

t γ -Open sets and Separation axioms

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Abstract:

In this paper by considering two types of operations on a topological space (X, τ) , t -operation and γ -operation, the researcher introduces the concept of $t\gamma$ -open set (which is analogue to the concept of semi open set), and we study the concepts of $t\gamma$ - T_i spaces ($i=0, \frac{1}{2}, 1, 2, b, d, g$) related to the new concept of $t\gamma$ -open sets and we study the relations between them.

1. Introduction.

Krishnan and Balachandran 1998 introduced the concept of γ -semi open sets in a topological space (by using the definition of operation γ as a mapping from τ into $P(X)$ with the condition $U \subseteq \gamma(U)$ for each $U \in \tau$) and the concepts of γ -semi T_i spaces ($i = 0, \frac{1}{2}, 1, 2$) and they studied the relation between them, also they introduced the concepts of γ - T_b and γ - T_d spaces using the concept of γ -gs.closed sets in a topological space.

The researcher in [1] gave a different definition of operation on a topological space (X, τ) and called it t -operation (as a mapping from $P(X)$ into τ with the condition $t(U) \subseteq U$ for each $U \subseteq X$ and $t(X)=X$), and he studied the concepts of t -open, t -closed, t -g.closed sets in (X, τ) , together with the concepts of t - T_i , t -semi T_i ($i = 0, \frac{1}{2}, 1, 2$), t - T_b and t - T_d spaces.

In this paper, in section 2, we give a necessary preliminaries consisting the main concepts and theorems coming in the papers of Krishnan and Balachandran 1998 and the researcher in [1] about γ -open and t -open sets with related concepts. In section 3 we define the $t\gamma$ -open set by considering a γ -operation and a t -operation on a topological space (X, τ) , and we study the related concepts; $t\gamma$ -closed set, $cl_{t\gamma}$, $t\gamma$ -generalized closed set, generalized $t\gamma$ -closed set investigating the relations between them. Finally we define $t\gamma$ - T_i spaces ($i=0, \frac{1}{2}, 1, 2, b, d, g$) and prove many theorems about them.

2 Preliminaries.

Throughout this work by $int A$ and $cl A$ we mean the interior and the closure sets of A with respect to the given topology τ respectively.

2.1 Definition [3]: A γ -operation γ on a topological space (X, τ) is a mapping from τ into the power set $P(X)$ such that $V \subseteq \gamma(V)$ for each $V \in \tau$.

2.2 Definition [1]: A t -operation t on a topological space (X, τ) is a mapping from $P(X)$ into τ such that $t(A) \subseteq A$ for each $A \subseteq X$ and $t(X)=X$.

2.3 Definition [3]: Let γ be a γ -operation on a topological space (X, τ) . A subset G of X is said to be γ -open set if for each x in G there exists $U \in \tau$ such that $x \in U$ and $\gamma(U) \subseteq G$. The collection of all γ -open sets in (X, τ) is denoted by τ_γ .

2.4 Remark. i) $\tau_\gamma \subseteq \tau$.

ii) $\emptyset, X \in \tau_\gamma$.

iii) If $G_\alpha \in \tau_\gamma, \alpha \in \Lambda$, then $\bigcup_{\alpha \in \Lambda} G_\alpha \in \tau_\gamma$.

iv) If $A, B \in \tau_\gamma$, it is not necessary that $A \cap B \in \tau_\gamma$.

(i.e. τ_γ does not form a topology on X in general)

v) If γ is defined by $\gamma(V)=V$ for each $V \in \tau$, then $\tau_\gamma = \tau$.

vi) $\gamma(V) \in \tau_\gamma$ for each $V \in \tau$.

2.5 Definition [1]: Let t be a t -operation on a topological space (X, τ) . A subset G of X is said to be t -open set if for each x in G there exists $A \subseteq X$ such that $x \in t(A) \subseteq G$. The collection of all t -open sets in (X, τ) is denoted by $t-O(X)$.

2.6 Remark . i) $t-O(X) \subseteq \tau$.

ii) $\emptyset, X \in t-O(X)$.

iii) If $G_\alpha \in t-O(X), \alpha \in \Lambda$, then $\bigcup_{\alpha \in \Lambda} G_\alpha \in t-O(X)$.

iv) If $A \cap B \in t-O(X)$, it is not necessary that $A \cap B \in t-O(X)$.

(i.e. $t-O(X)$ do not form a topology on X in general).

v) If t is defined by $t(A) = \tau\text{-int}(A)$ for each $A \subseteq X$, then $t-O(X) = \tau$.

2.7 Definition [3]: Let γ be a γ -operation on a topological space (X, τ) . A subset F of X is said to be γ -closed if $X - F \in \tau_\gamma$.

2.8 Definition [1]: Let t be a t -operation on a topological space (X, τ) . A subset F of X is said to be t -closed if $X - F \in t-O(X)$.

2.9 Definition [3]: Let γ be a γ -operation on a topological space (X, τ) , $A \subseteq X$.

The set $cl_\gamma A$ is defined by, $cl_\gamma A = \{x \in X \mid U \cap A \neq \emptyset, \text{ whenever } U \in \tau_\gamma \text{ and } x \in U\}$.

2.10 Remark . Let γ be a γ -operation on a topological space (X, τ) , $A, B \subseteq X$

and $\{A_\alpha\}$ an arbitrary family of subsets of X , then

i) $cl_\gamma A = \bigcap \{F \mid F \text{ is } \gamma\text{-closed and } A \subseteq F\}$

ii) $cl_\gamma \emptyset = \emptyset$ and $cl_\gamma X = X$ iii) $cl A \subseteq cl_\gamma A$.

iv) $A \subseteq B$ implies $cl_\gamma A \subseteq cl_\gamma B$.

v) $\bigcup cl_\gamma A_\alpha \subseteq cl_\gamma \bigcup A_\alpha$ and $cl_\gamma \bigcap A_\alpha \subseteq \bigcap cl_\gamma A_\alpha$ (where $\{A_\alpha\}$ is an arbitrary family of subsets of X).

vi) A is γ -closed iff $A = cl_\gamma A$.

2.11 Remark . A similar definition can be given and a similar remark can be proved about $cl_t A$, see [1].

2.12 Definition [1]: Let t be a t -operation on a topological space (X, τ) , $A \subseteq X$.

The set t -

$\text{int} A$ is defined by, $t\text{-int}(A) = \bigcup \{U \mid U \in t-O(X) \text{ and } U \subseteq A\}$.

2.13 Remark . Let t be a t -operation on a topological space (X, τ) , $A, B \subseteq X$

and $\{A_\alpha\}$ an arbitrary family of subsets of X , then

i) $A \subseteq B$ implies $t\text{-int}(A) \subseteq t\text{-int}(B)$.

ii) $\bigcup (t\text{-int} A_\alpha) \subseteq t\text{-int}(\bigcup A_\alpha)$.

iii) $t\text{-int}(\bigcap A_\alpha) \subseteq \bigcap (t\text{-int} A_\alpha)$.

iv) A is t -open if and only if $A = t\text{-int} A$.

v) $t\text{-int} A$ is a t -open set for each A .

vi) $t\text{-int} A \subseteq \text{int} A$.

2.14 Remark . A similar definition can be given and a similar remark can be proved about $\gamma\text{-int} A$, see [3].

2.15 Remark . Let γ be a γ -operation and t be a t -operation on a topological space (X, τ) , $A \subseteq X$. Then

i) $\gamma\text{-int}(X - A) = X - cl_\gamma A$ and $cl_\gamma(X - A) = X - (\gamma\text{-int} A)$.

ii) $t\text{-int}(X - A) = X - cl_t A$ and $cl_t(X - A) = X - (t\text{-int} A)$.

2.16 Definition [3]: Let γ be a γ -operation on a topological space (X, τ) , $A \subseteq X$.

A is said to be a γ -generalized closed (shortly γ -g.closed) set if $cl_\gamma A \subseteq U$ whenever $A \subseteq U$ and U is γ -open.

2.17 Remark . Every γ -closed set is γ -g. closed, but the converse is not true.

2.18 Definition [1]: Let t be a t -operation on a topological space (X, τ) , $A \subseteq X$.

A is said to be a t -generalized closed (shortly t -g.closed) set if $cl_t A \subseteq U$

whenever $A \subseteq U$ and U is t -open .

2.18 Remark . Every t -closed set is t -g. closed ,but the converse is not true .

3. $t\gamma$ -open sets

Throughout this section we will assume that γ is a γ -operation and t is a t -operation on the topological space (X,τ) .

3.1 Definition : A subset A of X is $t\gamma$ -open if $A \subseteq cl_\gamma (t-int A)$.The collection of all $t\gamma$ -open sets in (X,τ) is denoted by $t\gamma-o(X)$.

3.2 Remark . Since $A \in t-o(X)$ implies $A = t-int A$ (2.13 iv) and since $A \subseteq cl_\gamma A$ For each A (2.10 iii) ,therefore $t-o(X) \subseteq t\gamma-o(X)$.

3.3 Remark . The concepts of $t\gamma$ -open and τ -open sets are independent for example:

i) Let (X,τ) be any topological space , t a t -operation on (X,τ) defined by $t(A)=\phi$ if $A \neq X$, $t(X)=X$,and γ is a γ -operation on (X,τ) defined by $\gamma(V)=V$ for each $V \in \tau$, then $t-o(X) = \{\phi, X\}$, $\tau_\gamma = \tau$ and $t\gamma-o(X) = \{\phi, X\}$,that is τ is not subset of $t\gamma-o(X)$.

ii) Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}\}$, $\gamma(V)=V$ for each $V \in \tau$ and $t(A) = \phi$ if $A \notin \tau$, $t(A) = A$ if $A \in \tau$, then $t-o(X) = \tau_\gamma = \tau$ but $t\gamma-o(X) = \{\phi, X, \{a\}, \{a,b\}, \{a,c\}\}$ that is $t\gamma-o(X)$ is not subset of τ .

3.4 Remark . If γ is defined by $\gamma(V)=V$ for each $V \in \tau$, and t is defined by $t(A) = intA$ for each $A \subseteq X$, then by 2.4(v) and 2.6(v) $\tau_\gamma = t-o(X) = \tau$ and so $cl_\gamma A = clA$, $t-intA = intA$. Hence $t\gamma-o(X) = SO(X)$,the collection of all semi open sets in (X,τ) .

In general the concepts of $t\gamma$ -open set and semi open set are independent .

3.5 Definition. A set F is said to be $t\gamma$ -closed set if $X-F$ is $t\gamma$ -open .

3.6 Remark . F is $t\gamma$ -closed iff $\gamma-int(cl_t F) \subseteq F$.

Proof: F is $t\gamma$ -closed iff $X-F$ is $t\gamma$ -open (3.5)

iff $X-F \subseteq cl_\gamma (t-int (X-F))$ (3.1) , but $cl_\gamma (t-int (X-F)) = cl_\gamma (X-cl_t F) = X - (\gamma-int(cl_t F))$ (2.15) and finally $X-F \subseteq X - (\gamma-int(cl_t F))$ iff $\gamma-int(cl_t F) \subseteq F$.

3.7 Remark . The following statements are true:

- i) ϕ and X are $t\gamma$ -clop sets (both $t\gamma$ -closed and $t\gamma$ -open sets) .
- ii) If $A_\alpha \in t\gamma-o(X)$ $\alpha \in \Lambda$, then $\bigcup_{\alpha \in \Lambda} A_\alpha \in t\gamma-o(X)$.
- iii) The intersection of two $t\gamma$ -open sets need not be a $t\gamma$ -open set .
- iv) The intersection of any family of $t\gamma$ -closed sets is $t\gamma$ -closed set .
- v) The union of two $t\gamma$ -closed sets need not be a $t\gamma$ -closed set .

Proof : i) is clear.

ii) Since $A_\alpha \subseteq \bigcup_{\alpha \in \Lambda} A_\alpha$ for each α , by 2.10(iv) and 2.13(i),

$cl_\gamma (t-int A_\alpha) \subseteq cl_\gamma (t-int \bigcup_{\alpha \in \Lambda} A_\alpha)$ for each α ,so $A_\alpha \subseteq cl_\gamma (t-int A_\alpha)$ implies

$$\bigcup_{\alpha \in \Lambda} A_\alpha \subseteq cl_\gamma (t-int \bigcup_{\alpha \in \Lambda} A_\alpha) .$$

iv) Follows from the DeMorgan laws .

For (iii) and (v) see the following example.

3.8 Example . Let $X = \mathbb{R}$,the set of real numbers, τ the usual topology on \mathbb{R} , let $\gamma(V)=V$ for each $V \in \tau$ and $t(A) = intA$ for each $A \subseteq \mathbb{R}$.

If $A = (0,1]$ and $B = [1,2)$,then A and B are $t\gamma$ -open sets, but $A \cap B = \{1\}$ is not .

Also if $C = \mathbb{R} - (0,1]$, $D = \mathbb{R} - [1,2)$,then C and D are $t\gamma$ -closed sets, but $C \cup D$ is not.

3.9 Definition. Let $A \subseteq X$,the set $cl_{t\gamma} A$ is defined by

$$cl_{t\gamma} A = \{x \in X \mid U \cap A \neq \phi, \text{ whenever } U \in t\gamma-o(X) \text{ and } x \in U\} .$$

3.10 Remark. The following statements are true:

- i) $cl_{t\gamma} A = \bigcap \{F \mid F \text{ is } t\gamma\text{-closed and } A \subseteq F\}$.

- ii) $cl_{t\gamma}\emptyset=\emptyset$ and $cl_{t\gamma}X=X$ iii) $A \subseteq cl_{t\gamma} A$ and $A=cl_{t\gamma} A$ iff A is $t\gamma$ -closed .
- iv) $A \subseteq B$ implies $cl_{t\gamma} A \subseteq cl_{t\gamma} B$.
- v) $\cup cl_{t\gamma}A_\alpha \subseteq cl_{t\gamma}\cup A_\alpha$ and $cl_{t\gamma}\cap A_\alpha \subseteq \cap cl_{t\gamma}A_\alpha$ (where $\{A_\alpha\}$ is an arbitrary family of subsets of X).

Proof : trivial .

3.11 Remark. A is $t\gamma$ -open set iff for each $x \in A$,there exists $U \in t\gamma\text{-o}(X)$ such that $x \in U \subseteq A$.

3.12 Remark. Since $t\text{-o}(X) \subseteq t\gamma\text{-o}(X)$ (3.2) ,so $cl_{t\gamma} A \subseteq cl_t A$.

In example 3.3(ii) $cl_t \{c\}=\{b,c\}$, but $cl_{t\gamma}\{c\}=\{c\}$,that is $cl_{t\gamma}\{c\} \neq cl_t \{c\}$.

3.13 Definition. A set A is said to be $t\gamma$ -generalized closed set (shortly $t\gamma$ -g.closed) if $cl_{t\gamma} A \subseteq U$, whenever U is $t\gamma$ -open set and $A \subseteq U$.

3.14 Theorem. For each $x \in X$, either $\{x\}$ is $t\gamma$ -closed or $X-\{x\}$ is $t\gamma$ -g.closed set.

Proof : If $\{x\}$ is not $t\gamma$ -closed , then $X-\{x\}$ is not $t\gamma$ -open, that is X is the only $t\gamma$ -open set containing $X-\{x\}$ which is containing $cl_{t\gamma}(X-\{x\})$ too, hence $X-\{x\}$ is $t\gamma$ -g.closed set.

3.15 Remark. Any $t\gamma$ -closed set is a $t\gamma$ -g.closed set, but the converse is not true.

In example 3.3(ii) the set $\{a,b\}$ is a $t\gamma$ -g.closed set but not $t\gamma$ -closed set.

3.16 Remark. The concepts of γ -open and $t\gamma$ -open sets are independent .

In example 3.3 in (i) $t\gamma\text{-o}(X)$ is a proper subset of $\tau_\gamma = \tau$, and in (ii) τ_γ is a proper subset of $t\gamma\text{-o}(X)$.

3.17 Theorem. A set A is $t\gamma$ -open in (X,τ) iff $U \subseteq A \subseteq cl_\gamma U$, for some $U \in t\text{-o}(X)$.

Proof: If A is $t\gamma$ -open, take $U = t\text{-int } A$ then $U \subseteq A \subseteq cl_\gamma(t\text{-int } A) = cl_\gamma U$, where U is t -open set (by 2.13(v)).

On the other hand if $U \subseteq A \subseteq cl_\gamma U$ and U is t -open , then by (2.12) $U \subseteq t\text{-int } A$, hence $cl_\gamma U \subseteq cl_\gamma(t\text{-int } A)$ (2.10(iv)), that is, $A \subseteq cl_\gamma(t\text{-int } A)$ and A is $t\gamma$ -open.

3.18 Remark. The concepts of t -g.closed and $t\gamma$ -closed are independent, for example ,let $X=\{a,b,c,d\}$, $\tau =\{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$, define γ by $\gamma(V)=V$ for each $V \in \tau$ and t by $t(A)=\text{int } A$ for each $A \subseteq X$. Then $t\text{-o}(X)=\tau_\gamma = \tau$, and $t\gamma\text{-o}(X)=\{\emptyset, X, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}\}$.

$\{b, c\}$ is a $t\gamma$ -closed set which is not t -g.closed, $\{c, d\}$ is a t -g.closed set which is not $t\gamma$ -closed .

3.19 Definition. A set A is said to be generalized $t\gamma$ -closed (shortly $g. t\gamma$ -closed), if $cl_{t\gamma} A \subseteq U$, whenever U is t -open set and $A \subseteq U$.

3.20 Remark. If A is $t\gamma$ -g.closed set then it is $g. t\gamma$ -closed .(The converse is not true).

Proof : Follows from the fact that $t\text{-o}(X) \subseteq t\gamma\text{-o}(X)$ (3.2) . For the converse in example 3.3(ii) $\{a, b\}$ is a $g. t\gamma$ -closed (since the only t -open set containing it is X), but not $t\gamma$ -g.closed (since it is a $t\gamma$ -open set and $cl_{t\gamma}(\{a, b\})=X$ not subset of $\{a, b\}$).

3.21 Remark. If A is a t -g.closed set ,then it is a $g. t\gamma$ -closed .(The converse is not true).

Proof : Follows from the fact that $cl_{t\gamma} A \subseteq cl_t A$ (3.12) . For the converse in example 3.18 $\{b, c\}$ is a $g. t\gamma$ -closed set which is not t -g.closed .

3.22 Definition. Let γ be a γ -operation and t a t -operation on the topological space (X,τ) . Then (X,τ) is said to be:

- i) a $t\gamma$ - T_0 space if for each distinct points x and y in X there exists a $t\gamma$ -open set U containing one of them but not containing the other.
- ii) a $t\gamma$ - T_1 space if for each distinct points x and y in X there are two $t\gamma$ -open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.
- iii) a $t\gamma$ - T_2 space if for each distinct points x and y in X there are two disjoint $t\gamma$ -open sets U and V containing x and y respectively.
- iv) a $t\gamma$ - $T_{1/2}$ space if every $t\gamma$ -g.closed subset of (X,τ) is a $t\gamma$ -closed set.

- v) a $\tau\gamma$ - T_b space if every $g.\tau\gamma$ -closed set is t -closed.
- vi) a $\tau\gamma$ - T_d space if every $g.\tau\gamma$ -closed set is t - g -closed .
- vii) a $\tau\gamma$ - T_g space if every $g.\tau\gamma$ -closed set is $\tau\gamma$ - g -closed .

3.23 Theorem. i) (X,τ) is $\tau\gamma$ - T_1 space iff for each x in X , $\{x\}$ is $\tau\gamma$ -closed set.
 ii) (X,τ) is $\tau\gamma$ - $T_{1/2}$ space iff for each x in X , either $\{x\}$ is $\tau\gamma$ -closed or $\tau\gamma$ -open set .

Proof : i) Let (X,τ) be a $\tau\gamma$ - T_1 space and x in X , if $y \in X-\{x\}$, then by Def.3.22(ii), there exists $U_y \in \tau\gamma$ - $o(X)$ such that $y \in U_y$ and $x \notin U_y$, that is, $U_y \subseteq X-\{x\}$, therefore $X-\{x\}$ is $\tau\gamma$ -open and $\{x\}$ is $\tau\gamma$ -closed .
 Conversely assume that $\{x\}$ is $\tau\gamma$ -closed for each x in X , if $x \neq y$ then both of $X-\{y\}$ and $X-\{x\}$ are $\tau\gamma$ -open sets containing x and y respectively with $x \notin X-\{y\}$ and $y \notin X-\{x\}$, that is (X,τ) is $\tau\gamma$ - T_1 space .
 ii) Let (X,τ) be a $\tau\gamma$ - $T_{1/2}$ space and x in X , if $\{x\}$ is not $\tau\gamma$ -closed set, then $X-\{x\}$ is not $\tau\gamma$ - g -closed set (3.14), and by Def. 3.22(iv) $X-\{x\}$ is $\tau\gamma$ -closed, that is $\{x\}$ is $\tau\gamma$ -open .

Conversely assume that $\{x\}$ is either $\tau\gamma$ -closed or $\tau\gamma$ -open set for each x in X , and assume that A is a $\tau\gamma$ - g -closed set in (X,τ) , $x \notin A$.

Case1. $\{x\}$ is $\tau\gamma$ -open , then $X-\{x\}$ is $\tau\gamma$ -closed and $A \subseteq X-\{x\}$, so $cl_{\tau\gamma} A \subseteq X-\{x\}$, that is $x \notin cl_{\tau\gamma} A$.

Case2. $\{x\}$ is $\tau\gamma$ -closed , then $X-\{x\}$ is a $\tau\gamma$ -open set containing A , and since A is $\tau\gamma$ - g -closed, $cl_{\tau\gamma} A \subseteq X-\{x\}$ too, that is, $x \notin cl_{\tau\gamma} A$. Hence $x \notin A$ implies $x \notin cl_{\tau\gamma} A$, that is $cl_{\tau\gamma} A \subseteq A$, and A is $\tau\gamma$ -closed, which means (X,τ) is $\tau\gamma$ - $T_{1/2}$.

3.24 Theorem. If (X,τ) is t - T_i space , then (X,τ) is $\tau\gamma$ - T_i space, for $i=0,1/2,1,2$.

Proof : The proof for $i=0,1,2$, follows from the fact that t - $o(X) \subseteq \tau\gamma$ - $o(X)$ (3.2), and Def.3.22 .

When $i=1/2$, if (X,τ) is t - $T_{1/2}$ then for each x in X , $\{x\}$ is t -closed or t -open set (see [1],3.9) and so , $\{x\}$ is $\tau\gamma$ -closed or $\tau\gamma$ -open for each x in X (since t - $o(X) \subseteq \tau\gamma$ - $o(X)$). Hence by 3.23 (ii) (X,τ) is $\tau\gamma$ - $T_{1/2}$.

3.25 Theorem. If (X,τ) is $\tau\gamma$ - T_i space , then (X,τ) is a $\tau\gamma$ - $T_{i-1/2}$ space, for $i=1/2,1$.

Proof: Let $i=1/2$, If (X,τ) is $\tau\gamma$ - $T_{1/2}$, and $x, y \in X$, with $x \neq y$, then either $\{x\}$ is $\tau\gamma$ -closed and so $X-\{x\}$ is $\tau\gamma$ -open set containing y but not x , or $\{x\}$ is a $\tau\gamma$ -open set containing x but not y . Hence (X,τ) is $\tau\gamma$ - T_0 space.

Let $i=1$, If (X,τ) is $\tau\gamma$ - T_1 space, then for each x in X , $\{x\}$ is $\tau\gamma$ -closed, and so the statement " $\{x\}$ is $\tau\gamma$ -closed or $\tau\gamma$ -open for each x in X " is true , that is (X,τ) is $\tau\gamma$ - $T_{1/2}$ space (3.23(ii)) .

3.26 Theorem. If (X,τ) is $\tau\gamma$ - T_i space , then (X,τ) is a $\tau\gamma$ - T_{i-1} space, for $i=1,2$.

Proof: Follows from Def. 3.22 (i,ii,iii).

3.27 Theorem. If (X,τ) is $\tau\gamma$ - T_b space , then (X,τ) is a $\tau\gamma$ - T_d and $\tau\gamma$ - $T_{1/2}$ space.

Proof: If (X,τ) is $\tau\gamma$ - T_b space , then (X,τ) is $\tau\gamma$ - T_d by Def. 3.22(v,vi) and since any t -closed set is t - g -closed set (2.18).

If (X,τ) is $\tau\gamma$ - T_b space , and A is a $\tau\gamma$ - g -closed set , then by 3.20, it is $g.\tau\gamma$ -closed , hence it is t -closed since (X,τ) is $\tau\gamma$ - T_b and so A is $\tau\gamma$ -closed since t - $o(X) \subseteq \tau\gamma$ - $o(X)$. Therefore (X,τ) is a $\tau\gamma$ - $T_{1/2}$ space .

3.28 Remark. In example 3.3(ii) , (X,τ) is a $\tau\gamma$ - $T_{1/2}$ space and $\tau\gamma$ - T_d but not $\tau\gamma$ - T_b space, since $\{b\}$ (also $\{c\}$, $\{a,b\}$ and $\{a,c\}$) is $g.\tau\gamma$ -closed set which is not t -closed .

3.29 Remark. If (X,τ) is $\tau\gamma$ - T_b space , then (X,τ) is a $\tau\gamma$ - T_g .

Proof: Let A be $g.\tau\gamma$ -closed set, then by Def.3.22(v), A is t -closed, by

3.2, A is $\tau\gamma$ -closed, by 3.15, A is $\tau\gamma$ -g.closed .Hence (X, τ) is a $\tau\gamma$ - T_g .

3.30 Remark. By 3.27 and 3.29 , if (X, τ) is $\tau\gamma$ - T_b space ,then (X, τ) is a $\tau\gamma$ - T_d , $\tau\gamma$ - $T_{1/2}$ and $\tau\gamma$ - T_g .

3.31 Theorem. If (X, τ) is $\tau\gamma$ - T_b space ,then $t-o(X) = \tau\gamma-o(X)$.

Proof: Assume that (X, τ) is $\tau\gamma$ - T_b space and $A \in \tau\gamma-o(X)$,by 3.15 $X-A$ is a $\tau\gamma$ -g.closed set, by 3.20, $X-A$ is g. $\tau\gamma$ -closed set ,and by Def.3.22(v) $X-A$ is t-closed ,that is $A \in t-o(X)$,hence $t-o(X) = \tau\gamma-o(X)$.

3.32 Remark.

i) The converse of 3.31 is not true ,for example , let $X = \{a, b, c\}$, $\tau = P(X)$, $\gamma(V) = V$, and t is defined as following: $t(X) = X$, $t(A) = \emptyset$ if $a \notin A$, and $t(A) = \{a\}$ if $a \in A$. Then $t-o(X) = \{X, \emptyset, \{a\}\}$, $\tau_\gamma = \tau$ and $\tau\gamma-o(X) = t-o(X)$, but (X, τ) is not $\tau\gamma$ - T_b since any set other than $\{a\}$ is a g. $\tau\gamma$ -closed where the t-closed sets are X, \emptyset and $\{b, c\}$ only.

Note that (X, τ) is not t - $T_{1/2}$,not $\tau\gamma$ - $T_{1/2}$ but it is $\tau\gamma$ - T_g and $\tau\gamma$ - T_d .

ii) We will call the space (X, τ) in which $t-o(X) = \tau\gamma-o(X)$ as $\tau\gamma$ - T_t space and rewrite theorem 3.31: If (X, τ) is $\tau\gamma$ - T_b space ,then (X, τ) is $\tau\gamma$ - T_t space .

iii) By Theorem 3.17, (X, τ) is $\tau\gamma$ - T_t space if and only if every t-open set is a γ -closed set.

3.33 Theorem. If (X, τ) is $\tau\gamma$ - $T_{1/2}$ and $\tau\gamma$ - T_d space, then for each x in X , $\{x\}$ is either t-open or t-g.closed set.

Proof: If $\{x\}$ is not t-open, then it is not $\tau\gamma$ -open, by 3.23(ii) it is $\tau\gamma$ -closed (since (X, τ) is $\tau\gamma$ - $T_{1/2}$), by 3.15 it is $\tau\gamma$ -g.closed, by 3.20 it is g. $\tau\gamma$ -closed ,and by 3.22(vi) it is t-g.closed (since (X, τ) is $\tau\gamma$ - T_d) .

3.34 Theorem. (X, τ) is $\tau\gamma$ - T_b space if and only if it is t- $T_{1/2}$ and $\tau\gamma$ - T_d space.

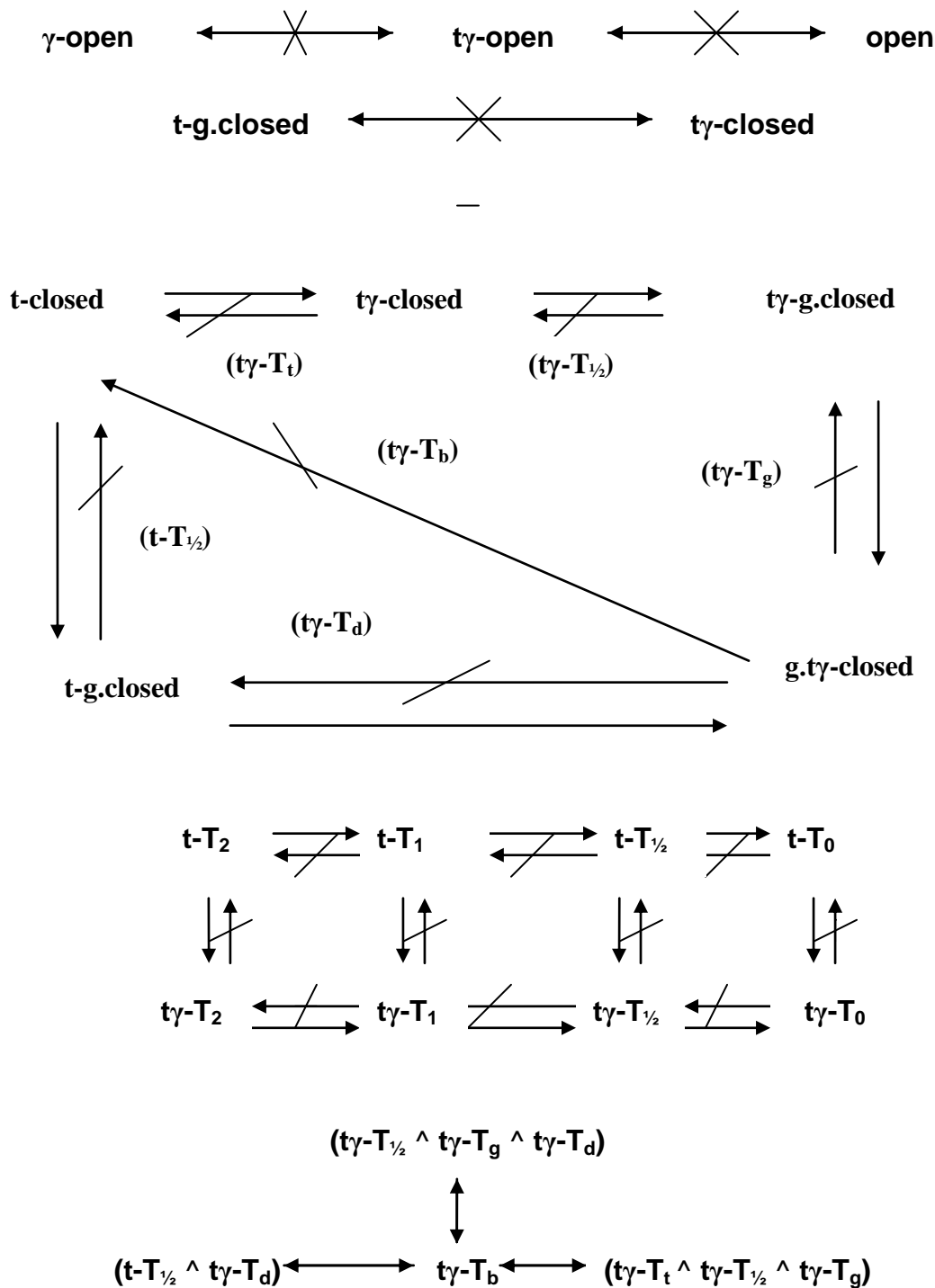
Proof: The sufficiency follows from the definitions of $\tau\gamma$ - T_d ,t- $T_{1/2}$ [1] and $\tau\gamma$ - T_b . Now if (X, τ) is $\tau\gamma$ - T_b space then by 3.31 it is $\tau\gamma$ - T_t ,by 3.27,3.29 it is $\tau\gamma$ - T_d , $\tau\gamma$ - $T_{1/2}$ and $\tau\gamma$ - T_g ,but $\tau\gamma$ - T_t and $\tau\gamma$ - $T_{1/2}$ imply t- $T_{1/2}$.Hence (X, τ) is t- $T_{1/2}$ and $\tau\gamma$ - T_d space

3.35 Theorem. If (X, τ) is $\tau\gamma$ - $T_{1/2}$, $\tau\gamma$ - T_g and $\tau\gamma$ - T_d space, then it is $\tau\gamma$ - T_b .

Proof: Let (X, τ) be $\tau\gamma$ - $T_{1/2}$, $\tau\gamma$ - T_g and $\tau\gamma$ - T_d space . Let $x \in X$, case1: $X - \{x\}$ is t-g.closed set, by 3.21 it is g. $\tau\gamma$ -closed ,by 3.22(vii) it is $\tau\gamma$ -g.closed (since (X, τ) is $\tau\gamma$ - T_g) ,and by 3.22(iv) it is $\tau\gamma$ -closed (since (X, τ) is $\tau\gamma$ - $T_{1/2}$), hence $\{x\}$ is $\tau\gamma$ -open and so it is t-open .(by 3.17) case2 : $X - \{x\}$ is not t-g.closed set, then $\{x\}$ is t-closed (see [1], 2.16).

We proved that for each x in X , $\{x\}$ is either t-open or t-closed, so (X, τ) is t- $T_{1/2}$ (see [1], 3.9). Hence by 3.33 (X, τ) is $\tau\gamma$ - T_b space .

Finally the following diagrams explain some of the results and relations between the deferent concepts that we were introduced.



Where $P \longrightarrow Q$ represents P implies Q , $P \longrightarrow / Q$ represents P does not imply Q ,

\longrightarrow and $P Q$ represents P and Q are independent.

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