Kriging of Groundwater Level - A Case Study of Dibdiba Aquifer in Area of Karballa-Najaf

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Abstract:
The application of the spatial statistical technique (kriging) is used in this research, for the spatial analysis of groundwater levels is shown. The data set consists of groundwater levels measured at about 13 hand dug wells were selected in the studied area for the observation water table (vary for ten month, from June 2002 to March 2003) in an area of 20x20 km^2 Dibdiba hydrologic basin; which lies between Karbala and Najaf provinces. With the use of measured elevations of the water table, experimental semivariograms were constructed that characterizes the spatial variability of the measured groundwater levels. The experimental semivariograms were fitted into many models as Spherical, exponential and gaussian semivariogram. The finally selected models were used to estimate the groundwater levels and estimation variance (which express the accuracy of the estimated groundwater levels) at the nodes of a square grid of 2.5km x 2.5km and to develop corresponding contour maps .Also, used the Inverse Square Distance (ISD) method in order to interpolate the groundwater levels for the study area. It was found that ISD method resulted in higher errors as compared to kriging method. The groundwater table maps resulted by kriging method were compared with the groundwater table maps prepared using the ISD method.

الخلاصة :
تم في هذا البحث تطبيق تقنية التخمين الاحصائي الكريكنك(Kriging) في التخمين المكاني لمستوى المياه الجوفية.اخذت مجموعة بيانات لمستوى المياه الجوفية المقاسة حقليا من (١٣) بئرا مختارةفي منطقة الدراسة(المراقبةشهريا ولمدة عشرة اشهر،من شهر حزيران لعام ٢٠٠٢ الى شهر اذار ٢٠٠٣).لمساحة (٤٠٠)كم(٢٠كم طول x ٢٠كم عرض)من حوض الدبدبة الرملي بين مدينتي كربلاء والنجف المقدستين.

باستخدام المناسيب المقاسة لمستويات المياه الجوفية تم انشاء مخططات (semivariograms)تجريبية والتي هي جزء من التغاير المكاني المقاس لمستوى المياه الجوفية. تم عمل تقريب ملائم(fitted) لعدة موديلات من (Semivariograms) منها (Spherical, exponential and Gaussian).

وبعد اجراء الاختبارات لعدة موديلات تم اختيار الموديل النهائي من هذه الموديلات لتخمين مستوى المياه الجوفية في المنطقة وكذلك مقدار التغاير (الذي يعبر عن دقة التخمين في مستوى المياه الجوفية)في نقاط لشبكة مربعة بابعاد (٢.٥كم x ٢.٥كم)ورسم خارطة كنتورية لمستوى المياه الجوفية.

كذلك تم تخمين مستوى المياه الجوفية لمنطقة الدراسة بطريقة مربع المسافات المعكوسة (Inverse Square Distance (ISD) method) حيث وجد ان نتائج هذه الطريقة تحوي على خطاء كبير اذا ماقورنت بنتائج التخمين بطريقة الكريكنك (Kriging) .كذلك تم مقارنة الخارطة الكنتورية لمستوى المياه الجوفية في المنطقة المرسومة بطريقة الكريكنك(Kriging) مع الخارطة الكنتورية التي رسمت بطريقة مربع المسافات المعكوسة.

Aim of the study
Using a new technique (kriging) to guess the fluctuating of groundwater levels for the study area (AL-Dibdiba Basin). Also confirm the accuracy of this method by comparing the results with other methods, such as Inverse Square Distance (ISD) method.

Introduction

Groundwater is one of the major sources of water. Management of this resource is very important to meet the increasing demand of water for domestic, agricultural and industrial use. Various management measures need to know the spatial and temporal behavior of groundwater. Observed groundwater levels serve as one of the main input data in studies related to groundwater simulation for various purposes as required in water balance studies, estimation of groundwater recharge potential, in the design of drainage structures etc. However, the measurement of groundwater levels are generally carried out at spatially random locations in the field, whereas, most of the groundwater models requires these measurement at a pre-specified grid. Some interpolation method is generally employed to get these values at grid nodes. The accuracy with which this interpolation can be carried out affects the accuracy of the model output.

Literature review

Basic concepts of the kriging technique and its application to natural phenomenon have been reviewed by the ASCE Task Committee (1990a, b). Kriging of groundwater levels was carried out by Delhomme (1978); Volpi and Gambolati (1978); Aboufirassi and Marino (1983); Virdee and Kottegoda (1984); Kumar (1996) and Kumar and Ahmed (2003); Kumar and Remadevi (2006). In this research, application of kriging to interpolate the groundwater levels, as observed in the part of Dibbdiba hydraulic basin has been shown.

Methodology

Kriging is a technique of making optimal, unbiased estimates of regionalized variables at unsampled locations using the structural properties of the semivariogram and the initial set of data values. Kriging takes into consideration the spatial structure of the parameter and hence score over other methods like arithmetic mean method, nearest neighbour method, distance weighted method, and polynomial interpolation. Also, kriging provides the estimation variance at every estimated point, which is an indicator of the accuracy of the estimated value. This is considered as the major advantage of kriging over other estimation techniques.

Spatial variations with interdependence are commonly described with a variogram (Isaaks and Srivastava, 1989).In geostatictics, the concept of variance from classic is extended to semi variance .Considering a transect with equally spaced samples and measurements of ground water level z, a set of values $z(x_1)$, $z(x_2)$ …$z(x_n)$ at location x_1, x_2…x_n were obtained .The experimental semivariance $\gamma^*(h)$ is estimated as:

$$\gamma^*(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[z(x_i) - z(x_i + h)\right]^2 \qquad (1)$$

Where:

$N(h)$ = the number of pairs separated by lag distance h;

$z(x_i)$ = measured variable value at point i; and

$z(x_i+h)$ = measured variable value at point i+h.

Experimental semivariograms were calculated for ten months between the years 2002-2003 using the computer software (Geostatistics for the environmental sciences) Ver. (5.1).A lag distance of 2.5km and a tolerance of 1.25km were used for the calculation of semivariogram.

The experimental semivariograms were fitted with various theoretical models like spherical, exponential, gaussian, linear and power by the weighted least square method. The theoretical model that gave minimum standard error is chosen for further analysis. The adequacy of the fitted models was checked on the basis of validation tests. In this method, known as jackknifing procedure, kriging is performed at all the data points, ignoring, in turn, each one of them one by one. Differences between estimated and observed values are summarized using the cross-validation statistics (de Marsily and Ahmed 1987): mean error (ME), mean squared error (MSE), and kriged reduced mean error (KRME), and kriged reduced mean square error (KRMSE). If the semivariogram model and kriging procedure adequately reproduce the observed value, the error should satisfy the following criteria.

$$ME = \frac{1}{N}\sum_{i=1}^{N}(z^*(x_i) - z(x_i)) \cong 0 \qquad (2)$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(z^*(x_i) - z(x_i))^2 \quad \text{Minimum} \qquad (3)$$

$$KRME = \frac{1}{N}\sum\left[(z^*(x_i) - z(x_i)) \Big/ \sigma_{ki}\right] \cong 0 \qquad (4)$$

$$KRMSE = \frac{1}{N}\sum_{i=1}^{N}\left[(z^*(x_i) - z(x_i))^2 \Big/ \sigma_{ki}^2\right] \cong 1 \qquad (5)$$

Where, z*(xi), z (xi) and σ_{ki}^2 are the estimated value, observed value and estimation variance, respectively, at points xi . N is the sample size. As a practical rule, the MSE should be less than the variance of the sample values and KRMSE should be in the range 1±2√2/N.

In all interpolation techniques, interpolated value of z at any point x_0 is given as the weighted sum of the measured values i.e.

$$z^*(x_0) = \sum_{i=1}^{N}\lambda_i z(x_i) \qquad i = 1, 2, 3, \ldots\ldots\ldots, N \qquad (6)$$

Where:

λ*i* is the weight for the observation z at location xi. In kriging, the weights λi are calculated by equation (7) so that z*(x_0) is unbiased and optimal (minimum squared error of estimation).

$$\sum_{j=1}^{N}\lambda_j\gamma(x_i, x_j) + \mu = \gamma(x_i, x_0) \qquad i = 1, 2, 3, \ldots\ldots\ldots, N$$

$$\sum_{j=1}^{N}\lambda_j = 1 \qquad (7)$$

Where:

μ = Lagrange multiplier

γ (xi, xj) = semivariogram between two points xi and xj.

The minimum squared error estimation is also a measure for the accuracy of estimates, which is known as estimation variance, or kriging variance, and is given by

$$\sigma_k^2(x_0) = \sum_{i=1}^{N}\lambda_i\gamma(x_i, x_0) + \mu \qquad (8)$$

Inverse Square Distance (ISD) method, widely used in geohydrology, was also employed to interpolate the groundwater level data. In this method, the weights λi are inversely proportional to the square of distance from the estimation point as:

$$\lambda_i = \frac{\frac{1}{(d_{oi})^2}}{\sum_{i=1}^{N}\frac{1}{(d_{oi})^2}} \qquad (9)$$

Where, d_{oi} is the distance between the sample point and the estimated point.

The case study

The Study area Al-Dibdiba basin (Fig. 1), it lies in the area between Karbala and Najaf provinces, Iraq. It located between longitude $43^0\ 30^/$ - $44^0\ 20^/$ E and latitude $32^0\ 00^/$ - $32^0\ 40^/$ N.
Tested the area within Al-Dibdiba basin by size 400 sq.km, (20km length x 20km width). This area is located between Khan Al-Rubu in the north to Khan Al-Nuss in the south and the Euphrates in the east to the strategic pipe line in the west which located in the (UTM) lines follows:
Longitude: 412 – 432
Latitude: 3575 – 3595

The area is falls within semi dry weather that characterized by rain shortage except for some of the rainy storms that covers the area from time to time (annual rainfall of about zero in August and 23.34 mm in January), with extremes of temperature (maximum upto $45C^0$ in July and minimum upto $6C^0$ in January), and very high potential evapotranspiration (6.67mm in January and 370.88mm in July), (Al-Ani, 2004).
The main soil types of the study area are Sedimentations of sand, gravel and gravelly sand with the existence of clayey lenses which are generally take the form of compacted clayey balls interfered with small amount of sand and gypsum working as agent material. The maximum depth of the formation is about 80 km in the western south of the basin, the depth decreases towards Al Saeed Tar, Al Najaf Tar and towards the river of Euphrates that considered as formation boundaries, (Al-Khateeb, 2001).

The water in the basin of Dibdiba is generally saline, but it still highly used in irrigation, since the soil is sandy which is do not allow the accumulation of slats in the upper layer of soil.
For this study, groundwater level data pertaining to ten months from June to March seasons over the years from 2002 to 2003 covering an area of 400 sq. km (Fig.1) were selected, [quoted by (Al-Ani,2004)]. Fig. 2 shows plan of study area and the location of observation wells. The descriptive statistics of the observed groundwater levels are shown in Table (1). Mean values of groundwater levels indicate decrease in groundwater level in summer season (minimum in August), and then starting rise in groundwater levels (maximum in March).There is very small change but that is due to as mean values are provided. Also study area receives very little rainfall.

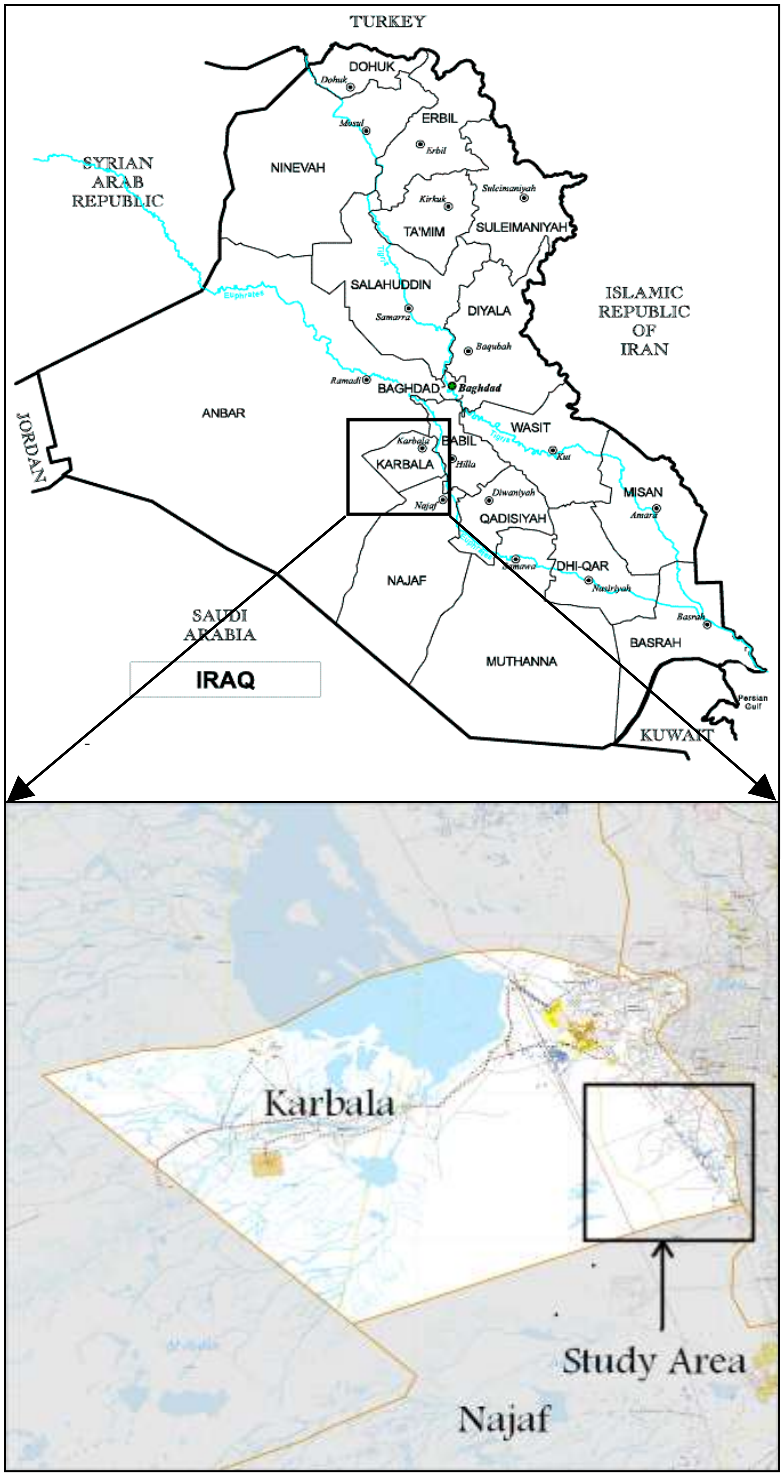


Fig.(1): Location map of study area

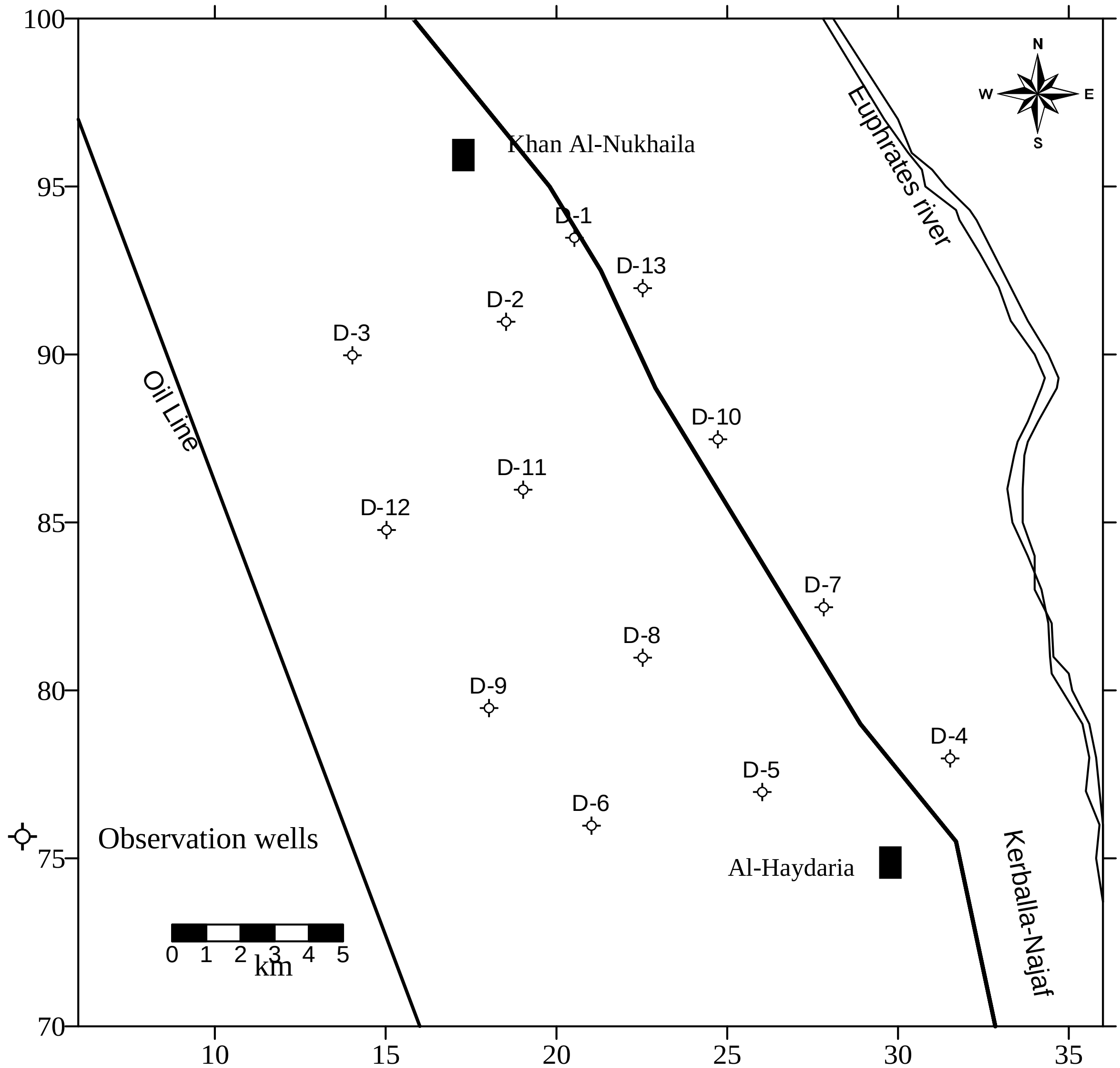


Fig. (2): Plan of study area and location of observation wells.

Table (1): Statistical parameters of select data .

No.	Data		No. of Wells	Mean (m)	Variance (m^2)	Coeff of Variance	Max. G.W.L (m.a.s.l.)	Min. G.W.L (m.a.s.l.)
	Year	Month						
1.	2002	Jun.	13	32.36	63.80	0.2468	42.5	16.5
2.		Jul.	13	32.35	63.90	0.2470	42.52	16.5
3.		Aug.	13	32.34	63.83	0.2469	42.44	16.51
4.		Sep.	13	32.38	64.15	0.2473	42.66	16.52
5.		Oct.	13	32.44	63.89	0.2464	42.7	16.62
6.		Nov.	13	32.47	64.03	0.2464	42.84	16.66
7.		Dec.	13	32.51	64.34	0.2467	42.92	16.6
8.	2003	Jan.	13	32.54	64.11	0.2460	42.99	16.72
9.		Feb.	13	32.62	64.43	0.2460	43.05	16.75
10.		Mar.	13	32.65	64.76	0.2464	43.15	16.73

Results and Discussion

Fig. (3) Shown the experimental semivariograms and the best-fitted theoretical model for all the all data sets. For all the data sets, Gaussian model resulted in the minimum standard error and so considered the best-fit model. The theoretical fitted gaussian semivariogram is of the form:

$$\gamma(h) = C_0 + C\left[1 - Exp\left(-\left(\frac{h}{a}\right)^2\right)\right] \qquad (10)$$

Where:

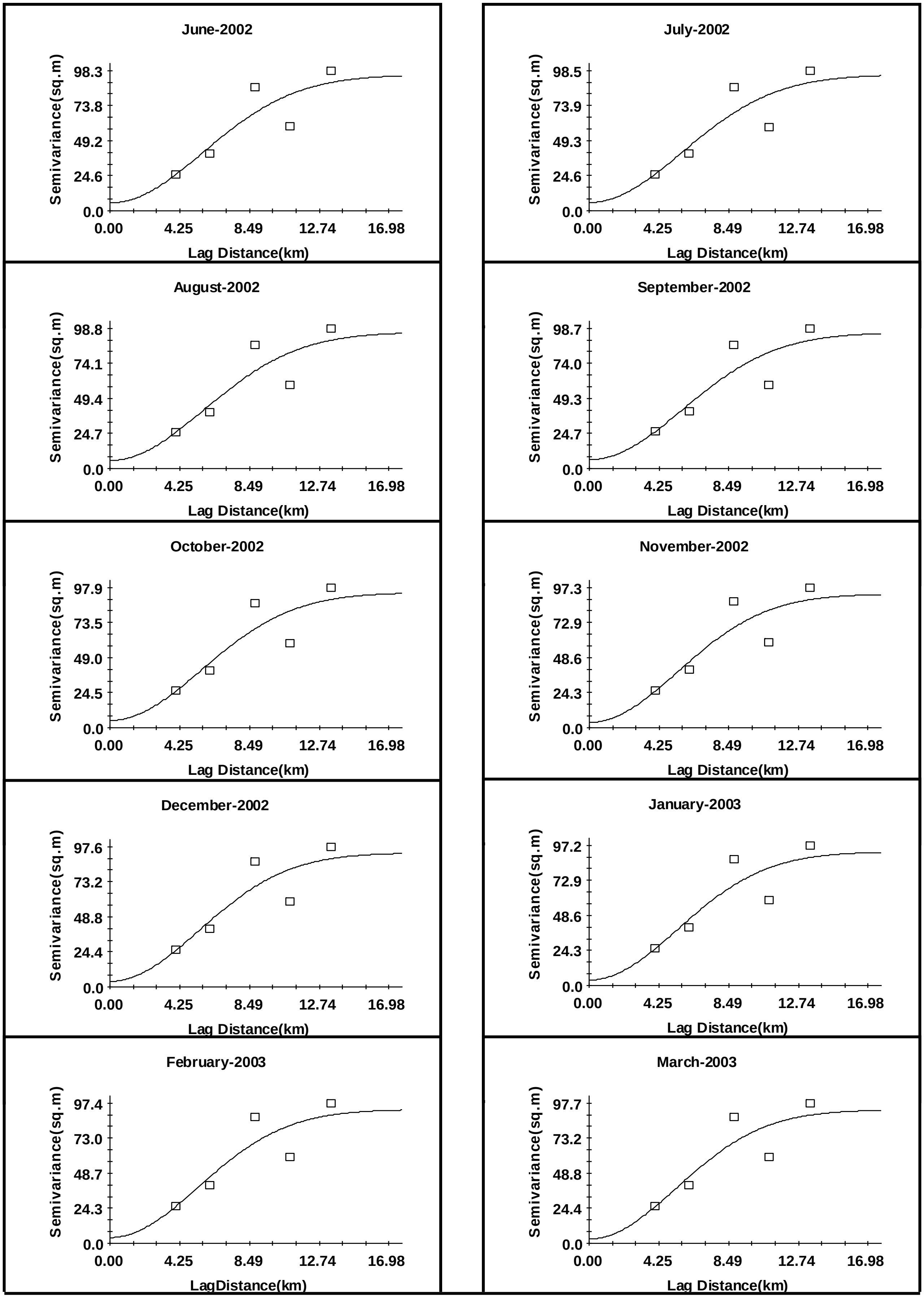
C_0= nugget effect (m^2),

C = intercept between sill and nugget effect (m^2),

a = range of influence (The distance at which samples become independent of each other) (km).

Then from the best fit model Fig.3, The theoretical fitted gaussian semivariogram for August 2002 data is of the form:

$$\gamma(h) = 5.7 + 95.98\left[1 - Exp\left(-\left(\frac{h}{8.06}\right)^2\right)\right] \qquad (11)$$



□ Experimental

—— Fitted (Gaussian)

Fig. (3): Experimental and fitted semivariogram for different data sets

The summary details of the best-fit gaussian model for ten months data set are given in Table 2. An important feature which has emerged from the best fit models (Table 2) is that while the gaussian model is the best fit for all the data set, the parameters have changed over the months. Nugget effect (C_0) shows random change between 3.0 and 6.2. The range in which the intercept C lies between 92.61 and 95.98. The range (a) exhibits constant increase through summer season (maximum in August) and it's a general decreasing trend in winter season (minimum in March).

Table (2): Parameters of fitted Gaussian models.

No.	Data		C_0	C	a
	Year	Month			
1.	2002	Jun.	5.5	95.24	7.99
2.		Jul.	5.8	95.53	8.01
3.		Aug.	5.7	95.98	8.06
4.		Sep.	6.2	95.55	8.01
5.		Oct.	5.1	94.31	7.85
6.		Nov.	3.7	92.70	7.60
7.		Dec.	3.9	93.20	7.63
8.	2003	Jan.	3.8	92.61	7.59
9.		Feb	4.0	92.85	7.58
10.		Mar.	3.0	92.85	7.49

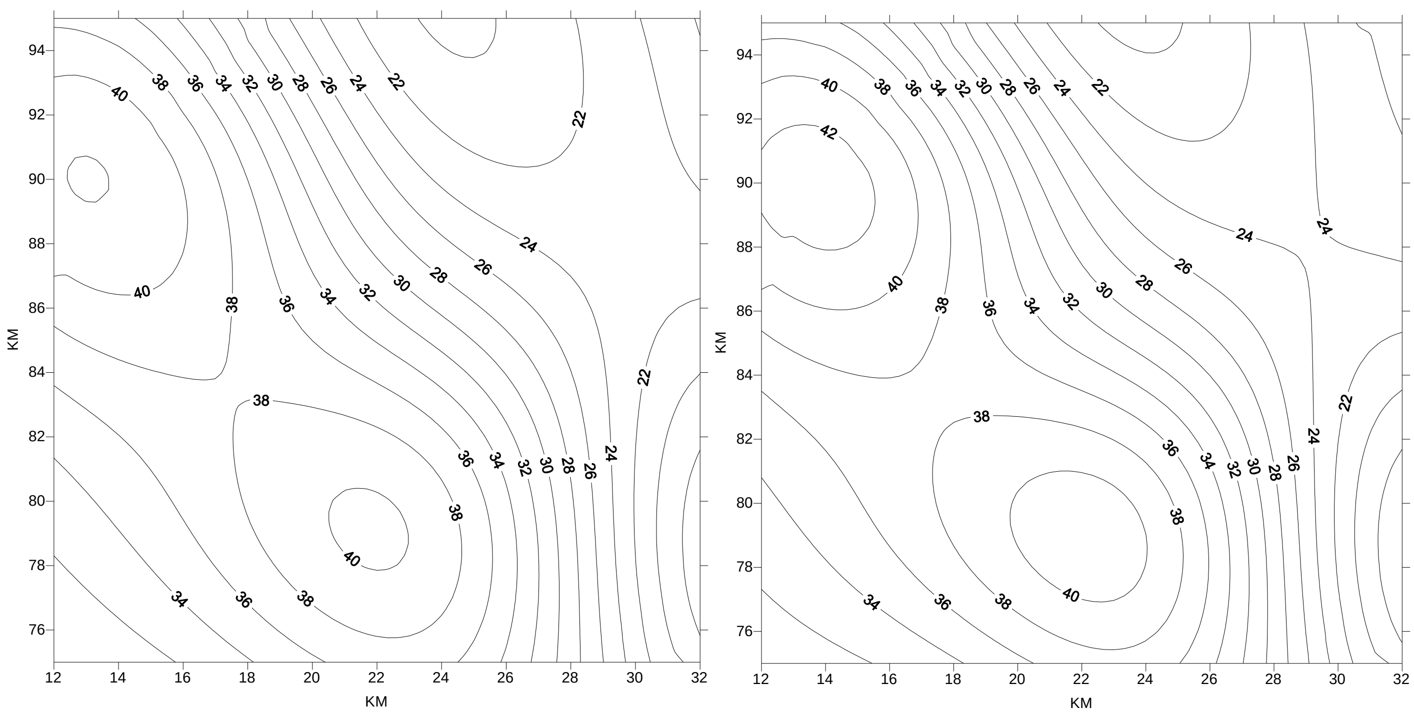
Table (3), shown the cross validation results for the ten months. Results of Jackknifing procedure for March 2003 data with the fitted gaussian model resulted in a mean error (ME) of 0.082, (which is very near to zero), mean square error (MSE) of 23.67, (which is very low as compared to the variance of the data), kriged reduced mean error (KRME) of 0.0162, (which is very near to zero) and a kriged reduced mean square error (KRMSE) of 0.934, (which is very near to 1). The above cross validation results show that the chosen model and its parameters are adequate.

Table (3): Cross validation results with gaussian model

No.	Data		ME[*1] (m)	MSE[*2] (m^2)	KRME[*3]	KRMSE[*4]
	Year	Month				
1.	2002	Jun.	0.137	23.55	0.0278	0.970
2.		Jul.	0.148	23.79	0.0300	0.983
3.		Aug.	0.137	23.54	0.0270	0.967
4.		Sep.	0.166	24.00	0.0339	0.997
5.		Oct.	0.137	23.40	0.0279	0.961
6.		Nov.	0.102	23.29	0.0204	0.939
7.		Dec.	0.108	23.65	0.0216	0.953
8.	2003	Jan.	0.108	23.25	0.0217	0.939
9.		Feb.	0.117	23.60	0.0236	0.954
10.		Mar.	0.082	23.67	0.0162	0.934

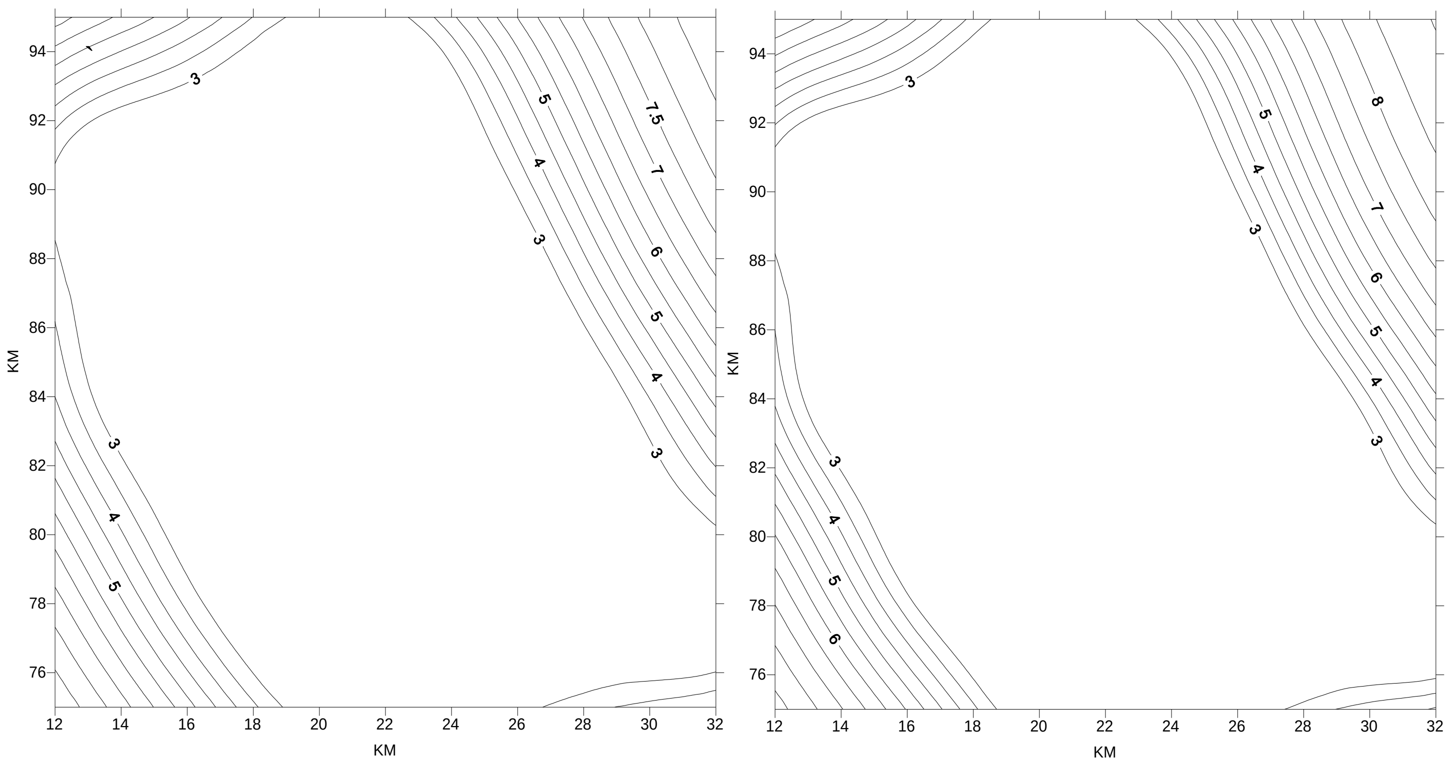
[*1] = Mean error, [*2] = Mean sq error, [*3] = Kriged reduced mean error,
[*4] = Kriged reduced mean sq error

Groundwater levels and estimation variances were calculated by kriging at the nodes of a square grid of 2.5km x 2.5km for August of 2002 and March of 2003 months. These estimated level values are used to draw the contour maps of groundwater levels and estimation variance. Fig. 4 and 5 are shown the contour maps of the groundwater levels and estimation variance obtained for August 2002 and March 2003 respectively. Fig. 5 can be interpreted as the map of the reliability of the kriged ground water level in Fig. 4. As seen from the Fig. 5, the estimation variance is low at $3m^2$ in the middle of the study area (where most of the observation points are located) and increase rapidly towards the boundaries, where no observation well is located. It indicates that the estimated groundwater level are highly reliable in the middle of the study area and at or near the boundary, these are to not reliable the same extent.



August 2002 **March 2003**

Fig. (4): Groundwater level contours maps of the study area (m) by Kriging Method.

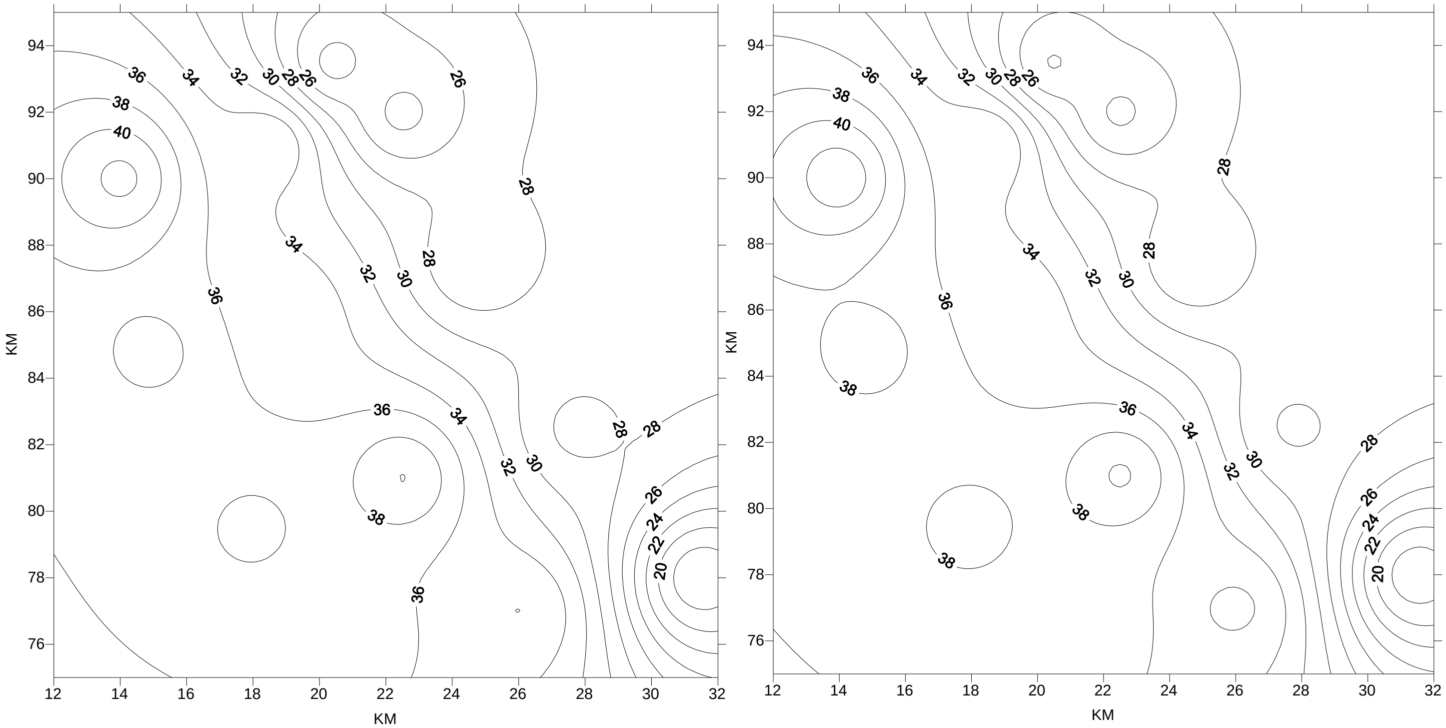


August 2002 **March 2003**

Fig. (5): Estimation variance (m^2) by Kriging method.

Fig. (6), shown the ground water level contour obtained by inverse square distance (ISD) method for August 2002 and March 2003. The contour map provided by two interpolation methods (Fig. 4 and 6) are different as kriging takes into consideration the spatial structure of the parameter and ISD method consider only distance between estimated and observed points. The comparison of ISD map with the map obtained by kriging (Fig. 4) indicated that kriged map provided a more regular gradient of the groundwater table, which seems more likely than the mound and valley combination provided by the inverse square distance method.

For more comparison of these two techniques was obtained by comparing the ME and MSE obtained by jackknifing procedure (Table 4). ISD resulted in a ME of 0.913m to 0.926m whereas kriging gave a ME of 0.082m to 0.166m. Similarly, ISD gave a MSE of 35.89m^2 to 36.32 m^2 and kriging 23.25 m^2 to 24.00 m^2. It is concluded that for this study, kriging performed better than the inverse square distance method and more importantly.



August 2002 **March 2003**

Fig. (6): Groundwater level contours (m) by Inverse Square Distance Method.

Table (4): Comparison of errors of two interpolation methods.

No.	Data		ME*1(m)		MSE*2(m^2)	
	Year	Month	ISD*3	K*4	ISD	K
1.	2002	Jun.	0.924	0.137	36.01	23.55
2.		Jul.	0.923	0.148	36.17	23.79
3.		Aug.	0.926	0.137	36.01	23.54
4.		Sep.	0.924	0.166	36.32	24.00
5.		Oct.	0.922	0.137	35.94	23.40
6.		Nov.	0.918	0.102	35.91	23.29
7.		Dec.	0.917	0.108	36.19	23.65
8.	2003	Jan.	0.913	0.108	35.89	23.25
9.		Feb.	0.919	0.117	36.12	23.60
10.		Mar.	0.915	0.082	36.22	23.67

*1 Mean error, *2 Mean square error, *3 Inverse square distance, *4 Kriging

Recommendation

1. Kriging method gave very good accurate guess fluctuations in groundwater levels (especially in center of the study area).
2. Kriging methods gave more accurate results through the Inverse Square Distance (ISD) method.
3. The level of groundwater near the surface in the west of the study area reach (42m.a.s.l) in the winter, and then sliding towards the east to reach a level (22m.a.s.l) in the summer.

Conclusions

In this study, kriging, a type of geostatistical techniques, is applied to the groundwater level data of minimum recharge season (August) and maximum recharge season (March) over a period of ten months in years (2002-2003). The gaussian model is found to the best model representing the spatial variability of groundwater level data over the months. The modeling results indicate that the kriged groundwater levels satisfactorily matched the observed groundwater levels. The degree of difference between the kriged values and the estimates using ISD are significantly high. Also, kriging out performs ISD in giving reliability indices and in the present study the reliability of the estimates is high as indicated by low level of variance.

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