Differential Equation For Higher Frequency Periodic Direction Function

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Abstract :

In this research, we will study an ordinary differential equation (n degree) for higher Frecuency periodic direction function (f_0, f_1).

ملخص البحث:

في بحثنا هذا تم دراسة دوال $(f_0\ and\ f1)$ الدورية الاتجاهية في معادلة تفاضلية اعتيادية من الدرجة (n) (وفي هذا الحالة تكون عالية التردد) حيث وضعنا في بحثنا أدناه شروط معية وخاصة ، حيث تميزنا بدراسة معادلة تفاضلية من الدرجة (n) ولأي دالتين دوريتين متجهتين وبهذا المعنى فان هذه الدراسة يمكن اعتبارها دراسة موسعة ومهمة.

Introduction :

In this research , f_0 *and* f_1 Vector Periodic in normal high order differential equations of order (8) with high frequency that a specific conditions are put in this research is differ from (Bateman H.1985 , Boyce W.E. , Di Prima R.C. 1977) . whom study a normal differential equation of order one for trigonometric function . While (Greenberg M.D.1994 and Struble R.A) deal with study of differential equation of order two for vector periodic function during which they put a specific conditions . While in this research we work studying a differential of order (n) for any two vector periodic function so by this work we consider this study is an important and expand study.

1- Basic Concept

Let m and *p*-natural number , and also n-even , $n = 2p$, and G_i , $i = 0,1,2,3,....., p$ organic domain in space R^m .We have to study problem $2\pi\omega^{-1}$periodic solution for differential equations to be formal n .

$$\frac{d^n u}{dt^n} = f_0(u, \frac{du}{dt}, ..., \frac{d^p u}{dt^p}, \omega t) + \omega^p f_1(u, \omega t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where ω-big parameter .We will be presupposed the following :

1.Vector functions $f_0(e,\tau)$ defined in the set $\Omega_0 = \{e, \tau, e \in G_0 \times G_1 \times .... \times G_p, \tau \in R\}$ *u* vector functions $f_1(u,\tau)$ defined in the set $\Omega_1 = \{u, \tau, u \in G_0, \tau \in R\}$, have meaning in R^m.

2. Vector functions $f_0(e,\tau)$ and $f_1(u,\tau)$ have continuously differentiable for any order with respect to *e* and *u* respectively .

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Asymptotic expansion solution equation (1) will be sought in the form

$$u_\omega(t) = \sum_{j=0}^{\infty} \omega^{-j} u_j + \sum_{j=p}^{\infty} \omega^{-j} v_j(\omega t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where $v_j(\tau)$ - 2π periodic vector functions have meaning in R^m . u_j-vector in R^m and

$$< v_j > = \frac{1}{2\pi}\int_0^{2\pi} v(\tau)d\tau = 0$$

We substitute equation (1) in place of $u, \frac{du}{dt}, \ldots\ldots, \frac{d^p u}{dt^p}$ expression (2) and we develop nonlinear f_0 and f_1 in Taylor series , as a result we have the following equation :

$$\sum_{j=p}^{\infty}\omega^{-j+n}\frac{\partial^n v_j}{\partial\tau^n} = f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau) + \frac{\partial f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)}{\partial e_0}\left[\sum_{j=1}^{\infty}\omega^{-j}u_j + \sum_{j=p}^{\infty}\omega^{-j}v_j\right] +$$

$$\frac{\partial f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)}{\partial e_1}\sum_{j=p}^{\infty}\omega^{-j+1}\frac{\partial^n v_j}{\partial\tau} + \ldots + f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)\sum_{j=p}^{\infty}\omega^{-j+p}\frac{\partial^p v_j}{\partial\tau^p} + \ldots +$$

$$\omega^p\{f_1(u_0,\tau) +$$

$$\frac{\partial f_1(u,\tau)}{\partial u}[\sum_{j=1}^{\infty}\omega^{-j}u_j + \sum_{j=1}^{\infty}\omega^{-j}v_j] + \frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}[\sum_{j=1}^{\infty}\omega^{-j}u_j + \sum_{j=p}^{\infty}\omega^{-j}v_j]^2 + \ldots\}\ldots\ldots\ldots \quad \ldots(3)$$

Where

$$\frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}[\sum_{j=1}^{\infty}\omega^{-j}u_j + \sum_{j=p}^{\infty}\omega^{-j}v_j]^2 =$$

$$\frac{1}{2!}\sum_{k,s=1}^{n}[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_k\partial u_s}[\sum_{j=1}^{\infty}\omega^{-j}u_{j_k} + \sum_{j=p}^{\infty}\omega^{-j}v_{j_k}][\sum_{j=1}^{\infty}\omega^{-j}u_{j_s} + \sum_{j=p}^{\infty}\omega^{-j}v_{j_s}]]$$

Equations coefficient with positive degree keep in mind:

$$\omega^p : \frac{\partial^n v_p}{\partial\tau^n} = f_1(u_0,\tau), \ldots\ldots\ldots\ldots(4)$$

$$\omega^{p-1} : \frac{\partial^n v_{p+1}}{\partial\tau^n} = f_1(u_0,\tau)u_1, \ldots\ldots\ldots\ldots(5)$$

$$\omega^{p-2} : \frac{\partial^n v_{p+2}}{\partial\tau^n} = \frac{\partial f_1(u_0,\tau)}{\partial u}u_2 + \frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}{u_1}^2, \ldots\ldots\ldots\ldots(6)$$

$$\omega^1 : \frac{\partial^n v_{n-1}}{\partial\tau^n} = \frac{\partial f_1(u_0,\tau)}{\partial u}u_{p-1} + \frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}\sum_{\substack{i+j=p-1\\ i,j\ge 1}}(u_i u_j) + \ldots + \frac{1}{(p-1)!}\frac{\partial^{p-1} f_1(u_0,\tau)}{\partial u^{p-1}}{u_1}^{p-1}, \quad \ldots\ldots(7)$$

Where

$$\frac{1}{2!}\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}\sum_{\substack{i+j=p-1\\ i,j\geq 1}}(u_i u_j)=\frac{1}{2!}\sum_{s,\ell=1}^{n}[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_s \partial u_\ell}\sum_{\substack{i+j=p-1\\ i,j\geq 1}}(u_{i_s}u_{j_\ell})]$$

And

$$\frac{1}{(p-1)!}\frac{\partial^{p-1} f_1(u_0,\tau)}{\partial u^{p-1}}u_1^{\,p-1}=\sum_{i_1,i_2,\ldots,i_{p-1}=1}^{n}\frac{\partial^{p-1} f_1(u_0,\tau)}{\partial u_{i_1}\partial u_{i_2}..\partial u_{i_{p-1}}}(u_{1j_1}u_{1j_2}..u_{1j_{p-1}}).$$

The equation (4) , where u_0 be considered parameter , it is well known have unique satisfying condition $<v_p(\tau)>=\frac{1}{2\pi}\int_0^{2\pi} v(s)ds=0$. We produce this solution in the from

$$v_p(\tau)=\varphi_p(u_0,\tau)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Analogous we have the solution equations (5)- (7) with zero mean :

$$v_s(\tau)=\frac{\partial\varphi_p(u_0,\tau)}{\partial u}u_{s-p}+F_s \quad s=p+1\ ,....,n+1\ \ldots\ldots\ldots\ldots\ldots(9)$$

Where F_s -expression depending on $u_i, 1\leq i\leq s-p-1$ .Now we equate in (3) coefficient expansion with ω^0 :

$$\frac{\partial^n v_j}{\partial\tau^n}=f_0(u_0,0,.....,0,\frac{\partial^p v_p}{\partial\tau^p},\tau)+\frac{\partial f_1(u_0,\tau)}{\partial u}(u_p+v_p)\ldots\ldots\ldots\ldots\ldots(10)$$

If we substitute expression v_p in equation (10) and from average , we have the equation:

$$\Phi(u)=\left\langle f_0(u_0,0,.....,0,\frac{\partial^p\Psi_p(u,\tau)}{\partial\tau^p},\tau)+\frac{\partial f_1(u,\tau)}{\partial u}\Psi_p(u,v)\right\rangle\ldots\ldots\ldots\ldots\ldots(11)$$

We will presuppose, the equation (11) has stationary solution $u=u_0$,that mean ,for vector function $\Phi(u)$ this equation is correct

$$\Phi(u_0)=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Where $\Phi(u_0)$ -invertible matrix .Here

$$\Phi'(u_0)=\left\langle\frac{\partial f_0(u_0,0,.....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0}+\frac{\partial f_0(u_0,0,.....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_p}\frac{\partial^{p+1}\Psi_p(u_0,\tau)}{\partial\tau^p\partial u}+\right.$$

$$\left.\frac{\partial f_1(u_o,\tau)}{\partial u}\frac{\partial\Psi_p(u_o,v)}{\partial u}+\frac{1}{2!}\sum_{i,k=1}^{n}[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_i\partial u_k}\psi(u_o,\tau)\right\rangle\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Equation coefficient at ω^{-1} , have to equation

$$\frac{\partial v_{n+1}}{\partial\tau^n}=\frac{\partial f_0(u_0,0,0,....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0}u_1+\frac{\partial f_0(u_0,0,0,....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}v_p}{\partial\tau^{p-1}}$$

$$+\frac{\partial f_0(u_0,0,0,....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_p}\frac{\partial^p v_{p+1}}{\partial\tau^p}+\frac{\partial f_1(u_0,\tau)}{\partial u}(u_{p+1}+v_{p+1})+\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_1v_p)\equiv$$

$$\Lambda_{p+1}(u_0+\tau)+\frac{\partial f_p(u_0,\tau)}{\partial u}u_{p+1}\ ,\ \text{.............................}(14)$$

Here

$$\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_1v_p)=\sum_{s,k=1}^{n}\frac{\partial^2 f_1(u_0,\tau)}{\partial u_k\partial u_s}(u_{1_s}v_{p_k}).$$

If we apply the equation (14) operation average with regard (9) we obtain:

$$\left\langle\frac{\partial f_0(u_0,0,0,....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0}\right\rangle u_1+\left\langle\frac{\partial f_0(u_0,0,0,....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}\Psi_p}{\partial\tau^{p-1}}\right\rangle+$$

$$\left\langle\frac{\partial f_0(u_0,0,0,....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_p}\frac{\partial^{p+1}\psi_p(u_0,\tau)}{\partial\tau^p\partial u}\right\rangle u_1+\left\langle\frac{\partial f_1(u_0,\tau)}{\partial u}\frac{\partial\ \psi_p(u_0,\tau)}{\partial u}\right\rangle u_1$$

$$+\left\langle\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}\psi_p(u_0,\tau)\right\rangle u_1=0$$

$$\Phi'(u_0)u_1=-\left\langle\frac{\partial f_0(u_0,0,.....,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}\Psi_p}{\partial\tau^{p-1}}\right\rangle\text{........................}(15)$$

By virtue equation (9) , the solution problem (14) have the meaning :

$$v_{n+1}(\tau)=\frac{\partial\psi_p(u_0,\tau)}{\partial u}u_{p+1}+\chi_{p+1}(u_0,\tau)\text{..................................}(16)$$

$<\chi_{p+1}>=0$ and $\frac{d^n\chi_{p+1}}{d\tau^p}=\Lambda_{p+1}(u_0,\tau)$, where χ_{p+1}-expression , depending on u_{s_1} and v_{s_s} at $0\le s_1\le 1$ and $p\le s_2\le p+1$. From (15) we have define u_1. From (9) we will find v_{p+1} :

$$v_{p+1}(\tau) = -\frac{\partial \psi_p(u_0,\tau)}{\partial u}\left[\Phi'(u_0)\right]^{-1}\left\langle \frac{\partial f_0(u_0,0,\ldots\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-1}} \frac{\partial^{p-1}\Psi_p}{\partial \tau^{p-1}} \right\rangle \ldots\ldots(17)$$

The equations from coefficients v_j , $j \geq n$ bear in mind :

$$\omega^{-2} : \frac{\partial^n v_{n+2}}{\partial \tau^n} = \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0} u_2 + \frac{1}{2!}\frac{\partial^2 f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^2} u_1^2 +$$

$$\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-2}}\frac{\partial^{p-2}v_p}{\partial \tau^{p-2}} + \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}v_{p+1}}{\partial \tau^{p-1}} +$$

$$\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_p}\frac{\partial^p v_{p+2}}{\partial \tau^p} + \frac{\partial f_1(u_0,\tau)}{\partial u}(u_{p+2}+v_{P+2}) + \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_2 v_P) +$$

$$\frac{\partial^3 f_1(u_0,\tau)}{\partial u^3}(u_1^2 v_P)$$

$$\equiv \Lambda_{p+2}(u_0+\tau) + \frac{\partial f_1(u_0,\tau)}{\partial u} u_{p+2} \ .$$

$$\omega^{-3} : \frac{\partial^n v_{n+3}}{\partial \tau^n} = \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0} u_3 + \frac{\partial^2 f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^2} u_1 u_2 + \frac{1}{3!}$$

$$\frac{\partial^3 f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^3} u_1^3 + \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-3}}\frac{\partial^{p-3}v_p}{\partial \tau^{p-3}} + \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-2}}$$

$$\frac{\partial^{p-2}v_{p+1}}{\partial \tau^{p-2}} + \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-1}}\frac{\partial^{p-1}v_{p+2}}{\partial \tau^{p-1}} + \frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_p}\frac{\partial^p v_{p+}}{\partial \tau^p}$$

$$\frac{\partial f_p(u_0,\tau)}{\partial u}(u_{p+3}+v_{P+3}) + \frac{\partial^2 f_p(u_0,\tau)}{\partial u^2}(u_3 v_P) + \frac{\partial^3 f_p(u_0,\tau)}{\partial u^3}(u_1 u_2 v_P) + \frac{\partial^4 f_0(u_0,\tau)}{\partial u^4}(u_1^3 v_P)$$

$$\equiv \Lambda_{p+3}(u_0+\tau)+\frac{\partial f_p(u_0,\tau)}{\partial u}u_{p+3}\ .$$

We show that description it is possible find any coefficients expansion (2) . presuppose , that we know $v_p, v_{p+1},\ldots\ldots.,v_{n+j-1}$ and $u_0,u_1,\ldots\ldots.,u_{j-1}$.

We cant find v_{n-j} and u_j . we have equation :

$$\omega^{-j}:\frac{\partial^n v_{n+j}}{\partial\tau^n}=\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0}u_j+\frac{1}{2!}\frac{\partial^2 f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0^2}\sum_{m+n=j}u_m$$

$$+\ldots+$$

$$\frac{1}{j!}\frac{\partial^j f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0^j}u_1^j+\sum_{s=0}^{j}\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-s}}\frac{\partial^{p-s}v_{p+j-s}}{\partial\tau^{p-s}}+$$

$$\frac{\partial f_1(u_0,\tau)}{\partial u}(u_{p+j}+v_{P+j})+\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_jv_P)+\ldots+\frac{1}{(j+1)!}\frac{\partial^{j+1}f_1(u_0,\tau)}{\partial u^{j+1}}(u_1^jv_P)-$$

$$-\left\langle\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0}\right\rangle u_j-\ldots-\left\langle\frac{1}{j!}\frac{\partial^j f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_0^j}\right\rangle u_1^j-$$

$$-\left\langle\sum_{s=0}^{j}\frac{\partial f_0(u_0,0,0,\ldots.,0,\frac{\partial^p\Psi_p(u_0,\tau)}{\partial\tau^p},\tau)}{\partial e_{p-s}}\frac{\partial^{p-s}v_{p+j-s}}{\partial\tau^{p-s}}\right\rangle--\left\langle\frac{\partial f_1(u_0,\tau)}{\partial u}v_{P+j}\right\rangle-\ldots.$$

$$-\left\langle\frac{1}{(j+1)!}\frac{\partial^{j+1}f_1(u_0,\tau)}{\partial u^{j+1}}(u_1^jv_P)\right\rangle$$

(18)

For v_{n+j} we get equation :

$$v_{n+j}(\tau)=\frac{\partial\psi_p(u_0,\tau)}{\partial u}u_j+\chi_j(u_0,\tau)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(19)$$

Where χ_j -expression , depending on u_{r_1} and v_{r_2} at $0\le r_1\le j-1$ and $p\le r_2\le n+j-1$. Coefficient $u_j, j\ge 1$ are solution linear problem :

$$\left\langle \frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0} \right\rangle u_j + \frac{1}{2!}\left\langle \frac{\partial^2 f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^2} \sum_{m+n=j}(u_m u_n) \right\rangle$$

$$\frac{1}{j!}\left\langle \frac{\partial^j f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_0^j} u_1^j \right\rangle + \ldots + \left\langle \sum_{s=0}^{j} \frac{\partial f_0(u_0,0,0,\ldots,0,\frac{\partial^p \Psi_p(u_0,\tau)}{\partial \tau^p},\tau)}{\partial e_{p-s}} \frac{\partial^{p-s} v}{\partial \tau^p} \right.$$

$$\left\langle \frac{\partial f_1(u_0,\tau)}{\partial u} v_{P+j} \right\rangle + \left\langle \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} \partial \psi_p(u_0,\tau) \right\rangle u_j + \ldots + \frac{1}{(j+1)!}\left\langle \frac{\partial^{j+1} f_1(u_0,\tau)}{\partial u^{j+1}} (u_1^j v_P) \right\rangle = 0$$

.

(From here , It is necessary equation) .

$$\Phi'(u_0)u_j = M_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Where M_j-expression as type that χ_j. We consider question about decidability constructed problems. Average problem (12) by condition has solution u_0 . substituting it in expression (8) we find $v_p(\tau)$. After that definable uniquely solution u_1 linear problems (20) with j=1 and by formula (9) at s=p+1 we can find v_{p+1} and etc.

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