) = \left\langle f_0(u_0, 0, \ldots.., 0, \frac{\partial^p \Psi_p(u,\tau)}{\partial \tau^p}, \tau) + \frac{\partial f_1(u,\tau)}{\partial u} \Psi_p(u,v) \right\rangle \ldots\ldots\ldots\ldots(11)$$

We will presuppose, the equation (11) has stationary solution $u = u_0$ ,that mean ,for vector function $\Phi(u)$ this equation is correct

$$\Phi(u_0) = 0 \ldots\ldots\ldots\ldots(12)$$

Where $\Phi(u_0)$ -invertible matrix .Here

$$\Phi'(u_0)=\left\langle \frac{\partial f_0(u_0,0,.....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0}+\frac{\partial f_0(u_0,0,.....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_p}\frac{\partial^{p+1}\Psi_p(u_0,\tau)}{\partial \tau^p \partial u}+\right.$$

$$\left.\frac{\partial f_1(u_o,\tau)}{\partial u}\frac{\partial \Psi_p(u_o,v)}{\partial u}+\frac{1}{2!}\sum_{i,k=1}^{n}[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_i \partial u_k}\psi(u_o,\tau)\right\rangle \quad \ldots\ldots(13)$$

Equation coefficient at $\omega^{-1}$ , have to equation

$$\frac{\partial v_{n+1}}{\partial \tau^n}=\frac{\partial f_0(u_0,0,0,....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0}u_1+\frac{\partial f_0(u_0,0,0,....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}v_p}{\partial \tau^{p-1}}$$

$$+\frac{\partial f_0(u_0,0,0,....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_p}\frac{\partial^p v_{p+1}}{\partial \tau^p}+\frac{\partial f_1(u_0,\tau)}{\partial u}(u_{p+1}+v_{p+1})+\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_1 v_p)\equiv$$

$$\Lambda_{p+1}(u_0+\tau)+\frac{\partial f_p(u_0,\tau)}{\partial u}u_{p+1}\ , \quad \ldots\ldots(14)$$

Here

$$\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_1 v_p)=\sum_{s,k=1}^{n}\frac{\partial^2 f_1(u_0,\tau)}{\partial u_k \partial u_s}(u_{1_s}v_{p_k})\,.$$

If we apply the equation (14) operation average with regard (9) we obtain:

$$\left\langle \frac{\partial f_0(u_0,0,0,....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0}\right\rangle u_1+\left\langle \frac{\partial f_0(u_0,0,0,....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}\Psi_p}{\partial \tau^{p-1}}\right\rangle+$$

$$\left\langle \frac{\partial f_0(u_0,0,0,....,0,\dfrac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_p}\frac{\partial^{p+1}\psi_p(u_0,\tau)}{\partial \tau^p \partial u}\right\rangle u_1+\left\langle \frac{\partial f_1(u_0,\tau)}{\partial u}\frac{\partial\ \psi_p(u_0,\tau)}{\partial u}\right\rangle u_1$$

$$+\left\langle \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}\psi_p(u_0,\tau)\right\rangle u_1=0$$

$$\Phi'(u_0)u_1=-\left\langle \frac{\partial f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}\Psi_p}{\partial\tau^{p-1}}\right\rangle \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

By virtue equation (9) , the solution problem (14) have the meaning :

$$v_{n+1}(\tau)=\frac{\partial\psi_p(u_0,\tau)}{\partial u}u_{p+1}+\chi_{p+1}(u_0,\tau) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(16)$$

$<\chi_{p+1}>=0$ and $\frac{d^n\chi_{p+1}}{d\tau^p}=\Lambda_{p+1}(u_0,\tau)$ , where $\chi_{p+1}$-expression , depending on $u_{s_1}$ and $v_{s_s}$ at $0\le s_1\le 1$ and $p\le s_2\le p+1$. From (15) we have define $u_1$. From (9) we will find $v_{p+1}$ :

$$v_{p+1}(\tau)=-\frac{\partial\psi_p(u_0,\tau)}{\partial u}\left[\Phi'(u_0)\right]^{-1}\left\langle \frac{\partial f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}\Psi_p}{\partial\tau^{p-1}}\right\rangle \ldots\ldots(17)$$

The equations from coefficients $v_j$ , $j\ge n$ bear in mind :

$$\omega^{-2}:\frac{\partial^n v_{n+2}}{\partial\tau^n}=\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0}u_2+\frac{1}{2!}\frac{\partial^2 f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0^2}u_1^2+$$

$$\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-2}}\frac{\partial^{p-2}v_p}{\partial\tau^{p-2}}+\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}v_{p+1}}{\partial\tau^{p-1}}+$$

$$\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_p}\frac{\partial^p v_{p+2}}{\partial\tau^p}+\frac{\partial f_1(u_0,\tau)}{\partial u}(u_{p+2}+v_{P+2})+\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_2v_P)+$$

$$\frac{\partial^3 f_1(u_0,\tau)}{\partial u^3}(u_1^2v_P)$$

$$\equiv \Lambda_{p+2}(u_0+\tau)+\frac{\partial f_1(u_0,\tau)}{\partial u}u_{p+2} \ .$$

$$\omega^{-3}:\frac{\partial^n v_{n+3}}{\partial \tau^n}=\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0}u_3+\frac{\partial^2 f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^2}u_1u_2+\frac{1}{3}$$

$$\frac{\partial^3 f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^3}u_1^3+\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-3}}\frac{\partial^{p-3}v_p}{\partial \tau^{p-3}}+\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-2}}$$

$$\frac{\partial^{p-2}v_{p+1}}{\partial \tau^{p-2}}+\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}v_{p+2}}{\partial \tau^{p-1}}+\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_p}\frac{\partial^p v_p}{\partial \tau^p}$$

$$\frac{\partial f_p(u_0,\tau)}{\partial u}(u_{p+3}+v_{P+3})+\frac{\partial^2 f_p(u_0,\tau)}{\partial u^2}(u_3v_P)+\frac{\partial^3 f_p(u_0,\tau)}{\partial u^3}(u_1u_2v_P)+\frac{\partial^4 f_0(u_0,\tau)}{\partial u^4}(u_1^3v_P)$$

$$\equiv \Lambda_{p+3}(u_0+\tau)+\frac{\partial f_p(u_0,\tau)}{\partial u}u_{p+3} \ .$$

We show that description it is possible find any coefficients expansion (2) . presuppose , that we know $v_p, v_{p+1},\ldots\ldots\ldots v_{n+j-1}$ and $u_0,u_1,\ldots\ldots\ldots u_{j-1}$ .

We cant find $v_{n-j}$ and $u_j$ . we have equation :

$$\omega^{-j}:\frac{\partial^n v_{n+j}}{\partial \tau^n}=\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0}u_j+\frac{1}{2!}\frac{\partial^2 f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^2}\sum_{m+n=}u$$

$$+\ldots+$$

$$\frac{1}{j!}\frac{\partial^j f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^j}u_1^j+\sum_{s=0}^{j}\frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-s}}\frac{\partial^{p-s}v_{p+j-s}}{\partial \tau^{p-s}}+$$

$$\frac{\partial f_1(u_0,\tau)}{\partial u}(u_{p+j}+v_{P+j})+\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_jv_P)+\ldots+\frac{1}{(j+1)!}\frac{\partial^{j+1} f_1(u_0,\tau)}{\partial u^{j+1}}(u_1^jv_P)-$$

$$-\left\langle \frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0} \right\rangle u_j - \ldots - \left\langle \frac{1}{j!} \frac{\partial^j f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^j} \right\rangle u_1^j -$$

$$-\left\langle \sum_{s=0}^{j} \frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-s}} \frac{\partial^{p-s} v_{p+j-s}}{\partial \tau^{p-s}} \right\rangle - -\left\langle \frac{\partial f_1(u_0,\tau)}{\partial u} v_{P+j} \right\rangle - \ldots$$

$$-\left\langle \frac{1}{(j+1)!} \frac{\partial^{j+1} f_1(u_0,\tau)}{\partial u^{j+1}} (u_1^j v_P) \right\rangle$$

.(18)

For $v_{n+j}$ we get equation :

$$v_{n+j}(\tau) = \frac{\partial \psi_p(u_0,\tau)}{\partial u} u_j + \chi_j(u_0,\tau) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(19)}$$

Where $\chi_j$ -expression , depending on $u_{r_1}$ and $v_{r_2}$ at $0 \le r_1 \le j-1$ and $p \le r_2 \le n+j-1$.

Coefficient $u_j, j \ge 1$ are solution linear problem :

$$\left\langle \frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0} \right\rangle u_j + \frac{1}{2!}\left\langle \frac{\partial^2 f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^2} \sum_{m+n=j}(u_m u_n) \right\rangle + \ldots$$

$$\frac{1}{j!}\left\langle \frac{\partial^j f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^j} u_1^j \right\rangle + \ldots + \left\langle \sum_{s=0}^{j} \frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-s}} \frac{\partial^{p-s} \ldots}{\partial \tau \ldots}\right.$$

$$\left\langle \frac{\partial f_1(u_0,\tau)}{\partial u} v_{P+j} \right\rangle + \left\langle \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} \partial \psi_p(u_0,\tau) \right\rangle u_j + \ldots + \frac{1}{(j+1)!}\left\langle \frac{\partial^{j+1} f_1(u_0,\tau)}{\partial u^{j+1}} (u_1^j v_P) \right\rangle = 0$$

.

(From here , It is necessary equation) .

$$\Phi'(u_0) u_j = M_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(20)}$$

Where $M_j$-expression as type that $\chi_j$. We consider question about decidability constructed problems. Average problem (12) by condition has solution $u_0$ . substituting it in expression (8) we find $v_p(\tau)$. After that definable uniquely solution $u_1$ linear problems (20) with j=1 and by formula (9) at s=p+1 we can find $v_{p+1}$ and etc.

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