Journal of Kerbala University , Vol. 6 No.1 Scientific .March. 2008

THE EFFECT OF TWO-BODY SHORT RANGE CORRELATION FUNCTIONS ON THE CHARGE DENSITY DISTRIBUTIONS OF SOME LIGHT NUCLEI

تأثير دالة ترابط الجسيمين ضمن المدى القصير على توزيعات كثافة الشحنة لبعض النوى الخفيفة

عادل خلف حمودي* و رعد عبدالكريم راضي* و فاضل إسماعيل شراد الطائي**

A. K. Hamoudi *, R. A. Radhi * and F.I. Shrrad Al-Taie **
*University of Baghdad-College of Science – Department of Physics
**University of Karbala-College of Science – Department of Physics

ABSTRACT:

The effects of short range correlation are taken into account in an effective way, that is, expressing the ground state wave function in terms of the occupation probabilites of singl particle orbits for various closed and open shell nuclei with N=Z.

The effect of the SRC's and the occupation probability (η) of higher states on the ground state 2BCDD's and the root mean square charge radii $\langle r^2\rangle^{1/2}$ are investigated. It is found that the inclusion of SRC's leads to enhance the probability of transferring the protons from the central region of the nucleus towards its surface since this causes to reduce the central part of the 2BCDD's significantly and increases the tail part of them slightly and consequently leads to increase the calculated values of $\langle r^2\rangle^{1/2}$ for ^{4}He, ^{12}C, ^{16}O, ^{28}Si, ^{32}S and ^{40}Ca nuclei. Considering the effect of higher occupation probabilities and the effect of SRC's are important in getting good agreement between the calculated 2BCDD's.

الخلاصة:

في هذه الدراسة تم الأخذ بنظر الاعتبار تأثير الترابط القصير المدى بين الجسيمين وكذلك احتمالية الأشغال لاشتقاق صيغة رياضية واضحة نحسب منها توزيعات كثافة الشحنة النووية ذو صيغة الجسيمين في الحالة الأرضية(2BCDD's). إن الصيغة الرياضية المشتقة يمكن تطبيقها على نوى مغلقة ومفتوحة القشرة شرط إن يكون فيها عدد البروتونات (Z) مساويا إلى عدد النترونات (N). درس تأثير كل من دالة ارتباط الجسيمين (SRC) واحتمالية الأشغال (η) للمستويات العالية على توزيعات كثافة الشحنة النووية (2BCDD's) و معدل الجذر ألتربيعي لنصف القطر $\langle r^2\rangle^{1/2}$. أظهرت هذه الدراسة بان إدخال دالة(SRC) في الحسابات يؤدي إلى زيادة احتمالية انتقال البروتونات من منطقة مركز النواة إلى السطح يرافقه اختزال واضح في الجزء المركزي وزيادة طفيفة في الجزء ألذيلي من توزيعات كثافة الشحنة النووية (2BCDD's) وبالتالي يؤدي ذلك إلى زيادة القيم المحسوبة لمعدل الجذر ألتربيعي لنصف القطر $\langle r^2\rangle^{1/2}$ولجميع النوى قيد الدراسة (^{4}He, ^{12}C, ^{16}O, ^{28}Si, ^{32}S and ^{40}Ca). إن إدخال تأثير كل من احتمالية الأشغال (η) للمستويات العالية و دالة (SRC) في الحسابات له أهمية كبيرة في الحصول على توافق جيد بين النتائج العملية والنظرية لكل من توزيعات كثافة الشحنة النووية (2BCDD's).

INTRODUCTION :

The charge density distribution $\rho_{ch}(r)$ is one of the many most important quantities in the nuclear structure which has been well studied experimentally over a wide range of nuclei. This interest in $\rho_{ch}(r)$ is related to the basic bulk nuclear characteristics such as the shape and the size of nuclei, their binding energies, and other quantities connected with $\rho_{ch}(r)$. Besides, the charge density distribution is an important object for experimental and theoretical investigations since it plays the role of a fundamental variable in nuclear theory [1]. From a theoretical point of view, most of the efforts concerning short-range correlations (SRC)have been concentrated in the study either of few-body systems (e.g. deuteron, triton, ^{3}He), where exact calculations can be performed, or of nuclear matter, a system which is obviously easier to study than finite nuclei [2]. A different and more phenomenological point of view can be adopted in order to search for experimental evidence of SRC effects. Instead of treating the energy of the system in a privileged way, as in variational approaches, one would like to have a phenomenological framework in which one can compute in a direct way the one- and two-body density matrices and thereby display the effects of correlations on various physical quantities. The method should be simple enough to be used for light as well as heavy nuclei. Such a programme seems to us appealing because it may help in finding which experimental quantities are more sensitive to SRC effects [2]. The inclusion of short – range and tensor correlation effects is rather a complicated problem especially for the microscopic theory of nuclear structure. Several methods were proposed to treat complex tensor forces and to describe their effects on the nuclear ground state [3,4,5].

A simple phenomenological method for introducing dynamical short range and tensor correlations has been introduced by Dellagiacoma et. al. [6]. In that method a two – body correlation operator is introduced to act on the wave function of a pair of particles. It resembles the earlier approaches of construcing the exact wave function ψ by means of an operator $\hat{F}$ such that $\hat{F}\Phi = \psi$, [7]or by a correlation Jastrow[8]and Jastrow – type [9,10] factor $\Pi \hat{f}$ such that $\Pi \hat{f}\Phi = \psi$ acting on the uncorrelated determinant wave function (Φ).

A similar correlation operator was proposed earlier by Da Proveidencia and Shakin [11]; Malecki and Picchi [12] for describing the short – range correlation effects.

The effect of the short range correlations due to the repulsive part of two-body interaction on the charge form factor of several p-shell nuclei has been analyzed in detail [13] with an independent particle model (IPM) generated in the harmonic oscillator (HO) well [14,15]. In ref [13], it was shown that the high-momentum parts (q>3 fm^{-1}) of the form factors calculated with and without correlations behave in completely different ways, which indicates that electron scattering at high momentum transfer could give useful information on the short-range correlations. Massen and Moustakidis [16,17] derived analytical expressions of the one and two body terms in the cluster expansion of the charge form factors and density distributions of sp- and sd- shell nuclei with Z=N. Those expressions were used for the systematic study of the effect of short range correlations on the form factors and densities, and they depend on the parameters b and β, which represent the harmonic oscillator parameter and the correlation parameter, respectively. These parameters were determined for various sp- and sd- shell nuclei by fitting the theoretical charge form factor to the experimental one.

THEORY:

In the present work, we assume that the nucleons of the nucleus behave as point particles. The particle density of a system (nucleus) consisting of A point – like particles can be described by means of the operator[1]

$$\hat{\rho}^{(1)}(\vec{r}) = \sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i) \qquad (1)$$

The one body density operator of equation (1) could be transformed into a two-body density form by the following transformation[18].

$$\hat{\rho}^{(1)}(\vec{r}) \Rightarrow \hat{\rho}^{(2)}(\vec{r})$$

$$\sum_{i=1}^{A}\delta(\vec{r}-\vec{r}_i)\equiv\frac{1}{2(A-1)}\sum_{i\neq j}\left\{\delta(\vec{r}-\vec{r}_i)+\delta(\vec{r}-\vec{r}_j)\right\} \qquad (2)$$

In fact, a further useful transformation can be made which is that of the coordinates of the two – particles, $\vec{r}_i$ and $\vec{r}_j$, to being in terms of that relative $\vec{r}_{ij}$ and center – of – mass $\vec{R}_{ij}$ coordinates [19] ,i.e.

$$\hat{\rho}^{(2)}_{ch}(\vec{r})=\frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}\left\{\delta\left[\sqrt{2}\,\vec{r}-\vec{R}_{ij}-\vec{r}_{ij}\right]+\delta\left[\sqrt{2}\,\vec{r}-\vec{R}_{ij}+\vec{r}_{ij}\right]\right\} \qquad (٣)$$

The effective two – body charge density operator of equation (3) , to be used with uncorrelated wave function , can be written as :

$$\hat{\rho}^{(2)}_{eff}(\vec{r})=\frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}f(r_{ij})\left\{\delta\left[\sqrt{2}\,\vec{r}-\vec{R}_{ij}-\vec{r}_{ij}\right]+\delta\left[\sqrt{2}\,\vec{r}-\vec{R}_{ij}+\vec{r}_{ij}\right]\right\}f(r_{ij}) \qquad (4)$$

where the functions $f(r_{ij})$ are the two – body short range correlation (SRC).Since $f(r_{ij})$ are central functions of the separation between the pair of particles which reduce the two–body wave function at short distances, where the repulsive core forces the particles apart, and heal to unity at large distances where the interactions are extremely weak . In this work, a simple model form of short range correlation of Ref. [20] will be adopted, i.e.

$$f(r_{ij})=1-\exp[-\beta(r_{ij}-r_c)^2] \qquad (5)$$

where r_c is the radius of a suitable hard – core and β=25 fm^{-2}[20] is a correlation parameter. The 2BCDD of closed shell nuclei is given by the expectation values of the effective two-body charge density operator of eq(4) and expressed as

$$\langle\Psi\,|\,\hat{\rho}^{(2)}_{eff}(\vec{r})\,|\,\Psi\rangle=\sum_{i\langle j}\langle i\,j|\,\hat{\rho}^{(2)}_{eff}(\vec{r})\,\left[|\,ij\,\rangle-|\,ji\,\rangle\right] \qquad (6)$$

where i and j are all the required quantum numbers, i.e.

$i\equiv n_i,\ell_i,j_i,m_i,t_i,m_{t_i}$ and $j\equiv n_j,\ell_j,j_j,m_j,t_j,m_{t_j}$

It is important to remark that our derived effective two-body charge density matrix elements of eq(6) are of the form

$$\left\langle\hat{\rho}^{(2)}(\vec{r})\right\rangle_{eff}\equiv\langle\Psi\,|\,\hat{\rho}^{(2)}_{eff}(\vec{r})\,|\,\Psi\rangle$$

or

$$\left\langle\hat{\rho}^{(2)}(\vec{r})\right\rangle_{eff}\equiv\langle\Phi_{corr}\,|\,\hat{\rho}^{(2)}(\vec{r})\,|\,\Phi_{corr}\rangle \qquad (7)$$

where $|\Phi_{corr}\rangle=\left|f(r_{ij})\Psi\right\rangle$ (8)

and $f(r_{ij})$ is the two body correlation functions of eq(5). Here we wish to indicate that the uncorrelated slater determinant wave function Ψ is correctly normalized while the correlated wave function Φ_{corr} of eq(8) is not. To address this matter we adopt the following renormalization scheme

$$\int_0^\infty 4\pi r^2\left\langle\hat{\rho}^{(2)}(\vec{r})\right\rangle_{eff}dr=\tilde{Z} \qquad (9)$$

here $\widetilde{Z} \neq Z$, (Z is the number of protons).While the real two-body charge density matrix elements $\left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{real}$ should give a correct number of protons,i.e.

$$\int_0^\infty 4\pi r^2 \left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{real} dr = Z \qquad (10)$$

Thus the matrix elements $\left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{eff}$ and $\left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{real}$ of eq's (9) and (10), respectively, can be related to each other as

$$\left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{real} = \frac{Z}{\widetilde{Z}} \left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{eff} \qquad (11)$$

So that a correct number of charges (protons) can now be reproduced by introducing eq(11) into eq(10), i.e.

$$\int_0^\infty 4\pi r^2 \frac{Z}{\widetilde{Z}} \left\langle \hat{\rho}^{(2)}(\vec{r}) \right\rangle_{eff} dr = Z \qquad (12)$$

We also wish to mention that we have written all computer programs needed in this study by the languages of Fortran 90 power station .

RESULTS, DISCUSSION AND CONCLUSIONS

In figures (1) to (6) we present the dependence of the ground state 2BCDD's (in fm^{-3}) on r (in fm) for ^{4}He, ^{12}C, ^{16}O, ^{28}Si, ^{32}S and ^{40}Ca nuclei, respectively. Parts (a) and (b) of these figures are the calculated distributions based on case 1 (it based on the prediction of the simple shell model) and case 2 (it have included the higher occupation probabilities) of tables (1) and (2), respectively. The dotted symbols are the experimental results whereas the dashed and solid curves are the calculated 2BCDD's without and with the inclusion of the two body SRC's, respectively. As it is evident from parts (a) of these figures, with the exception of figure (2), that the calculated 2BCDD's of case 1 deviated clearly from those of the experimental results especially at the region of small r (i.e. $0 \leq r \leq 2$ fm). Introduction of the two-body SRC's in the calculations causes to reduce these deviations in ^{4}He, ^{32}S and ^{40}Ca nuclei and increase them in ^{16}O and ^{28}Si nuclei as seen in the solid distributions of these figures. While part (a) of figure (2) shows a very nice agreement between the dashed curve and the dotted symbols throughout all values of r. It also shows a deviation between the solid curve and the dotted symbols in the region of small r since the inclusion of the two-body SRC's leads to underestimate the experimental data at this region. However, these deviations presented in the above figures are attributed to the necessity of introducing the occupation probabilities of higher states, in addition to those predicted by the simple shell model of case 1. So that, in parts b (case 2) of the above figures we have included the higher occupation probabilities of $\eta_{1p_{3/2}}$ in ^{4}He nucleus, $\eta_{2s_{1/2}}$ in ^{12}C, ^{16}O and ^{28}Si nuclei, $\eta_{1d_{3/2}}$ in ^{32}S nucleus and $\eta_{2p_{3/2}}$ in ^{40}Ca nucleus and considered them as free parameters to be adjusted in order to obtain a satisfactory results for the 2BCDD's and $\left\langle r^2 \right\rangle_{cal}^{1/2}$ in comparison with those of experimental data. It is important to point out that these higher occupation probabilities must be zero in case 1 and different from zero in case 2 as seen in tables (1) and (2), respectively. In general, an improvement results for the calculated 2BCDD's, in the region of small r (i.e. $0 \leq r \leq 2$ fm), is obtained in part b (case 2) of the above figures since the calculated 2BCDD's with the inclusion of the two-body SRC's are now much closer to those of experimental data than before, i.e. the quality of agreement between the solid curves and dotted symbols is better reproduced in part (b) than part (a) of the above figures. It is concluded from these figures that the dominant influence of the change of the shape of the 2BCDD's at the central region (i.e. $0 \le r \le 2$ fm) is the occupation probability of higher states considered in table (2) for various nuclei. In the above figures the contributions of the SRC's to the 2BCDD's $\rho_{SRC}(\mathrm{r}) = \rho_{\mathrm{r}_c=0.5}(\mathrm{r}) - \rho_{\mathrm{r}_c=0}(\mathrm{r})$ are also shown. We conclude from these figures that the inclusion of the two-body SRC's has the feature of reducing the central part of distributions significantly and increasing the tail part of the distributions slightly, i.e. considering of the two-body SRC's leads to increase the probability of transferring the protons from the central part into the tail part of the distribution and this will make the nucleus to be less rigid than before (i.e. the case with $\mathrm{r}_c = 0$). Thus an increase in the calculated $\left\langle \mathrm{r}^2 \right\rangle_{cal}^{1/2}$ of the nucleus is expected with the inclusion of the two-body SRC's as seen in table (1) and (2) of case 1 and 2, respectively.

REFERENCES:

[1]A.N.Antonov,P.E.Hodgson and I.Zh.Petkov,"Nucleon Momentum and Density Distribution in Nuclei",Clarendon Press, Oxford, (1988).
[2] M .DAL RI, S. Stringari and O.Bohigas ; Nucl. Phys.,**A376**,81,(1982).
[3] H.Bethe,B.H.Brandow and A.G.Petschek; Phys. Rev.,**129**,225,(1963)
[4] T.T.S.Kuo and G.E.Brown ; Nucl. Phys., **A85**,40,(1966).
[5] T.T.S.Kuo,S.Y.Lee and K.F.Ratefcliff ; Nucl. Phys.,**A176**,65,(1971).
[6]F.Dellagiacoma,G.Orlandiniand and M.Traini;Nucl.Phys,**A393**,95 (1983).
[7]K.A. Brueckner ,R.J.Eden and N.C.Francis ; Phys. Rev., **98**,1445, (1955).
[8]R. Jastrow ; Phys. Rev., **98**,1479,(1955).
[9]G.Ripka and J.Gillespie ; Phys. Rev. Lett., **25**,1624,(1970).
[10]S.Fantoni and V.R.Pandharipande ; Nucl. Phys., **A427**,473,(1984).
[11] J.Da Providencia and C.M.Shakin ; Ann. Phys., **30**,95,(1964).
[12]A.Malecki and P.Picchi ; Phys. Rev. Lett., **21**,1395,(1968).
[13]C.Ciofi Degli Atti; Phys. Rev., **175**,1256 , (1968).
[14]F.C.Khanna; Phys. Rev. Lett., **20**,871, (1968).
[15]C.Ciofi Degli Atti and N.M.Kabachnik; Phys. Rev., **C1**,809, (1970).
[16] S.E.Massen and C.Moustakidis ; Phys. Rev., **C60**, 024005, (1999).
[17]C.Moustakidis and S.E.Massen ; Phys. Rev., **C62**, 034318, (2000).
[18]S. Gartenhaus and C. Schwartz; Phys. Rev., **108**,482, (1957).
[19]R.d.Lawson, " Theory of the Nuclear Shell Model ", Clarendon Press, Oxford, (1980).
[20]J.Fiase, A.Hamoudi,J.M.Irvine and F.Yazici; J. Phys, **G14**,27,(1988).
[21]J.S.McCarthy,I.Sick and R.R.Whitney; Phys. Rev .,**C15**, 1396, (1977).
[22] Atomic Data and Nuclear Data Tables, **36**, 3, (1987).

Table (1)

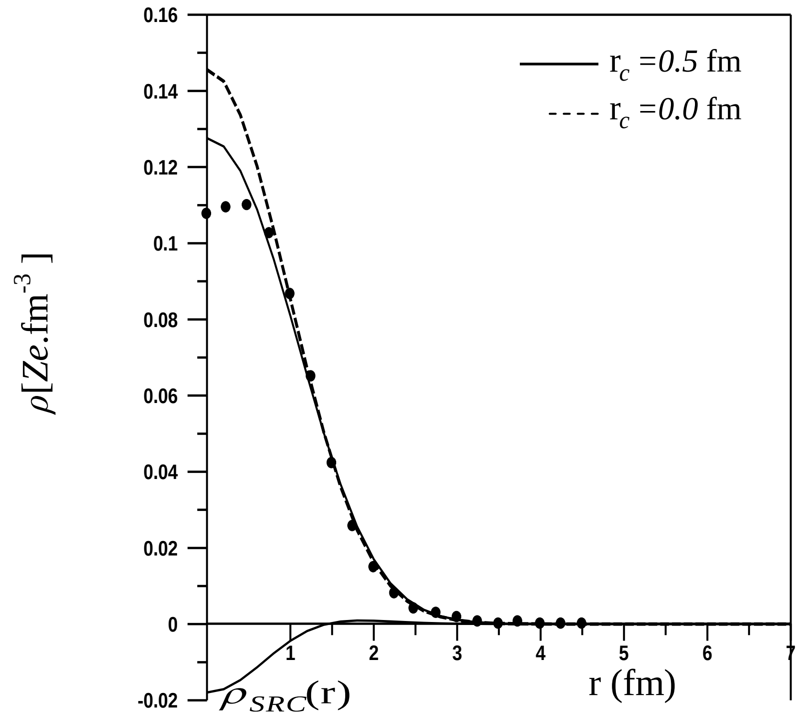
The values of harmonic oscillator spacing parameters ($\hbar\omega$) and the occupation probabilities *used in* the calculation of case 1 togther with correspoding results of $\langle r^2\rangle^{1/2}_{r_c=0}$, $\langle r^2\rangle^{1/2}_{r_c=0.5}$ and $\langle r^2\rangle^{1/2}_{SRC}$ and those of $\langle r^2\rangle^{1/2}_{exp}$ [16] for all considered nuclei.

Nucleus	^{4}He	^{12}C	^{16}O	^{28}Si	^{32}S	^{40}Ca
$\hbar\omega(MeV)$	23	15	12.6	12	11	10
$\eta_{1S_{1/2}}$	1	1	1	1	1	1
$\eta_{1P_{3/2}}$	-	1	1	1	1	1
$\eta_{1P_{1/2}}$	-	-	1	1	1	1
$\eta_{1d_{5/2}}$	-	-	-	1	1	1
$\eta_{2S_{1/2}}$	-	-	-	-	1	1
$\eta_{1d_{3/2}}$	-	-	-	-	-	1
$\langle r^2\rangle^{1/2}_{r_c=0}$ fm	1.644	2.437	2.715	3.071	3.267	3.504
$\langle r^2\rangle^{1/2}_{r_c=0.5}$ fm	1.678	2.454	2.728	3.082	3.278	3.513
$\langle r^2\rangle^{1/2}_{SRC}$ fm	0.336	0.288	0.266	0.260	0.259	0.251
$\langle r^2\rangle^{1/2}_{exp}$ [16]	1.676(8)	2.471(6)	2.730(25)	3.086(18)	3.248(11)	3.479(3)

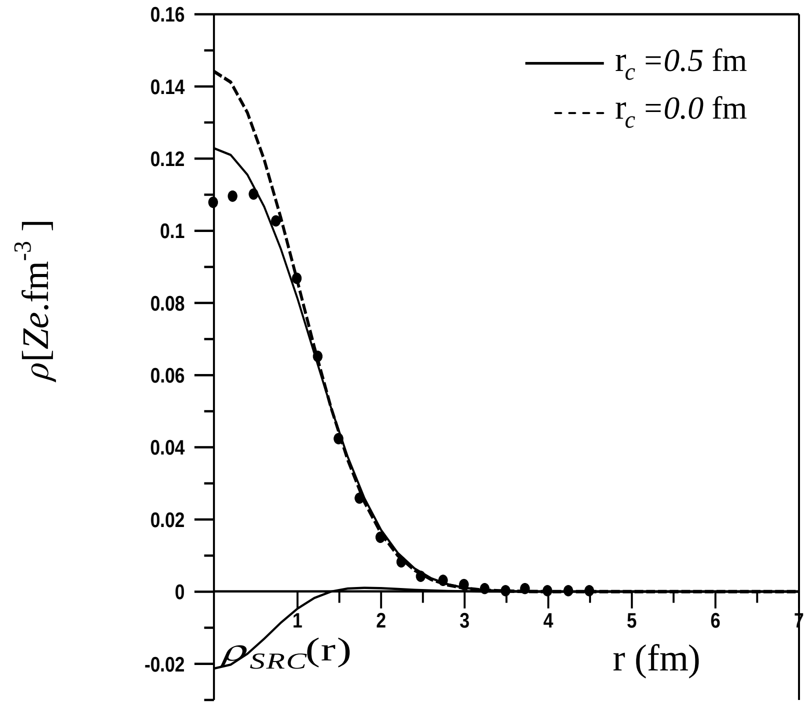
Table (2)

The values of harmonic oscillator spacing parameters ($\hbar\omega$)and the occupation probabilities used in the calculation of case 2 togther with correspoding results of $\langle r^2\rangle^{1/2}_{r_c=0}$, $\langle r^2\rangle^{1/2}_{r_c=0.5}$ and $\langle r^2\rangle^{1/2}_{SRC}$ and those of $\langle r^2\rangle^{1/2}_{exp}$ [16] for all considered nuclei.

Nucleus	^{4}He	^{12}C	^{16}O	^{28}Si	^{32}S	^{40}Ca
$\hbar\omega(MeV)$	26	14.665	12.5	11.63	10.9	10
$\eta_{1S_{1/2}}$	0.6	1	1	1	1	1
$\eta_{1P_{3/2}}$	0.2	0.95	1	1	1	1
$\eta_{1P_{1/2}}$	-	-	0.97	1	1	1
$\eta_{1d_{5/2}}$	-	-	-	0.8	1	1
$\eta_{2S_{1/2}}$	-	0.1	0.03	0.6	0.7	0.7
$\eta_{1d_{3/2}}$	-	-	-	-	0.15	1
$\eta_{2P_{3/2}}$	-	-	-	-	-	0.15
$\langle r^2\rangle^{1/2}_{r_c=0}$ fm	1.633	2.454	2.723	3.069	3.255	3.490
$\langle r^2\rangle^{1/2}_{r_c=0.5}$ fm	1.669	2.472	2.737	3.081	3.266	3.499
$\langle r^2\rangle^{1/2}_{SRC}$ fm	0.344	0.297	0.276	0.271	0.267	0.250
$\langle r^2\rangle^{1/2}_{exp}$ [16]	1.676(8)	2.471(6)	2.730(25)	3.086(18)	3.248(11)	3.479(3)

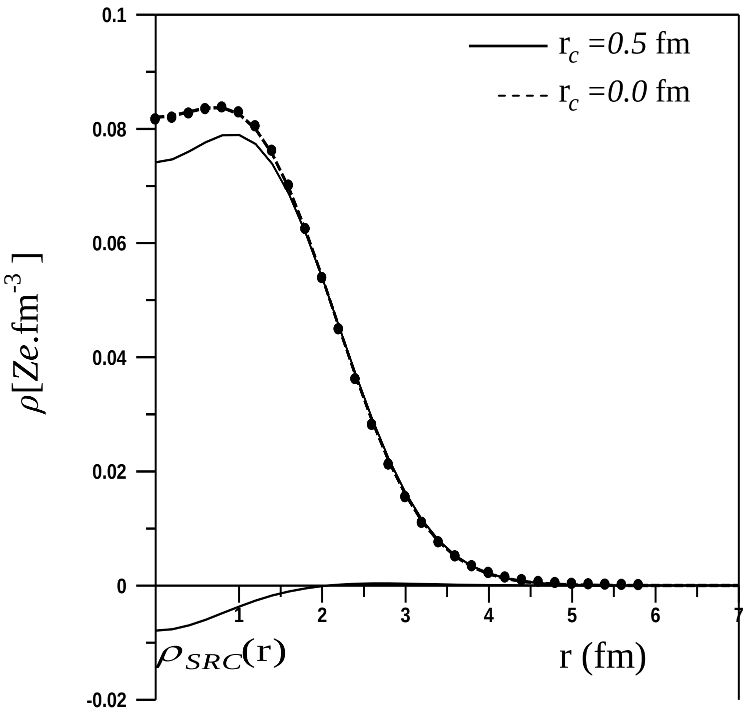


(a)- case 1

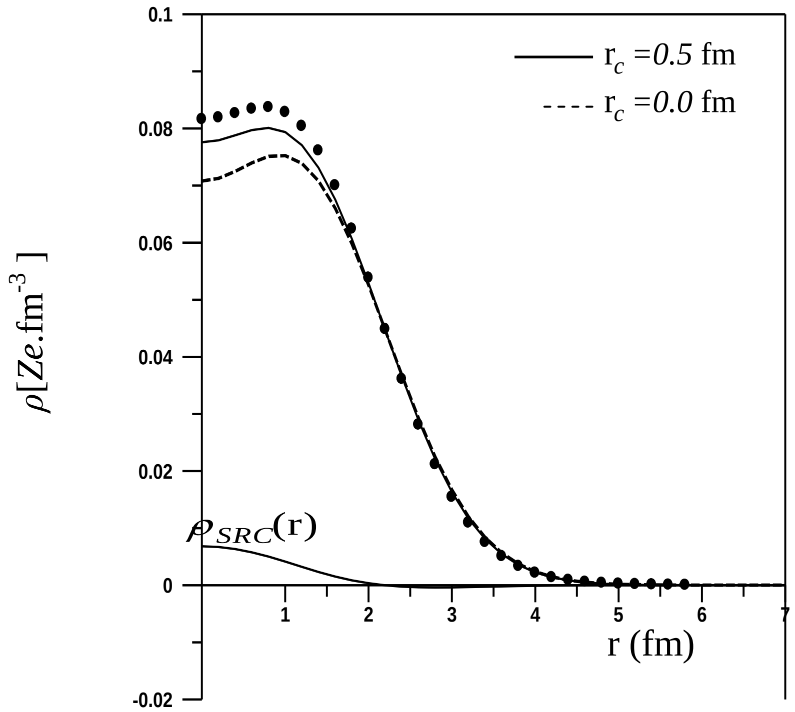


(b)- case 2

Figure (١): Dependence of the 2BCDD on (r) for ^{4}He nucleus.
The dotted symbols are the experimental data of Ref [21].

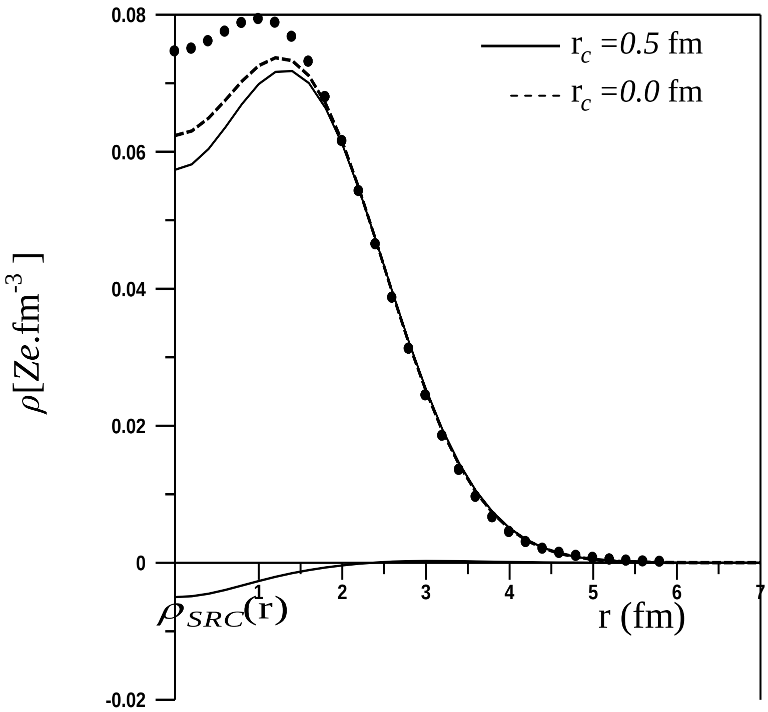


(a)- case 1

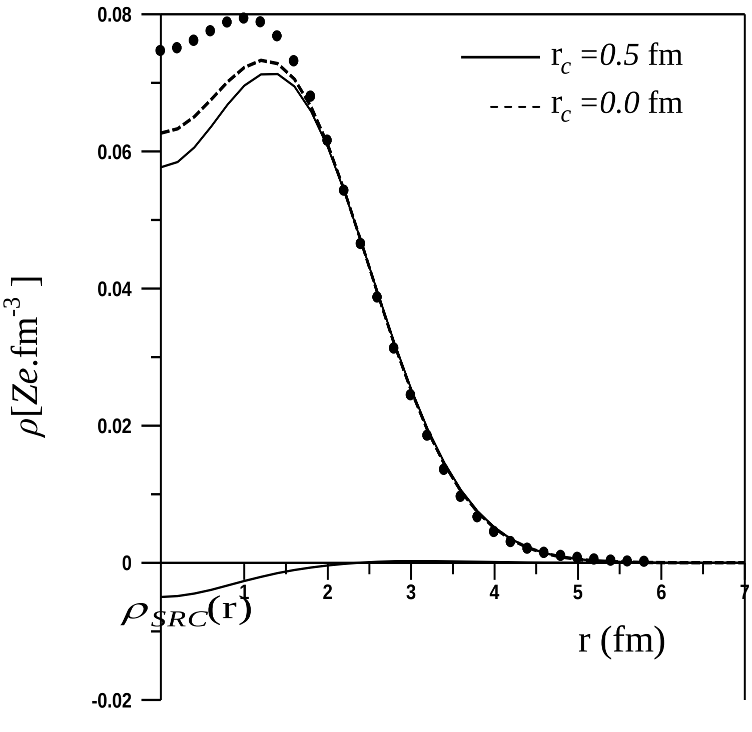


(b)- case 2

Figure (٢): Dependence of the 2BCDD on (r) for ^{12}C nucleus.
The dotted symbols are the experimental data of Ref [22].

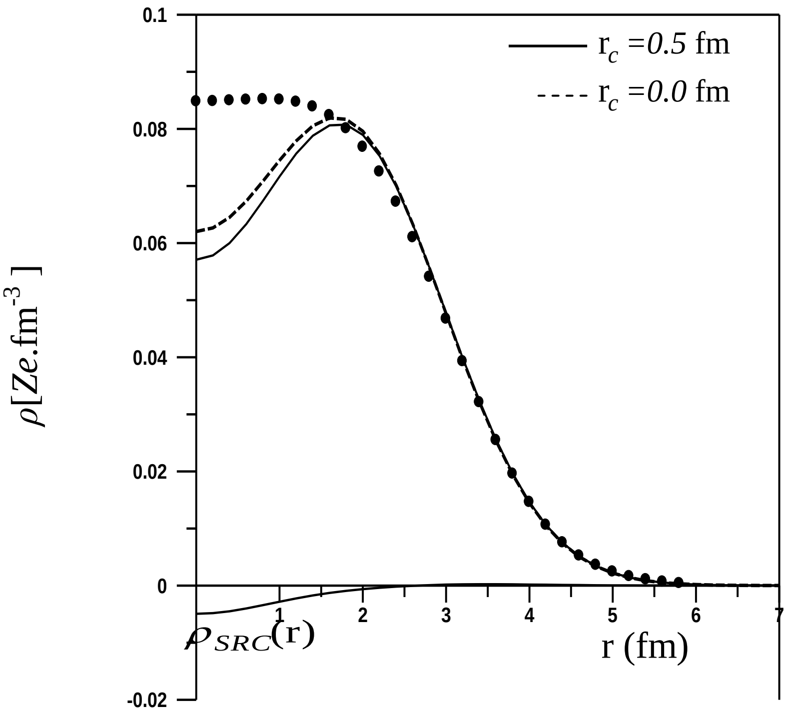


(a)- case 1

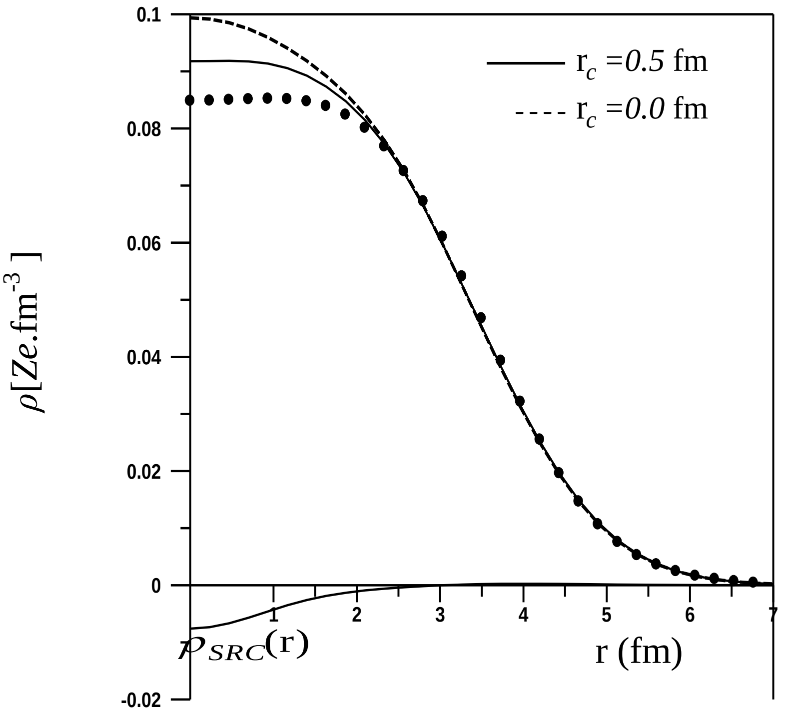


(b)- case 2

Figure (٣): Dependence of the 2BCDD on (r) for ^{16}O nucleus.
The dotted symbols are the experimental data of Ref [22].

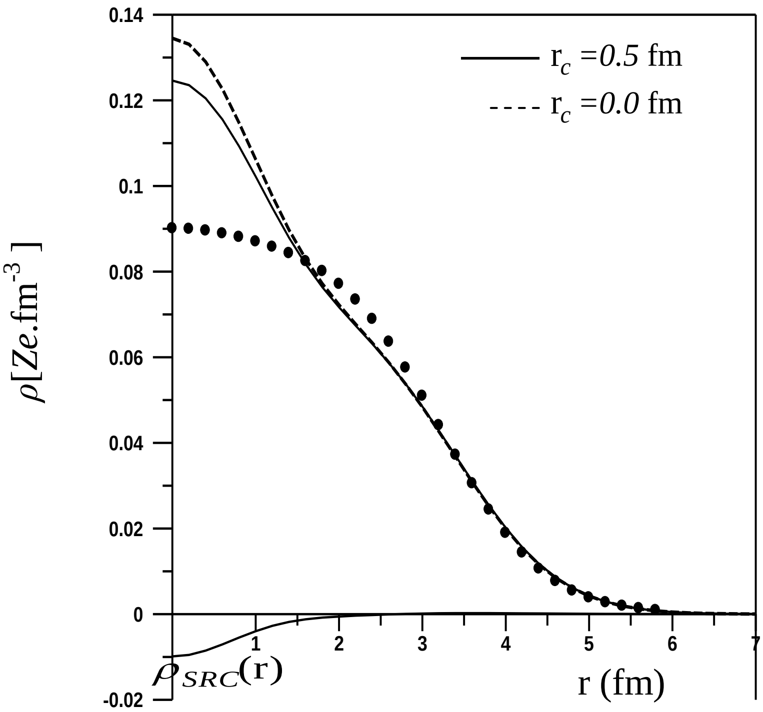


(a)- case 1

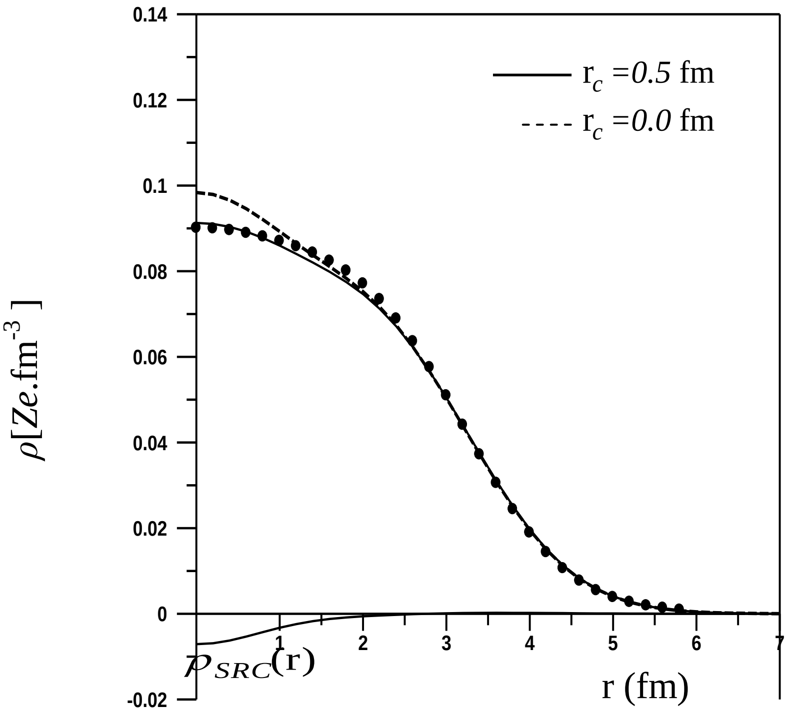


(b)- case 2

Figure (٤): Dependence of the 2BCDD on (r) for ^{28}Si nucleus.
The dotted symbols are the experimental data of Ref [22].

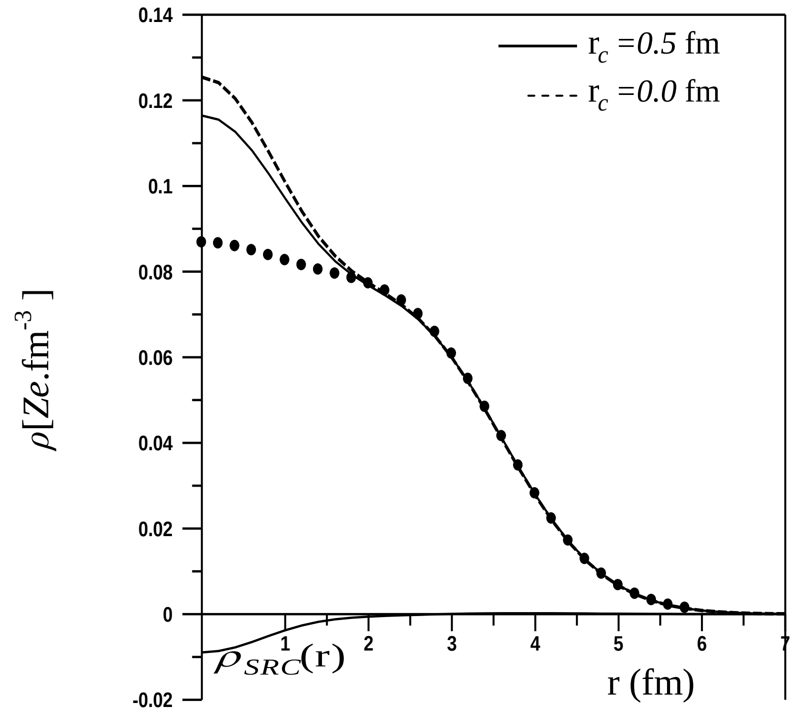


(a)- case 1

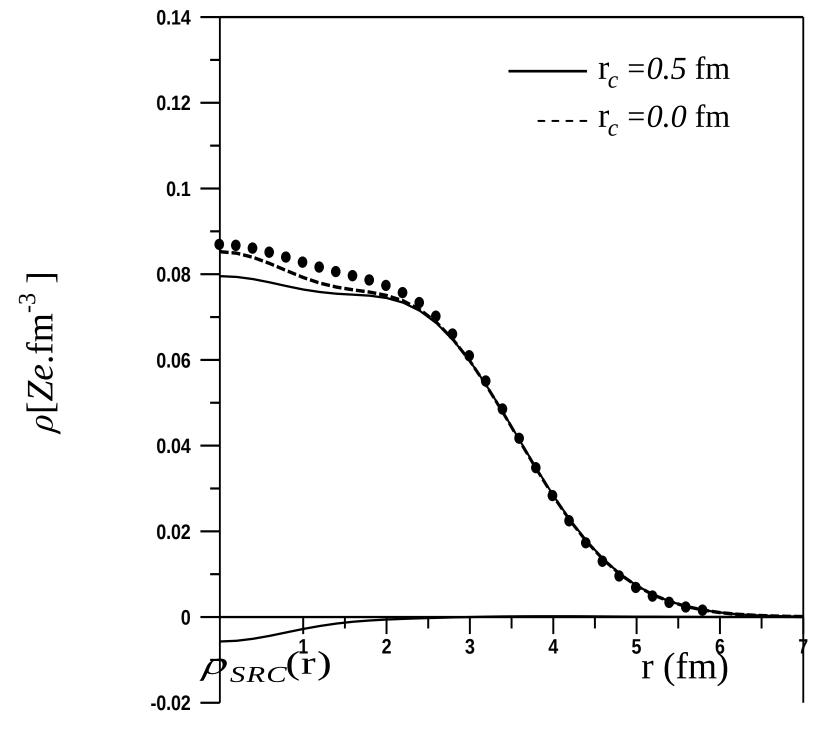


(b)- case 2

Figure (5): Dependence of the 2BCDD on (r) for ^{32}S nucleus.
The dotted symbols are the experimental data of Ref [22].



(a)- case 1



(b)- case 2

Figure (6): Dependence of the 2BCDD on (r) for ^{40}Ca nucleus.
The dotted symbols are the experimental data of Ref [22].