Journal of Kerbala University , Vol. 6 No.4 Scientific. 2008

Nonlinear polarization density behaviour simulation in media by electromagnetic wave influence

محاكاة كثافة الاستقطاب في الأوساط اللاخطية بتأثير الموجة الكهرومغناطيسية

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Abstract :

Bloch's equations have been solved numerically by Runge-Kutta-Fehlberg method to evaluate the medium nonlinear polarization density behaviour. The medium has been represented as a two- level system according to the quantum theory. In this representation, matrix density formula describes the two levels coupling, under rotating wave approximation (RWA). The effect of frequency detuning (Δ) on the nonlinear polarization density shows the nonlinear polarization density decreases when the interaction becomes far away from exact resonance interaction. The response of time for nonlinear medium is increase for certain values of η (relative ratio of longitudinal to transverse relaxation time).Then, the responses will be very little as it increase η. **Subject term: Wave-Matter interaction: Nonlinear Optics: Nonlinear Polarization**

الخلاصة:

تم حل معادلات بلوخ عدديا بطريقة رانك – كوته- فلبرك (Runge – Kutta-Fehlberg) لتقييم ميزات كثافة الاستقطاب للوسط اللاخطي ،ولذلك تم تمثيل الوسط بنظام المستويين اعتمادا على النظرية الكمية ، في هذا التمثيل وصفت صيغة مصفوفة الكثافة للاستقطاب بازدواج المستويين في تقارب الموجة الدوارة (Rotating wave approximation) ، وقد ظهرت نتائج تأثير تردد فض الموجة (Frequency detuning) (Δ) على كثافة الاستقطاب اللاخطية واتضح أن كثافة الاستقطاب اللاخطية تتناقص عندما يكون التأثير بعيدا عن تفاعل الرنين التام .و:ذلك أن زمن الأستجابة للوسط اللاخطي يزداد لقيم محددة من □ ومن ثم تكون الستجابة صغيرة جدا عند زيادة □

1. Introduction

Since a long time and for different varieties phenomena it had been believed that it was sufficient to consider the wave – matter interaction as linear. The nonlinear effect of the wave – matter in purely optical fields remained undiscovered until the first laser appeared in the 1960’s, which provided sufficiently powerful and coherent light sources. Many important observations related to the wave-matter interaction are representing the beginning of entire new field in physics called Nonlinear Optics (NLO). Recently, many advances in nonlinear optics have been made; they become the basis for different technological areas. A large number of researches and applications have been found in this field to describe the electromagnetic wave propagation or the nonlinear interaction between light and matter which leads to a wide spectrum of phenomena [1-7]. All these researches and applications require nonlinear optics materials NLOM which respond to an applied optical field, the nonlinear polarization is an important picture of the optical response. Theoretically, many important models can provide better understanding to describe the mechanism of interaction between these NLOM and the wave [8–10]. This mechanism shows the average response of the NLOM media by polarization that is the important phenomena for these media. .So, in this work The effect of frequency detuning (Δ) on the nonlinear polarization density and the effect of η (relative ratio of longitudinal to transverse relaxation time), on the response time of the nonlinear medium and how long does the medium obey the nonlinearity has been studied

2. Theory

By the time dependent Schrödinger equation; for the two-level system, the motion equation can be expressed in terms of the matrix elements of the density matrix as: -

$$i\hbar\frac{d\rho_{aa}}{dt}=\left[\hat{H}_o,\hat{\rho}\right]_{aa}+\left[\hat{H}_I,\hat{\rho}\right]_{aa}+\left[\hat{H}_R,\hat{\rho}\right]_{aa} \quad (1a)$$

$$i\hbar\frac{d\rho_{ab}}{dt}=\left[\hat{H}_o,\hat{\rho}\right]_{ab}+\left[\hat{H}_I,\hat{\rho}\right]_{ab}+\left[\hat{H}_R,\hat{\rho}\right]_{ab} \quad (1b)$$

$$i\hbar\frac{d\rho_{bb}}{dt}=\left[\hat{H}_o,\hat{\rho}\right]_{bb}+\left[\hat{H}_I,\hat{\rho}\right]_{bb}+\left[\hat{H}_R,\hat{\rho}\right]_{bb} \quad (1c)$$

For the element (□$_{ba}$), the solution for this element immediately follows from $\rho_{ba}=\rho_{ab}^{*}$.Where $\hat{H}_o$ is free Hamiltonian operator with out external field effect describing the internal working of the isolated two- level system, $\hat{H}_I$ is the Hamiltonian operator describing the interaction between the system and the external field, $\hat{H}_R$ is the relaxation Hamiltonian operator describes all of the processes that return the ensemble to the thermal equilibrium. The most important of these processes are spontaneous emission, collision and the coupling between rotational, vibrational and electronic excitations of the molecules [11].The spontaneous emission processes that result in decay from state $|b\rangle$ to state $|a\rangle$ is described by the diagonal element of density matrix (ρ_{aa},ρ_{bb}).The element ρ_{bb} decay to ρ_{aa} element by longitudinal relaxation time *(LT)*, and the element ρ_{ba} is decaying to ρ_{ab} element of density matrix by the transverse relaxation time (TT).[12-14]

A physical and mathematical approximations have been considered such as the two states $|a\rangle$, $|b\rangle$ to be similar [11,15] and assuming the light quasi monochromatic, that means

$$E(r,t)=\mathrm{Re}\left[E_{w_o}(t)e^{-iw_o t}\right]=E_{w_o}(t)\cos(w_o t) \quad (2)$$

Where the spatial envelope of the electric field is $E_{w_o}(t)$ and w_o is the frequency of light wave. Also the optical Stark shift (δ E_a) and (δE_b) are neglected (absence the strong static magnetic field).

$$r_{ab}=r_{ba} \quad (3\text{-a})$$

Then

$$er_{ab}E_{w_o}(t)=er_{ba}E_{w_o}(t)=\mu E_{w_0}(t), \quad (3\text{-b})$$

Where μ= er_{ba} is the transition dipole moment. For a slowly varying amplitude of the off – diagonal elements, we rewrite *(*ρ_{ab}*)* and (ρ_{ba}*)* in the following form [16].

$$\rho_{ab}=\rho_{ab}^{w}\,e^{i(w_{ba}-\Delta)t}=\rho_{ab}^{w}\,e^{+iw_o t} \quad (4)$$

$$\rho_{ba}=\rho_{ba}^{w}\,e^{-i(w_{ba}-\Delta)t}=\rho_{ba}^{w}\,e^{-iw_o t} \quad (5)$$

Where $(\Delta=w_{ba}-w_o)$ is the detuning of the angular frequency of the light from the transition frequency $w_{ba}=(E_b-E_a)/\hbar$.

At typical optical frequencies, the terms oscillating rapidly at ($2w_o t$) are causing a small shift called Bloch-Siegret shift which is negligible in most cases [15]. Then we can separate out rapidly oscillating terms of frequencies $(w_{ba}-\Delta)$, and neglect these terms compared with more slowly varying terms; this is the idea of rotating wave approximation (RWA) [15-17]. The motivation for this approximation is that whenever high-frequency components appear in the equations of motions, the high frequency terms will contain large denominators while integration and will hence be minor in comparison with terms with slow variations. Then the terms with oscillatory dependence of $\exp((w_{ba}-\Delta)t)$ are neglected. Then by applying slowly varying amplitude of the off-diagonal elements and rotating wave approximation, the equations of motion become: -

$$\frac{d}{dt}\rho_{aa} = \frac{i}{2}\left(\rho_{ba}^{w} - \rho_{ab}^{w}\right)\Omega - \left(\rho_{aa} - \rho_{o}(a)\right)/T_{a} \qquad \text{(6-a)}$$

$$\frac{d}{dt}\rho_{ab}^{w} = i\Delta\rho_{ab}^{w} + \frac{i}{2}\Omega\left(\rho_{bb} - \rho_{aa}\right) - \rho_{ab}^{w}/TT \qquad \text{(6-b)}$$

$$\frac{d}{dt}\rho_{bb} = -\frac{i}{2}\left(\rho_{ba}^{w} - \rho_{ab}^{w}\right)\Omega - \left(\rho_{bb} - \rho_{o}(b)\right)/T_{b} \qquad \text{(6-c)}$$

Consider the variables *U*, *V*, and *W* as the Bloch vectors [11, 15, and 18].

$$U = \rho_{ba}^{w} + \rho_{ab}^{w}, \quad V = i\left(\rho_{ba}^{w} - \rho_{ab}^{w}\right), \quad W = \rho_{bb} - \rho_{aa} \qquad (7)$$

Where *U* is related to the dispersion component of the polarization density of the medium, *V* is related to the absorption component of the polarization density of the medium and W describes the population inversion of the two – level system.

2.1The normalized form of the optical Bloch equations

We define τ as normalized time (τ = t / TT), and consider the following parameters $\delta = \Delta * (TT)$, $\gamma(t) = \Omega * (TT)$, $\eta = LT/TT$ . . Then the normalized optical Bloch equations formulated as a form:

$$\frac{d}{d\tau}U = -\delta V - U \qquad \text{(8-a)}$$

$$\frac{d}{d\tau}V = \delta U + \gamma W - V \qquad \text{(8-b)}$$

$$\frac{d}{d\tau}W = -\gamma(t)V - \left(W - W_{o}\right)/\eta \qquad \text{(8-c)}$$

2.2 Macroscopic Polarization

The macroscopic polarization *P(r,t)* (coul/m^2) is usually introduced in order to describe the influence of electromagnetic fields in a dielectric medium. The molecules are mutually non-interacting, identical and similarly oriented, then the macroscopic polarization density of the medium can be written as: -

$$P(r,t) = \frac{1}{V}\sum_{m=1}^{M}\langle e\hat{r}\rangle = N\langle e\hat{r}\rangle \qquad (9)$$

As $N = M/V$ is the number density of molecules, $\langle e\hat{r}\rangle$ the expectation value of electric dipole moment operator.

The theory of the density matrix for two-level system can be applied to calculate the electric polarization density of a medium, consisting of (N) identical molecules per unit volume, as follow [11,19,20] :-

$$P(r,t) = NT_{r}\left[\hat{\rho}\, e\hat{r}\right] = \mathrm{N}\sum_{K=a,b}\sum_{j=a,b}\left(\rho_{Kj}\, e\hat{r}_{jK}\right)$$

$$P(r,t) = N\left\{\rho_{ab}\cdot e\hat{r}_{ba} + \rho_{ba}\cdot e\hat{r}_{ab}\right\} \qquad (10)$$

Using the mathematical definition of *U*, *V* in the set of equations (7) with equation (4) and equation (5) respectively, we get the following:-

$$\rho_{ab} = \frac{1}{2}\left(U + iV\right)\cdot e^{iw_o t} \qquad (11)$$

$$\rho_{ba} = \frac{1}{2}\left(U - iV\right)\cdot e^{-iw_o t} \qquad (12)$$

Substituting equation (11) and (12) into equation (10), then comparing the result equation with the following equation [19].

$$P(r,t) = \mathrm{Re}\left[P_{wo}(r,t)e^{-iw_o t}\right] \qquad (13)$$

; We get the following

$$\therefore\ P_{w_o}(r,t) = \frac{1}{2}N\left(U - iV\right)\cdot e\,\hat{r}_{ab} \qquad (14)$$

By applying equation (13) one can obtain the polarization density in terms of the Bloch vector (*U*, *V*) such as

$$P_{wo}(r,t) \approx \left(U - iV\right) \qquad (15)$$

The Bloch vectors (*U*) and (*V*) in equation (15) represent the dispersion and absorption components of the interaction between light and matter, respectively.

3. Numerical simulation results and discussion

The set of equation (8) have been solved numerically by Runge-Kutta-Fehelberg technique to describe the wave - matter interaction. The nonlinear polarization density (NLPD) is proportional to $|U(\tau) - iV(\tau)|$, so one can follow the time profile of NLPD of the nonlinear media. The exact values of the NLPD depend on the value of a proportional factor, which is related to the medium and wave parameters. Certain conditions have been taken into account to fit the physical behaviour of the variables. So at initial time according to equation(7),where (t = 0.0), *U* and *V* are equal to zero, and *W* is approximately equal to (-1.0).The nonlinear polarization density behaviour has been simulated as function of frequency detuning(Δ),it affects the nonlinear polarization density and that depends on the value of the detuning. This value specifies three cases of the wave- matter interaction ;(a) Exact resonance interaction (Δ=0), (b) Near exact resonance or slightly off – resonance, and (c) Far off exact resonance (Δ is significant).The study shows, similar behaviour of nonlinear polarization density at near exact resonance interaction for different values of Δ. This is concerning the fast build-up and damping, as shown in figure (1). In all cases, the nonlinear polarization density reaches its maximum value during the same build – up time (τ = 0.4), the maximum values decreases while the increase of Δ, as it appears in figure (2). This is due to the decrease of the medium absorption which passively affects the transferred energy to the system. This is clear from the decrease of the population inversion as detuning goes far away from Δ=0. This behaviour illustrated in figure (3). The decrease in the population inversion will be referred to decrease the medium response, which leads to the decrease in the nonlinear polarization density maxima values. The medium nonlinear response time as function of η revealed, the nonlinear response time of the medium is affected by the ratio of longitudinal to transverse relaxation time (η). While increasing η, the nonlinear response time will be increased in both exact and non-exact resonance interaction. It means that, the nonlinear interaction stands for long time while increasing η, this is disregarded the interaction intensity. This behaviour is standing clearly at certain values of η, above it the nonlinear response time will be affected slightly while increasing η and the behaviour of response will be far away from no linearity, as it is clearly shown in figure (4) and table.1. This is due to the increase of η, which means the increase of the upper level life time that leads to the increase of population inversion. That will decrease most particles contribution in the interaction, so the reaction will be damped.

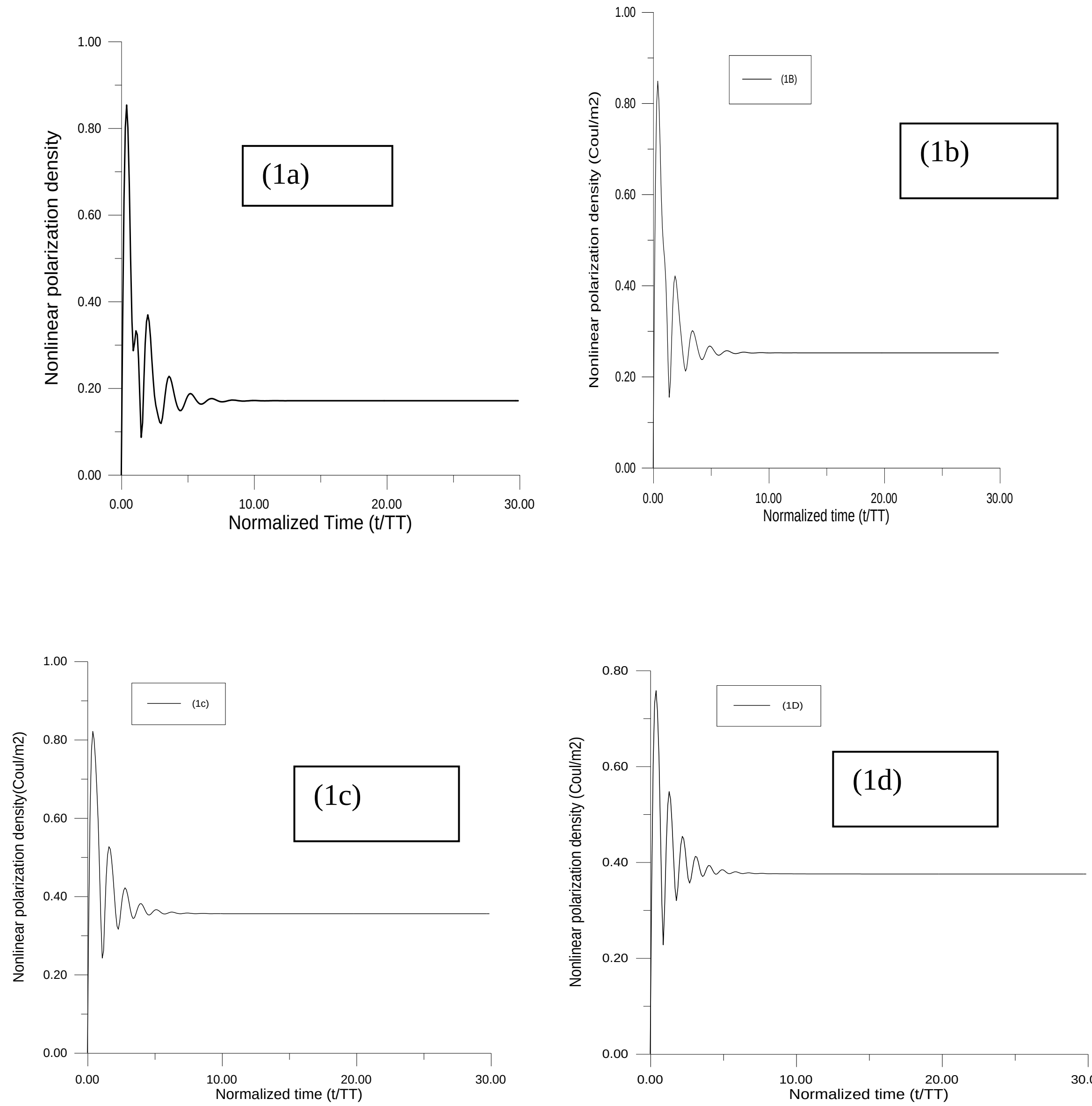

Fig.(1) Profiles of nonlinear polarization density, (η=2) ; (1a)Δ=1, (1b) Δ=2, (1c) Δ=4 and (1d) Δ=6)

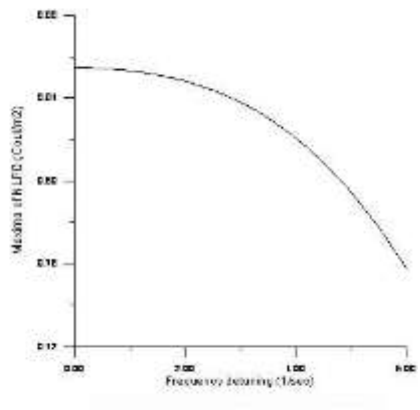

Fig. 2: Nonlinear polarization density as a function of frequency detuning (Δ)

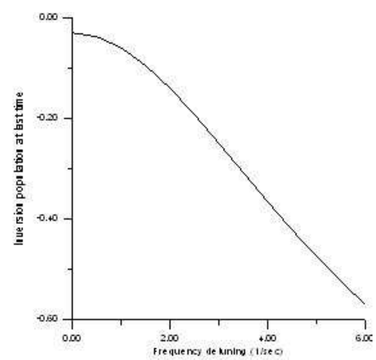

Fig. 3: Inversion population (w) as a function of frequency detuning(Δ)

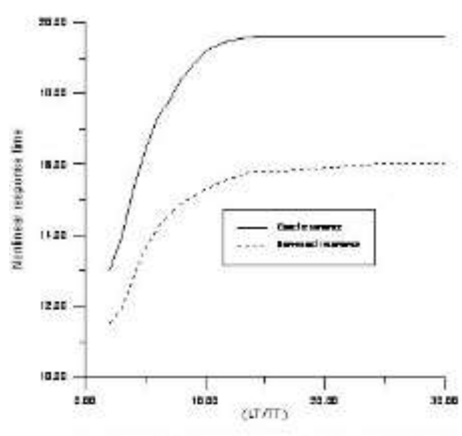

Fig. 4: The nonlinear response time of medium as a function of (LT/TT) for exact and non-exact resonance.

4. Conclusions

The nonlinear behaviour time and the maximum value of nonlinear polarization density are decreasing while the wave-matter interaction is far away from exact resonance interaction. It has been observed from the study that at certain limit values of η, the response time will increase while increasing η. After these values, the increasing of η will possess very little effect on the nonlinear response time, and the behaviour will be far away from nonlinearity and approaches steady state.

Table (1) The Nonlinear response time of medium and inversion population, as a function of η.

η	Exact resonance (Δ=0)		Non-exact resonance (Δ=2)	
	τ	W	τ	W
2	13.0	-0.029403	11.5	-0.139738
3	13.9	-0.019628	11.9	-0.097801
4	15.3	-0.014854	12.8	-0.074992
5	16.4	-0.012026	13.6	-0.060978
6	17.3	-0.009900	14.2	-0.051252
7	17.8	-0.008578	14.6	-0.044372
8	18.4	-0.007525	14.9	-0.039047
9	18.8	-0.0066627	15.1	-0.034615
10	19.2	-0.005978	15.3	-0.031391
12	19.5	-0.005031	15.6	-0.026277
14	19.6	-0.004334	15.8	-0.022637
16	19.6	-0.003802	15.8	-0.019860
20	19.6	-0.003056	15.9	-0.015973
24	19.6	-0.002559	16.0	-0.013372
30	19.6	-0.002061	16.0	-0.010733

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