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Study of some topological concepts in bitopological spaces

دراسة بعض المفاهيم التبولوجية في الفضاء ثنائي التبولوجي

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Abstract:
Anew definition of bitopological space is introduce in this paper with its separation axiom and continuity . connected and compact set are defined with some theorems and also $T_{1/2}$,semi $T_{1/2}$,semi $T_{1/3}$, spaces are defined in this paper .

ملخص البحث :
يقدم هذا ا البحث تعريف جديد للمجموعة المفتوحة في الفضاء ثنائي التبولوجي (X,P_1,P_2)في هذا الفضاء عرفنا الترابط والتراص و الاستمرارية وقمنا بدراسة بعض العلاقات بينها، بلاضافة إلى ذلك قمنا بتعريف فضاء $T_{1/2}$ و فضاء semi$T_{1/2}$ و فضاء semi$T_{1/3}$ في الفضاء ثنائي التبولوجي ودراسة بعض خواصها فيه .

1- introduction :

A bitopological space (X,p_1,p_2)[J.C.Kelly "bitopological spaces", 1963] is anon-empty set X with two topologies p_1 and p_2 on X .

[Caldas,M, "semi-$T_{1/2}$-space" 1994] defined the concept of anew class of topological spaces called semi –$T_{1/2}$ spaces.

[Kumar ,"Between semi closed set and semi-priclosed set" 1991] introduced a new class of spaces namely semi- $T_{1/3}$ spaces he proved that the class of semi T183 spaces properly contains the class of semi-$T_{1/2}$ spaces and he defined anew maps namely ψ-continuity and he discuss the relation between this map and semi –continuity [Biswas,N "characterization of semi-continuous function",1970], g-continuity[Caldas,M,"on g-closed sets and g-continuous mapping" ,1993], [R.Devi, H.Maki and K.Balachandran,"semi-generalized closed map and generalized closed map" ,1993], [K.Balachandran ,P.Sundaram and H.Maki,"semi-generalized continuous maps and semi-$T_{1/2}$", 1991], sg-continuity[P.sundaram, H.Maki, K. Balachandran,"on generalized continouos maps in topological spaces",1991], [P.Bhattacharya and B.K.Lahiri,"semi-generalized closed sets in topology" 1987] and gs-continuity[Miguel Caladas Cueva and Ratnesh Kumar saraf ,"A reaserch on charcterization of semi-$T_{1/2}$-spaces " 2000].

The purpose of this paper that is give a new definition of these concepts in Bitopological spaces.

2- Basic definitions and theorems

We would like to point out that all the definitions provided in this research has been formulated by researchers by adoption of their counterparts in topological spaces.

Definition (2-1)

Let (X,p_1,p_2) be a bitopological space then a subset A of X is said to be open iff there exists T_i –open set U such that $U\subseteq A$ and $\cap cl_{pi}(U) \subseteq A$,I=1,2 , this open set denoted by ∂-open set

Example(2-2):

let X={a,b,c,d} ,T_1=X,ϕ,{a},{b},{a,b}} ,T_2={X,ϕ,{a},{c},{a,c}} then
∂-open set={X,ϕ,{a,b,d},{b,c,d},{a,c,d}}

Remark(2-3)

The intersection of two ∂-open sets is not necessary ∂-open while the union is ∂-open set

Proof: let $\{A_\lambda : \lambda\in\Lambda\}$ be any arbitrary collection of ∂-open set , then there exist T_i-open set U_λ suc that $U_\lambda \subseteq A_\lambda$ and $\cap cl_{pi}(U_\lambda) \subseteq A_\lambda$,I=1,2 for each λ .

Since

$\cup_{\lambda\in\Lambda}(\cap_{i=1,2}clp_i(U_\lambda))$

$=(cl_{p1}(\cup(U_\lambda))\cap(clp_2((\cup(U_\lambda))=\cap_{i=1,2}clp_i((\cup_{\lambda\in\Lambda}(U_\lambda))\subseteq\cup_{\lambda\in\Lambda}A_\lambda$

Remark (2-4)

-The set of all ∂-open sets is not a topological space.
- if a is pi-closed set for i=1,2 then A is ∂-open set

Example(2-5):

Let X=a,b,c,d} and T={Xϕ , {a},{b,c}} , T={X, ϕ,{a},{c,d}}
∂-open set ={ X,ϕ , {a}, {a,b} ,{a,c} ,{a,d} , ,{a,b,c} , {a,b,d} , {b,c,d}}
Since {a,b,d}$\cap${c,b,d}={{b,d} which is not ∂-open set then the set of all ∂-open sets is not topological space .

Notation (2-6)

let (X,p_1,p_2) be a bitopological space and let Y be a subset of X then a subset B of Y is said to be ∂_Y-open (the open set in (Y,p_{1Y},p_{2Y})) iff there exist p_{iY} open set W such that $W\subseteq B$ and $\cap\ clp_{iY}(W)\subseteq B$

Theorem (2-7)

if A is ∂-open set and B is ∂_Y-open set where Y is a subset of X which is not ∂-open set then $A\cap B$ is ∂_Y-open set and $A\cup B$ is ∂-pen set

proof:- since A is ∂-open set then there exist T_i-open set U such that $U\subseteq A$ and $\cap cl_{pi}(U)\subseteq A$ and since B is ∂_Y-open set then there exist T_{iY}-open set such that $W\subseteq B$ and $\cap cl_{piY}(W)\subseteq B$, then $[\cap cl_{pi}(U)]\cap[\cap cl_{piY}(W)]\subseteq A\cap B$ and then $clp_1(U\cap W)\cap Y\cap clp_2(U\cap W)\cap Y\subseteq(clp_1(U)\cap clp_1(W))\cap Y\cap(clp_2(U)\cap clp_2(W))\cap Y\subseteq A\cap B$, from the last statement we get the result .

$[\cap cl_{pi}(U)]\cup[\cap cl_{piY}(W)]\subseteq A\cup B$, then

$(clp_1(U)\cap clp_2(U))\cup(clp_{1Y}(W)\cap clp_{2Y}(W))$

$=(clp_1(U)\cup clp_{1Y}(W))\cap(clp_1(U)\cup clp_{2Y}(W))\cap(clp_2(U)\cup clp_{1Y}(W))\cap(clp_2(U)\cup p_{2Y}(W))$

$=[clp_1(U\cup W)\cap clp_2(U\cup W)]\cap[clp_1(U)\cup Y]\cap[clp_2(U)\cup Y]$

$\cap[clp_1(U)\cup clp_{2Y}(W)]\cap[clp_2(U)\cup clp_{1Y}(W)]\subseteq A\cup B$ and then

$\cap cl_{pi}(U\cup W)\subseteq A\cup B$.

Remark (2-8)

By theorem (1-5) we can define the subspace Y of a topological space X as follow:
if Y is a subset of X such that Y is ∂-open set then ∂_Y-open={$G\cap Y$: G is ∂-open set in X }

Notation (2-9)

Let (X,p_1,p_2), (Y,w_1,w_2) are two bitopological spaces , a function
f: $(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is said to be ∂-continuous iff $f^{-1}(V)$ is ∂-open(∂-closed) set in X for each V is ∂-open(∂-closed) set in Y

Theorem (2-10)

A function f: $(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is said to be
1- ∂-open mapping iff f(U) is ∂-open whenever U is ∂-open set in X
2- ∂-closed iff f(U) is ∂-closed in Y whenever U is ∂-closed set in X
3- ∂-bi continuous iff f is ∂-open and ∂-continues
4- ∂-homeomorphism iff f is bijection mapping , ∂-continuous and f^{-1} is ∂-continuous

Proof: we using the same proof in [1] with replacing the open set(closed set) by ∂-open(∂-closed) set respectively.

Notation (2-11)

Let (X,p_1,p_2) be a bitopological and A,C are two subsets of X we say that A and C are separated iff $A\cap cl_{pi}(C)=\varnothing$ and $C\cap cl_{pi}(A)=\varnothing$ for I=1,2

Remark(2-12)

If A ,C are pi-separated, for I=1 or 2 then it is not necessary that A,C are separated in (X,p_1,p_2)

Example(2-13)

In example(2-5) {a} and {b,c} which are T_1-separated but are not separated in (X,p_1,p_2)

Theorem(2-14)

let Y be a subspace of a bitopological space X such that Y is ∂-open set and let A,C be two subset of Y . then A , C are pi-separated if and only if they are p_{iY}-separated.

Proof: by using the relation clip(A)$\cap$Y=$clpi_Y$(A) and clip(C)$\cap$Y=$clpi_Y$(C) the result exist

Notation(2-15)

Let (X,p_1,p_2) be a bitopological space . a subset A of X is said to be ∂-disconnected if and only if it is the union of two non-empty pi-separated subsets for I=1,2. A is said to be ∂-connected iff it is not ∂-disconnected .

Example(2-16)

X={a,b,c} T_1={X,ϕ,{a},{b,c}} T_2={X,ϕ,{a,c}} then X is ∂-disconnected and {a,c} is ∂-connected

Theorem(2-17)

Let Y be a subspace of a bitopological space X and A subset of Y such that Y is ∂-open set then A is ∂-disconnected iff it is ∂_Y-disconnect and A is said ∂-connected iff it is ∂_Y-connected

Proof: by theorem (2-14) A is union of pi-separated sets iff it is pi_Y-separated sets and hence the result .

Notation (2-18)

Let (X,p_1,p_2) be a bitopological space and A be a subset of X is said to be compact iff every pi-open cover of A has pi-finite sub cover for I=1 or 2

Example(2-19)

Let X={a,b,c} , T_1={X,ϕ,{a},{c},{a,c}}, T_2={X,ϕ,{a},{b,c}} then (X,p_1,p_2)
Is compact with respect to T1 or T2.

Remark(2-20)

If (X,p_1) or (X,p_2) is compact then (X,p_1,p_2) is compact

Proof : by the definition of compactness in this paper clearly that if we take any open cover of X then this cover we take from p_1 or p_2 and hence the open cover containing a finite sub cover.

Theorem(2-21)

Let Y be subspace of a bitopological space X such that Y is ∂-open set and let D$\subset$Y. then D is compact relative to X if and only if D is compact relative to Y.

Let D is compact relative to X and $\{G_{\lambda\,:\,\lambda\in\Lambda}\}$ is pi_Y-open cover of A in Y then A$\subseteq\cup\{G_{\lambda\,:\,\lambda\in\Lambda}\}$, then there xist V_λ which is pi-open sets in X such that $G_\lambda=V_\lambda\cap Y$ for every $\lambda\in\Lambda$ and then D$\subseteq\cup\{V\lambda :\lambda\in\Lambda\}$ then $\{V\lambda :\lambda\in\Lambda\}$ is open cover of D in X and since D is compact relative to X then there exist $\lambda1,\lambda2$,-----, λ_n such that

D $\subseteq V\lambda_1\cup V\lambda_2\cup$-----$\cup$ $V\lambda_n$ and then D $\subseteq[V\lambda_1\cup V\lambda_2\cup$-----$\cup$ $V\lambda_n]\cap Y$ = $G\lambda_I$ and there for D is compact relative to Y. and in the same way we proof the converse.

Theorem(2-22)

Let (X,p_1,p_2)be a compact bitopological space then a pi-closed F of X is compact for i=1or2 respectively. subset

Proof:

Let C=$\{G_\lambda : \lambda\in\Lambda\}$ is pi-open cover of F where F is pi-closed set in X let
D=$\{ G_\lambda : \lambda\in\Lambda\}\cup\{X-F\}$ form pi-open cover of X since X is compact the D has a finitely sub collection of D covers X and then covers F, and then F is compact.

Notation (2-23)

a bitpological space (X,p_1,p_2) is say to be

1- ∂-T_0 iff for each x , y in X thee exist ∂-open set U such that x$\in$U , y$\notin$U
2- ∂-T_1 iff for each two distinct point x,y there exist two ∂-open sets U,W such that x$\in$U , y$\notin$W

3- ∂-T_2 space iff for each two distinct point x, y there exist two ∂-open sets U,W such that $x\in U$, $y\notin W$ and $U\cap W=\varnothing$
4- ∂-$T_{2\ 1/2}$ space iff any two points of X can be separated by ∂-closed neighborhood
5- ∂-regular iff for each ∂-closed set F and $y\notin F$ there exist two ∂-open sets U,W such that $F\subseteq U$, $y\in W$ and $U\cap W=\varnothing$
6- ∂-T3 iff it is ∂-T_1 and regular
7- ∂-completely regular space if and only if for an ∂-closed set F in X and x any point in X not in F there exist ∂-continuous map $f:X\to[0,1]$ such that $f\{F\}=\{1\}$, $f(X)=0$
8- ∂-$T_{31/2}$ space iff this space is ∂-completely regular and ∂-housdorff
9- ∂-normal iff for each two disjoint pi-closed sets F,H for i=1,2 there exist two ∂- open set U,W such that $F\subseteq U, H\subseteq W$ and $U\cap W=\varnothing$
10- T_4 space if and only if X is ∂-T_1 and ∂-normal space
11- completely normal space if and only if for a separated sets A,C there exist two ∂-open sets U,W such that $A\subseteq U$ and $C\subseteq W$, $U\cap W=\varnothing$ and we denoted this space by [CN]
12- ∂-T_5 space iff it is both ∂-completely normal and hausdorff

Theorem(2-24)

Let (X,p_1,p_2)be a bitopological space then every ∂-complete normal space is ∂-normal
Proof:
Let A,B are two disjoint pi-closed sets then $A\cap clpi(B)=\phi$ and $B\cap clpi(A)=\phi$ and there for A,B are two disjoint separated sets in X and since X is ∂-complete normal space there exist two disjoint ∂-open set U,W such that $A\subseteq U$ and $B\subseteq W$, then (X,p_1,p_2) is ∂-normal space.

Theorem (2-25) let (X,p_1,p_2) be a bitopological space and Y be a subset of X such that $Y\in\partial$-open(X) then ∂-T_0 , ∂-T_1 , ∂-T_2 , ∂-regular , ∂-T_3 , ∂-T_5 are hereditary property
Proof: see [1] with replacing every open set (closed set) by ∂-open set (∂-closed set) respectively .

3- new main results

Notation (3-1)
Let (X,p_1,p_2) be a bitopological space then a subset A of X is said to be
a- semi-open set if $A\subseteq cl_{pi}(int_{pi}(A))$ and semi-closed if $int_{pi}(cl_{pi}(A))\subseteq A$ for i=1or2.
b- generalized closed set (briefly g-closed set) iff $cl_{pi}(A)\subseteq U$ where $A\subseteq U$ and U is ∂-open set in (X,p_i) for i=1or2.
c- semi-generalized closed (briefly sg-closed set) if $scl_{pi}(A)\subseteq U$ wheneverA$\subseteq$U and U is semi-open set in (X,p_i) for i=1or2
d- generalized semi-closed(briefly gs-closed) iff $scl_{pi}(A)\subseteq U$,where $A\subseteq U$ and U is ∂-open set in (X,p_i) for i=1or2.
e- ψ-closed set if $scl_{pi}(A)\subseteq U$ whenever $A\subseteq U$ and U is sg-open set in (X,p_i) for i=1or2

Example(3-2)
Let X={a,b,c} and $T_1=\{X, \phi,\{a,b\}\}$ $T_2=\{X ,\phi, \{a\},\{b,c\}\}$ then {a,c} is ψ-closed
And {b} is sg-open set and sg-closed set but {b,c} is not ψ-closed set .

proposition (3-3)
A subset A of X is semi-open set (respectively, semi-closed , sg-closed, gs-closed ,g-closed,ψ-closed) in (X,p_i) for i=1or2 then it is semi-open set(respectively ,semi-closed ,sg- closed , gs-closed ,g-closed , ψ-closed) in (X,p_1,p_2)
proof:
let A is pi-semi-open (respectively, pi-semi-closed , pi-sg-closed, pi-gs-closed ,pi-g-closed,pi-ψ-closed)set then $A\subseteq cl_{pi}(int_{pi}(A))$[respectively, $int_{pi}(cl_{pi}(A))\subseteq A$, $scl_{pi}(A)\subseteq U$ where $A\subseteq U$ and U is pi-∂-open set, $scl_{pi}(A)\subseteq U$ where $A\subseteq U$and U is pi-semi-open set, $cl_{pi}(A)\subseteq U$ where $A\subseteq U$ and U is pi-∂-open set , $scl_{pi}(A)\subseteq U$ whenever $A\subseteq U$ and U is pi-sg-open set] and then A is semi-open set (respectively, semi-closed , sg-closed, gs-closed ,g-closed,ψ-closed) in (X,p_1,p_2).

Notation (3-4)

A bitopological space (X,p_1,p_2) is said to be
a- $T_{1/2}$ space if every g-closed set is ∂-closed set
b-Semi-$T_{1/2}$ space [Caldas,M.,1994] if every sg-closed set is semi- closed set
c- semi-$T_{1/3}$ space [M.K.R.S.Veera Kumar,1991] if every ψ-closed set in it is semi-closed

Example(3-5)

Let $X=\{a,b,c\}$ and $T_1=\{X,\phi,\{a\},\{b,c\}\}$ $T_2=\{X,\phi,\{b\},\{b,c\}\}$ then (X,p_1,p_2) is semi-$T_{1/3}$ space and semi $T_{1/2}$ space

proposition (3-6)

If (X,p_i) is semi-$T_{1/3}$ space (respectively $T_{1/2}$ space, semi- $T_{1/2}$space) for i=1or2 then (X,p_1,p_2) is semi-$T_{1/3}$ space (respectively $T_{1/2}$ space, semi -$T_{1/2}$ space)

proof: let A be a ψ-closed set (respectively, g-closed set , sg-closed set) in(X,p_1,p_2) by theorem (3-5) A is ψ-closed set (respectively , g-closed set , sg-closed set) in (X,p_i) , i=1or2 and since (X,p_i) is semi $T_{1/3}$ space (respectively $T_{1/2}$ space , semi-$T_{1/2}$ space) A is semi- closed set (respectively, ∂-closed set , semi close set) then (X,p_1,p_2) is semi $T_{1/3}$ space (respectively , $T_{1/2}$ space , semi $T_{1/2}$ space) .

Remark:(3-7)[7]

1-every semi closed set and thus every ∂-closed set is ψ-closed set
2- every ψ-closed [M.K.R.S.Veera Kumar,1991]is sg-closed set and also gs-closed set 3- every semi-closed set is sg-closed set

proposition (3-8)

every semi-$T_{1/2}$ space is semi- $T_{1/3}$ space

proof:- let A be a ψ-closed set in X , by remark (3-7) A is sg-closed set and since X is semi $T_{1/2}$ space A is semi closed set and then (X,p_1,p_2) is semi $T_{1/3}$ space.
the converse is not true as we show in the following example

Example(3-9)

let $X=\{a,b,c\}$, $p_1=\{X,\emptyset,\{a\},\{b,c\}\}$, $p_2=\{X,\emptyset,\{a,b\}\}$
Then (X,p_1,p_2) is not semi-$T_{1/2}$ space since $\{a,c\}$ is sg-closed but not semi closed set ,however (X,p_1,p_2) is semi $T_{1/3}$ space .

Definition (3-10)[6]

For any subset E of (X,p_1,p_2) , $scl_{pi}*(E)= \cap\{A:E\subseteq A$ such that $A\in sd(X,p_1,p_2)\}$ where $sd(X,p_1,p_2)=\{A:A\subset X$ and A is sg-closed in (X,p_1,p_2) } and $SO(X,p_1,p_2)^* =\{B:scl_{pi}*(B^c)=B^c\}$ for i=1or2.

Proposition (3-11)

A bitopological space (X,p_1,p_2) is a semi-$T_{1/2}$-space if and only if
$SO(X,p_1,p_2)=SO(X,p_1,p_2)^*$.

Proof

Since the semi closed sets and the sg-closed sets are coincide by assumption $scl_{pi}(E)=scl_{pi}*(E)$ holds for every subset E of (X,p_1,p_2)there for we have $SO(X,p_1,p_2) =SO(X,p_1,p_2)^*$

Conversely let A is sg-closed set of (X,p_1,p_2) . then we have $A=scl_{pi}*(A)$ and hence $A^c\in SO(X,p1,p2)$ thus A is semi close set there for (X,p_1,p_2) is semi $T_{1/2}$ space

Proposition (3-12)

A bitopological space (X,p_1,p_2) is semi $T_{1/2}$ space if and only if for each $x\in X$, $\{x\}$ is semi open or semi closed

Proof

Suppose that for some $x \in X$, $\{x\}$ is not semi closed .since X is the only semi open set containing $\{x\}^c$, the set $\{x\}^c$ is sg-closed set so it is semi closed set in the semi$T_{1/2}$ space (X,p_1,p_2) ,therefore $\{x\}$ is semi open set

Conversely , since $SO(X,p_1,p_2) \subseteq SO(X,p_1,p_2)^*$ holds by theorem (3-11)it is enough to prove that $SO(X,p_1,p_2)^*\subseteq SO(X,p_1,p_2)$. let $E \in SO(X,p_1,p_2)^*$ .Suppose that $E \notin SO(X,p_1,p_2)$.then $scl_{pi}*(E^c)=E^c$ and $scl_{pi}(E^c)\neq E^c$ hold. There exist a point x of x such that $x\in scl_{pi}(E^c)$ and $x \notin E^c(=scl_{pi}*(E^c))$.since

$x \notin sc_{pi}*(E^c)$ there exist sg-clsed set A such that $x \notin A$ and $E^c \subset A$ .by the hypothesis the singleton $\{x\}$ is semi-open set or semi-closed set.

Now if $\{x\}$ is semi-open set ,since $\{x\}^c$ is semi closed set with $E^c \subset \{x\}^c$, we have $scl_{pi}(E^c) \subset \{x\}^c$, i.e, $x \notin scl_{pi}(E^c)$. this contradicts the fact that $x \in scl_{pi}(E^c)$. therefore $E \in SO(X,p_1,p_2)$.

If $\{x\}$ is semi closed set , since $\{x\}^c$ is semi-open set containing the sg-closed set A ($\supset E^c$) we have $scl_{pi}(E^c) \subset scl_{pi}(A) \subset \{x\}^c$. therefore $x \notin sclpi(E^c)$. this is contradiction . therefore $E \in SO(X,p_1,p_2)$.

Hence in both cases we have $E \in SO(X,p_1,p_2)$, i.e, $SO(X,p_1,p_2)^* \subseteq SO(X,p_1,p_2)$.

Proposition (3-13)

(X,p_1,p_2) is semi-$T_{1/2}$ space if and only if every subset of X is the intersection of all semi-open sets and all semi-closed sets containing it.

Proof

Let (X,p_1,p_2) be a semi-$T_{1/2}$ space with $B \subset X$ arbitrary . then $B = \cap\{\{x\}^c : x \notin B\}$ is an intersection of semi-open sets and semi-closed sets by the above theorem the results follow.

Conversely , for each $x \in X$, $\{x\}^c$ is the intersection of all semi-open sets and all semi-closed sets containing it. Thus $\{x\}^c$ is either semi-open sets or semi-closed set and hence X is semi-$T_{1/2}$ space.

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