DYNAMICS AND HEAT TRANSFER OF GAS BUBBLES IN ACOUSTIC FIELD

Abbas Z. AL-Asady

Mechanical Engineering Dept., Engineering College, Basrah University, Iraq

Abstract

A new model of bubble dynamics in acoustic field is constructed, including effect of heat transfer at bubble wall. The temperature inside the bubble is calculated by solving the energy equation of the gas inside the bubble (using finite difference method). The liquid temperature at bubble wall is numerically calculated by assuming an exponential profile of liquid temperature. It is including effect of thermal conduction at bubble wall. Calculations are performed for adiabatic model.

The results reveal that the effect of heat transfer is considerable on bubble dynamics. The calculated results fit with the experimental data of radius-time curve much more satisfactorily than those by the adiabatic model (without heat transfer). It is clarified that the effect of heat transfer stabilizes bubble oscillations.

حركة و انتقال الحرارة لفقاعات غازية في وسط صوتي

عباس زكي الأسدي

قسم الهندسة الميكانيكية - كلية الهندسة - جامعة البصرة - العراق

الخلاصة

تم بناء نموذج جديد لحركة فقاعة في وسط صوتي متضمناً تأثير انتقال الحرارة عند جدار الفقاعة. احتسبت درجة الحرارة داخل الفقاعة بواسطة حل معادلة الطاقة للغاز الموجود داخل الفقاعة (باستخدام طريقة الفروقات المحددة). احتسبت درجة حرارة السائل عند جدار الفقاعة عددياً بواسطة فرض شكل أسي لها. كانت متضمنة تأثير التوصيل الحراري عند جدار الفقاعة. أجريت الحسابات للنموذج الأديباتيكي.

تبوح النتائج أن تأثير انتقال الحرارة مهم في حركة الفقاعة. تطابق النتائج المحسوبة مع البيانات التجريبية لمنحني القطر - الزمن أكثر مرضي من تلك النتائج المحسوبة للنموذج الأديباتيكي (بدون انتقال حرارة). وضحت النتائج أن تأثير انتقال الحرارة يجعل تذبذب الفقاعة مستقراً.

1. Introduction

The study of oscillations of gas bubbles in a liquid is of considerable practical interest specifically in regard to the question of the possible use of bubble screens for damping shock waves and the use of acoustic disturbances for intensification of technological processes.

In gas-liquid flows, the mass, force and energy interactions between phases originate on the interface surface. These interactions can be significantly altering the flow velocity, pressure and temperature fields. A correct specification of the inter-phase heat and mass requires the knowledge of interaction of single inclusions with the carrier phase [1].

At present a number of publications are available in which different aspects of the problem of oscillations of gas bubbles in a liquid are studied. It has been revealed that the gas bubbles in a liquid are studies. It has been revealed [2] that in the case of small oscillations of a bubble within a wide range of the equilibrium values of its radius the heat transfer dominates over other dissipation mechanisms, *i.e.* velocity and compressibility of the liquid. The problem of heat transfer in the course of nonlinear oscillations of a gas bubble was studied experimentally [3]. The results of numerical solution for the nonlinear problem of thermal and dynamic interactions of a gas bubble with the liquid induced by a sudden pressure change in the liquid are presented in ref. [4]. A number of studies deal with the study of growth and collapse of vapor bubbles in a liquid (see ref. [5]). The assumptions of the temperature uniformity in the bubbles and of the thinness of a thermal boundary layer in the liquid, adopted in the majority of these studies, considerably simplify the problem but hold under certain restrictions only. The heat transfer effects on the vapour bubble dynamics with account for the temperature nonuniformity in it is considered in ref. [6].

In this paper, we construct a mathematical formulation that enables us to study the motion of a bubble in a liquid and the effects of heat conduction, shear viscosity, compressibility, surface tension, temperature non-uniformity in the bubble, and variation of liquid temperature at bubble wall on their dynamical behavior. The formulation is specifically designed to describe the motion of a bubble that expands to some maximum radius and then contracts violently. This formulation consists of a set of nonlinear equations that can be solved numerically.

In section (2), the adiabatic case is described, which is frequently employed in the study of bubble dynamics in liquids. Also, the new case is described in which effects of thermal conduction inside the bubble and variation of liquid temperature at bubble wall. In section (3) we present and discuss results and in section (4) a number of conclusions are presented. Followings are the description of the cases.

2. Mathematical Formulation

The author has already constructed a model of bubble dynamics in acoustic field including effect of thermal conduction inside and outside a bubble [7]. In this paper, effect of variation of liquid-temperature is newly included in the study of bubble dynamics. This paper gives the results of investigation of the heat transfer effect on the dynamics of gas bubbles as well as of the reverse effect of the dynamics of radial bubble motion on the enhancement of heat transfer between the bubbles and liquid.

2.1 Case(1) Adiabatic Model

In this case, pressure and temperature are assumed to be spatially uniform in a bubble. It is a good approximation only when $\dot{R} << c_g$, where $\dot{R}$ is the speed of the bubble wall and c_g is the speed of sound in the bubble [8]. The liquid temperature on the external side of the bubble wall is assumed to be constant (T_∞) during bubble oscillations. The temperature discontinuity (ΔT) exists at the bubble wall ($\Delta T = T_g - T_\infty$). In this case, it is assumed that no heat exchange is taken into account between a bubble and the surrounding liquid.

As an equation of bubble radius (R), eq. (1) is employed in which the effect of compressibility of liquid is taken into account (the derivation of eq. (1) was given by the author in ref. [7]).

$$R\ddot{R}\left[1-\frac{2\dot{R}}{c}+\frac{23}{10}\frac{\dot{R}^2}{c^2}\right]+\frac{3}{2}\dot{R}\left[\dot{R}-\frac{4\dot{R}^2}{3c}+\frac{7\dot{R}^3}{5c^2}\right]+\frac{1}{\rho_\infty}\left[\begin{array}{l}(P_{L\infty}-P_{L2})-\frac{R}{c}\frac{dP_{LR}}{dt}+\frac{1}{c^2}\\ \left\{\begin{array}{l}2R\dot{R}\frac{dP_{LR}}{dt}+(P_{L\infty}-P_{LR})\\ \left[\frac{1}{2}\dot{R}^2+\frac{3}{2}\frac{(P_{L\infty}-P_{LR})}{\rho_\infty}\right]\end{array}\right\}\end{array}\right]=0 \quad ...(1)$$

where $$P_{L2}=P_{LR}-\frac{4\mu}{3c^2\rho_\infty}\frac{dP_{LR}}{dt} \quad ...(2)$$ و

P_{LR} is the liquid pressure on the external side of the bubble wall and $P_{L\infty}$ is the sum of the ambient static pressure (P_o) and a nonconstant ambient pressure component such as a sound field. When a bubble is irradiated by an acoustic wave [9],

$$P_{L\infty}=P_o-P_m\sin\omega t \quad ...(3)$$

where P_m is the pressure amplitude of the acoustic wave and ω its angular frequency. P_{LR} is related to the internal bubble pressure $P_g(t)$ by [10],

$$P_{LR} = P_g(t) - \frac{2\sigma}{R} - \frac{4\mu \dot{R}}{R} \qquad \ldots(4)$$

The dynamics of a gas bubble in a liquid is strongly dependent on the pressure of the gas contained in it. A situation where the bubble interior contains an incondensable gas can be simply represented by a polytropic law of compression by [11]

$$P_g = P_{go}\left(\frac{R_o}{R}\right)^{3\gamma} \qquad \ldots(5)$$

where γ is the polytropic exponent and

$$P_{go} = P_o + \frac{2\sigma}{R_o} \qquad \ldots(6)$$

is the internal pressure corresponding to the rest radius R_o.

2.2 Case(2) Heat Transfer Model

The difference between this case and the adiabatic case is the inclusion of the effects of heat transfer inside the bubble and variation of liquid temperature at bubble wall. In this case, pressure is assumed to be spatially uniform in a bubble as in the adiabatic case. The liquid temperature on the external side of the bubble wall is assumed to be variable (T_{LR}) during bubble oscillations. Followings are the different points as compared with the adiabatic case, but eq. (1) is used for the motion of bubble radius as in the previous case.

The internal pressure P_g is found by integrating [12]

$$\frac{dP_g}{dt} = \frac{3}{R}\left[(\gamma - 1)k_g \left.\frac{\partial T_g}{\partial r}\right|_R - \gamma P_g \dot{R}\right] \qquad \ldots(7)$$

where γ is the ratio of the specific heats of the gas and k is the gas thermal conductivity. The bubble interior is then described by an ordinary differential equation for the pressure and a partial differential equation, the energy equation, which is written in the following form (the gas temperature field T_g (r,t)).

$$\begin{aligned} &\frac{\gamma}{\gamma - 1}\frac{P_g}{T_g}\left[\frac{\partial T_g}{\partial t} + u(r,t)\frac{\partial T_g}{\partial r}\right] = \\ &\qquad \frac{dP_g}{dt} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(k_g\, r^2 \frac{\partial T_g}{\partial r}\right) \end{aligned} \qquad \ldots(8)$$

with the following analytical expression for u(r, t):

$$u(r,t) = \frac{1}{\gamma P_g}\left[(\gamma - 1)\, k_g \frac{\partial T_g}{\partial r} - \frac{1}{3}\, r \frac{dP_g}{dt}\right] \quad .(9)$$

The continuity of energy flux at the bubble wall is given by [13]

$$k_L \left.\frac{\partial T_L}{\partial r}\right|_R = k_g \left.\frac{\partial T_g}{\partial r}\right|_R \qquad \ldots (10)$$

where $T_L(r)$ is the liquid temperature at radius r and k_L is the thermal conductivity of liquid. k_L and k_g depend on the liquid and gas temperature, respectively. The formulas of them are reported as an Appendix in ref. [8]. In this case, $\left.\frac{\partial T_L}{\partial r}\right|_R$ is calculated by eq. (10). The temperature is continuous at the interface:

$$T_L (R,t) = T_g (R,t) \qquad \ldots (11)$$

The spatial distribution of the liquid temperature (T_L=T_L (r)) should satisfy the following boundary conditions.

$$T_L(R) = T_{LR} \qquad \ldots (12)$$

$$\left.\frac{\partial T_L (r)}{\partial r}\right|_R = \left.\frac{\partial T_L}{\partial r}\right|_R \qquad \ldots (13)$$

$$T_L (r \to \infty) = T_\infty \qquad \ldots (14)$$

$$\left.\frac{\partial T_L (r)}{\partial r}\right|_R = 0 \qquad \ldots (15)$$

In the present case, the temperature profile in the liquid (T_L=T_L (r)) is assumed to be exponential (eqs. (16) and (17)) [14].

When $(T_{LR} - T_\infty) \left.\frac{\partial T_L}{\partial r}\right|_R < 0$

$$T_L (r) = (T_{LR} - T_\infty) \exp\left[- \frac{\left.\frac{\partial T_L}{\partial r}\right|_R (r - R)}{(T_\infty - T_{LR})} \right] + T_\infty \qquad \ldots (16)$$

When $\left(T_{LR} - T_\infty\right) \left.\frac{\partial T}{\partial r}\right|_R > 0$

$$T (r) = A \exp\left[-B(r-C)^2\right] + T_\infty \qquad \ldots (17)$$

where

$$A = (T_{LR} - T_\infty) \exp(B e_1^2) \qquad \ldots (18)$$

$$B = \frac{\left.\frac{\partial T}{\partial r}\right|_R}{2(T_{LR} - T_\infty) e_1} \qquad \ldots (19)$$

$$C = R + e_1 \qquad \ldots (20)$$

$$e_1 = e_0 \left| \frac{T_{LR} - T_g}{\left.\frac{\partial T_L}{\partial r}\right|_R} \right| \qquad \ldots (21)$$

where e_0 is a parameter with the numerical value ($e_0 = 1 \times 10^{-3}$). Both eqs. (16) and (17) satisfy the boundary conditions given by (eqs. (12)~(15)).

In the present case, a boundary layer is assumed in liquid phase near a bubble. The thickness of the layer (δ_L) is assumed as given in eqs. (22) and (23).

When $(T_{LR} - T_\infty) \left.\frac{\partial T_L}{\partial r}\right|_R < 0$

$$\delta_L = \frac{T_\infty - T_{LR}}{\left.\frac{\partial T_L}{\partial r}\right|_R} \qquad \ldots (22)$$

When $(T_{LR} - T_\infty) \frac{\partial T_L}{\partial r} > 0$

$$\delta_L = \frac{1}{\sqrt{B}} + e_1 \qquad \ldots (23)$$

The variation of the liquid temperature at bubble wall (T_{LR}) is calculated by the following equation:

$$T_{LR}(t + \Delta t) = T_{LR}(t) + \frac{4\pi R^2 \Delta t\, q_1' - 4\pi (R + \delta_L)^2 \Delta t\, q_2'}{\frac{4}{3}\pi \left[(R + \delta_L)^3 - R^3\right] \rho_{LR} \cdot cp_L} \qquad \ldots (24)$$

where q_1' (q_2') is the energy flux at r = R ($r = R + \delta_L$) per unit area and unit time, and cp_L is the specific heat of liquid water at constant pressure. q_1' and q_2' are calculated by eqs. (25) and (26), where $q_1' = -(k_L)_R \left[\frac{\partial T_L}{\partial r}\right]_R \qquad \ldots (25)$

$$q_2' = -(k_L)_{R+\delta}\left[\frac{\partial T_L}{\partial r}\right]_{R+\delta} \quad \ldots (26)$$

The physical quantities of liquid at interface depend on the liquid temperature (T_L) and the liquid pressure (P_L). Formulas of these quantities employed in the calculations are described in the next section.

Eqs. (1), (7), (8) and (24) are collectively dealt with in a single computer program and solved numerically. Since eqs. (1), (7), and (24) are ordinary differential equations, while eq. (8) is a partial differential equation. A finite-difference, second-order method [15,16], were employed to solve them numerically.

3. Results and Discussion

Calculations are performed under a condition of T_o=20 ^oC and Ro=10.5 (μs), where T_o and R_o are the ambient liquid temperature and the initial bubble radius, respectively. The frequency and the amplitude of the acoustic field are chosen to be 26.5 kHz and 1 bar, respectively. The undisturbed pressure is taken to be P_o=1 bar. Calculations start from the time t=0 (μs) with the initial conditions:

$$R = R_o, \dot{R} = 0, T_g = T_{LR} = T_o, P_{L\infty} = P_o$$

(A) case (1): Adiabatic Model

Under the physical conditions employed in the calculations described above, a periodic solution is obtained by numerical calculations as is shown in Figs. (1~4) for one acoustic cycle by the adiabatic model.

The pressure of the acoustic field applied on a bubble and employed in the calculation is a function of time. The bubble radius (R) is shown in Fig. 1 as a function of time. The bubble wall velocity $(\dot{R})$ is shown in Fig. 2 as a function of time. Both the radius and the wall velocity of the bubble change with time periodically.

In Fig. 3, the pressure inside the bubble (P_g) is shown as a function of time with logarithmic scale for vertical axis. In Fig. 4, the temperature inside the bubble (T_g) is shown as a function of time with linear vertical axis. Both the pressure and temperature change with time periodically. At the slow expansion phase in a bubble oscillation, the pressure and the temperature inside the bubble is slightly less than the ambient liquid pressure and temperature, respectively. On the other hand, at collapse stage the bubble, P_g and T_g increase suddenly, followed by oscillations due to the bounces of bubble radius (see Fig. 1).

(B) Case (2): Heat Transfer Model

The results of the calculation using the model with thermal conduction both inside and outside the bubble (hereafter it is called" the model with HT') stated in section 5 are shown in Figs. 5~9. The results of the calculation shown in these figures indicate that the ratio of specific heats of the gas (air) inside the bubble of order of 1.4, the specific heat at constant pressure of liquid (water) is taken to be 4.2 kJ/kg.K.

As is shown in the figures 5~9, all the physical quantities of a bubble change with time periodically with the frequency of the acoustic field applied on the bubble. In Fig. 5, the bubble radius (R) is shown as a function of time. In Fig. 6, the bubble wall velocity ($\dot{R}$) is shown as a function of time.

In Figs. 7 and 8 the pressure (P_g) and the temperature (T_g) inside the bubble are shown as a function of time, respectively. At the slow expansion phase in a bubble oscillation, Pg is slightly less than the ambient liquid pressure (P_o) and T_g approaches (~ equal) liquid temperature (T_o) (isothermal process). On the other hand, at collapse stage of a bubble, P_g and T_g increase drastically, followed by small oscillations due to the small bounces of bubble radius (see Fig. 5). The liquid-temperature at the bubble wall (T_{LR}) is shown in Fig. 9 as a function of time. It is concluded from Fig. 9 that T_{LR} is almost identical to T_o during bubble oscillations except at strong collapses. At strong collapse, T_{LR} increases due to the thermal conduction from the heated interior of the bubble to the surrounding liquid.

In Fig. 10, a comparison is given between the radius-time curve calculated by the model with HT (line) and those calculated by the model without HT (dash) for one acoustic cycle (~ 40 ms). The solid circles are the experimental data by Barber and Putterman [17]. It can be seen from the figure that the radius-time curve calculated by the model with HT (line) fits the experimental data (solid circles) more satisfactorily than that calculated by the model without HT (dash).

Comparisons for the two cases of the temperature (T_g) and pressure (P_g) inside the bubble are shown in Fig. 11 and Fig. 12, respectively. It is concluded from the figures that the effect of heat transfer is considerable on bubble dynamics.

4. Conclusion

A new model of bubble dynamics is proposed in which heat transfer (thermal conduction both inside and outside the bubble) is included. The assumption of the spatial uniformity of temperature in a bubble is no more a realistic one at the strong collapse of the bubble, while in this study the temperature in the bubble is calculated by solving (numerically) the energy equation of the gas (air) in the bubble. In the present model, the liquid temperature profile near bubble wall is assumed to be exponential.

From the calculations, it is concluded that the effect of heat transfer is considerable on bubble dynamics in the acoustic field. The calculated results fit with the experimental data of radius-time curve much more satisfactorily than those by a model without heat transfer. It is concluded that the effect of heat transfer at bubble wall stabilizes bubble oscillations. It is also concluded that a bubble transducers the energy of the acoustic wave into heat.

5. Nomenclature

Symbol	Definition	Unit
c	Sound speed in the liquid at infinity	m/s
cp_L	Heat capacity of liquid at constant pressure	J/kg.K
f	Acoustic field frequency	Hz
k	Thermal conductivity	W/m.K
P	Pressure	Pa
$P_{L\infty}$	Pressure of the static and acoustic pressure	Pa
P_o	Ambient liquid pressure	Pa
P_m	Acoustic pressure amplitude	Pa
P_{go}	Equilibrium pressure in the bubble	Pa
r	Radial distance from the bubble	m
R	Bubble radius	m
$\dot{R}$	Bubble wall velocity	m/s
$\ddot{R}$	Second derivative of the bubble radius	m/s^2
t	Time	s
T	Temperature	K
T_{LR}	Liquid temperature at bubble wall	K
T_∞	Ambient liquid temperature	K
u	Velocity	m/s
q'_1	Energy flux at $r = R$	$J/m^2.s$
q'_2	Energy flux at $r = R+\delta_L$	$J/m^2.s$
γ	Ratio of specific heats for gas	
δ	Thickness of the liquid layer	m
μ	Liquid viscosity	$N.s/m^2$
ρ	Liquid density	kg/m^3
ρ_∞	Ambient liquid density	kg/m^3
σ	Surface tension	N/m
ω	Angular frequency ($2\pi f$)	rad/s

Subscripts

g : Refers to gas in the bubble

L : Refers to liquid

LR : Refers to liquid at bubble wall

o : Refers to the equilibrium value

∞ : Refers to the condition at a great distance from the bubble.

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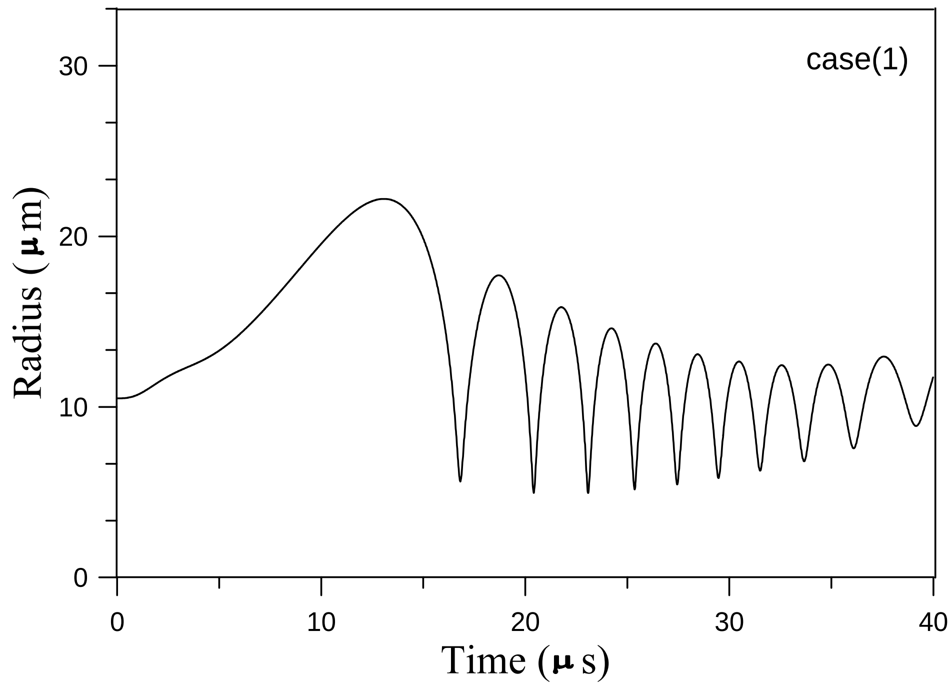

Fig. 1. The bubble radius (R) as a function of time.

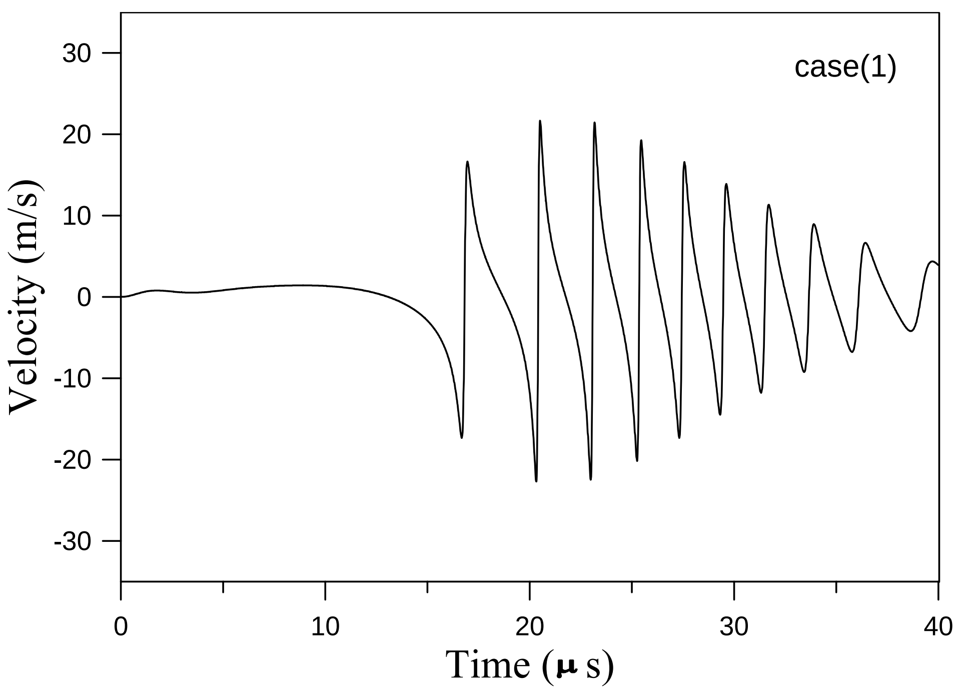

Fig. 2. The bubble wall velocity ($\dot{R}$) as a function of time.

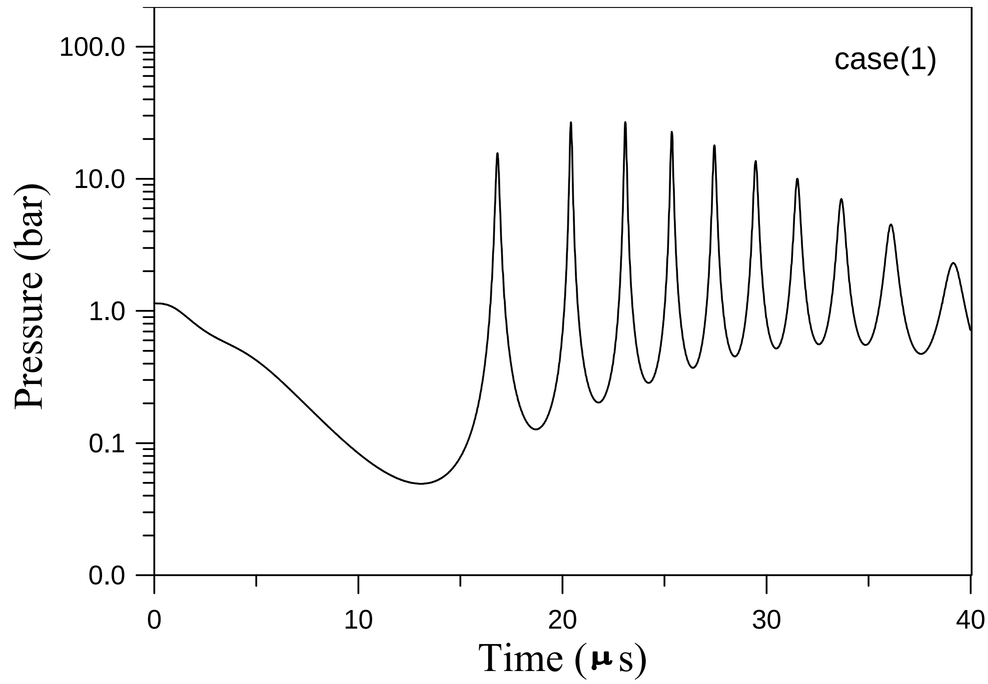

Fig. 3. The pressure inside the bubble (P_g) as a function of time with logarithmic vertical axis.

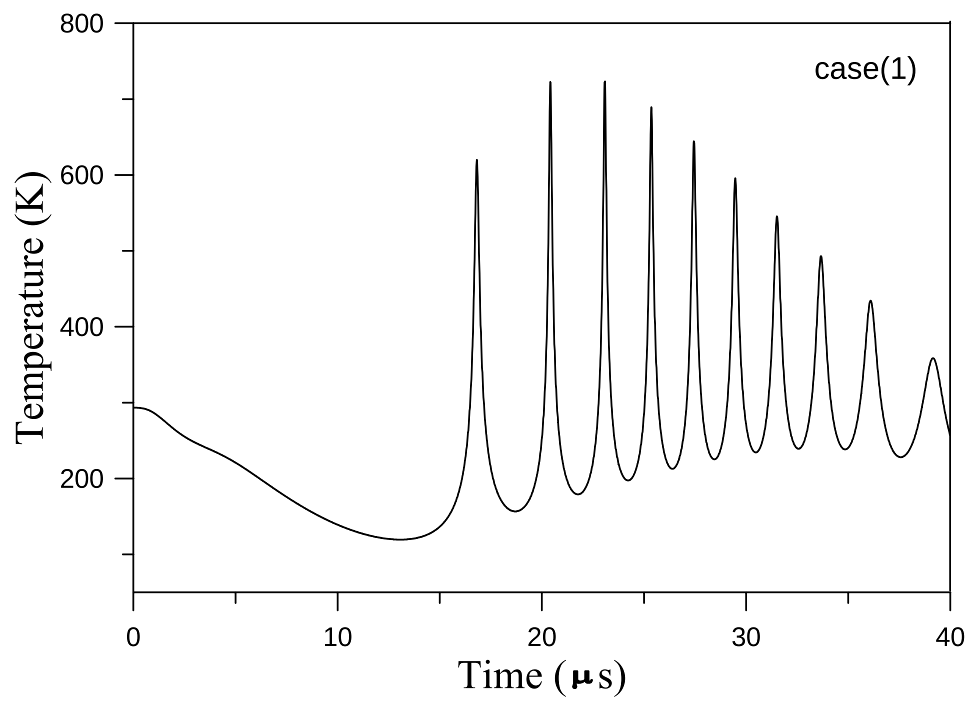

Fig. 4. The temperature at the bubble center as a function of time.

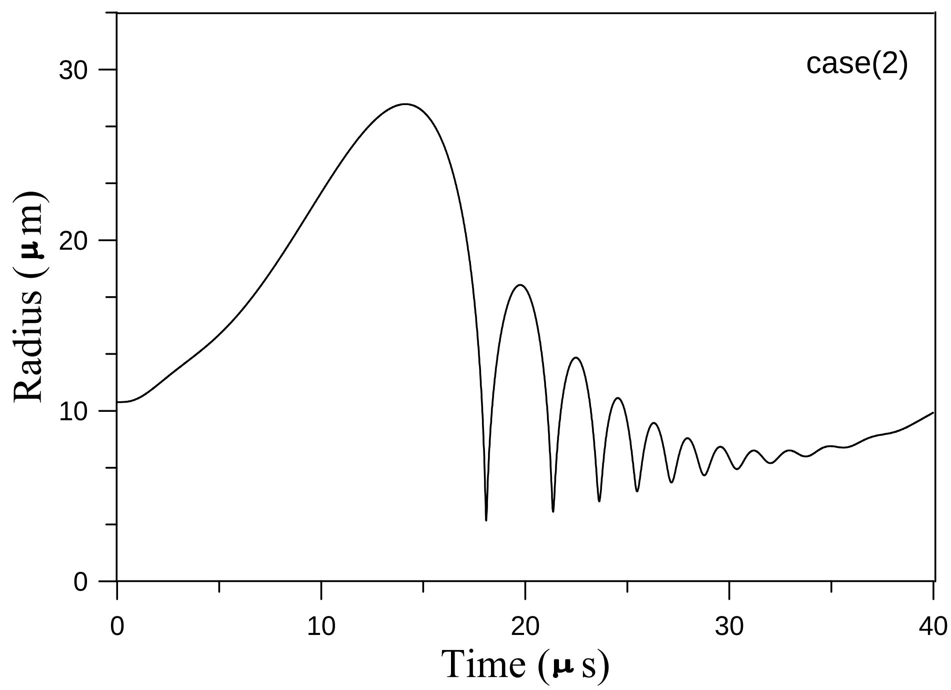

Fig. 5. The bubble radius (R) as a function of time.

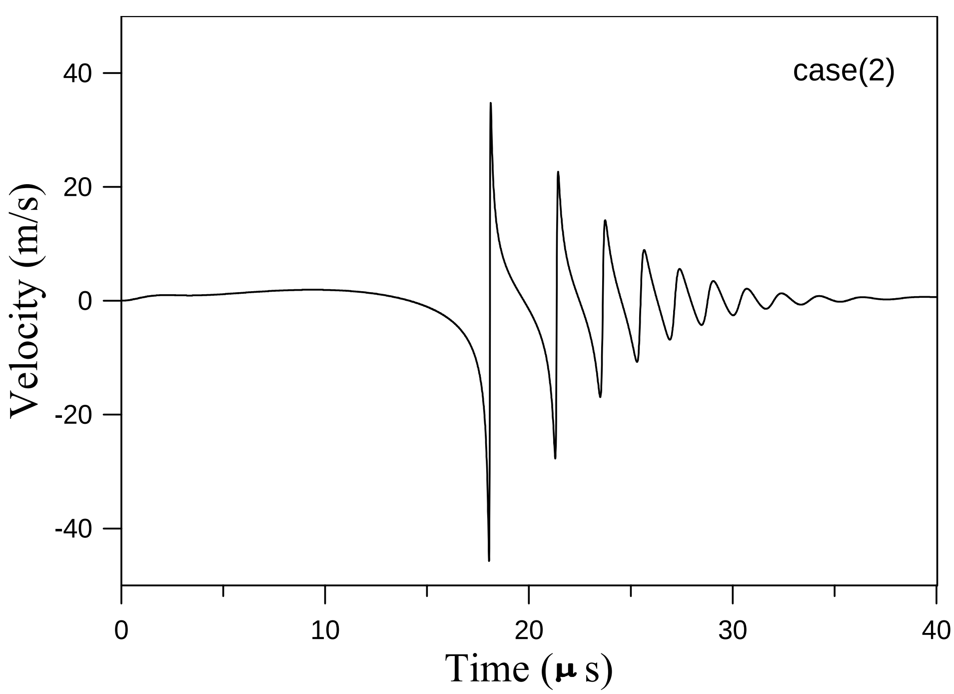

Fig. 6. The bubble wall velocity ($\dot{R}$) as a function of time.

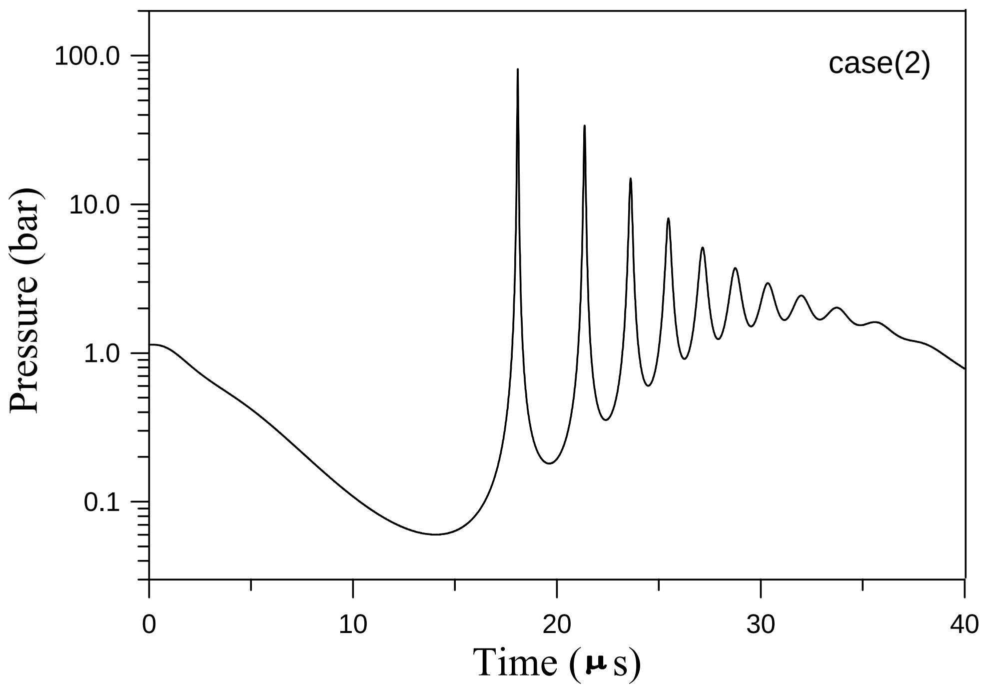

Fig. 7. The pressure inside the bubble (P_g) as a function of time with logarithmic vertical axis.

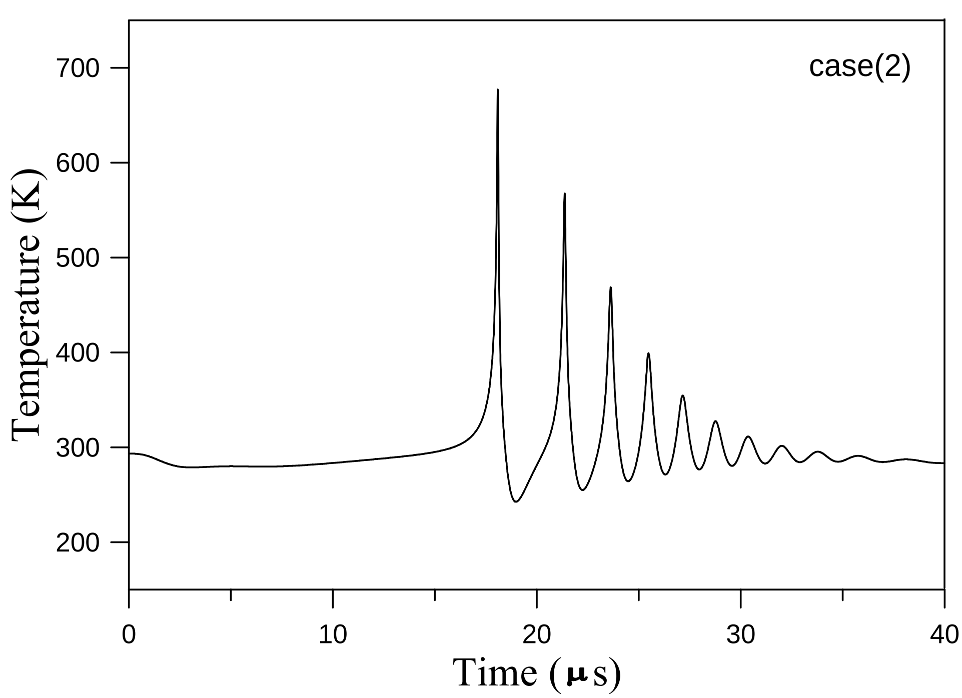

Fig. 8. The temperature at the bubble center as a function of time.

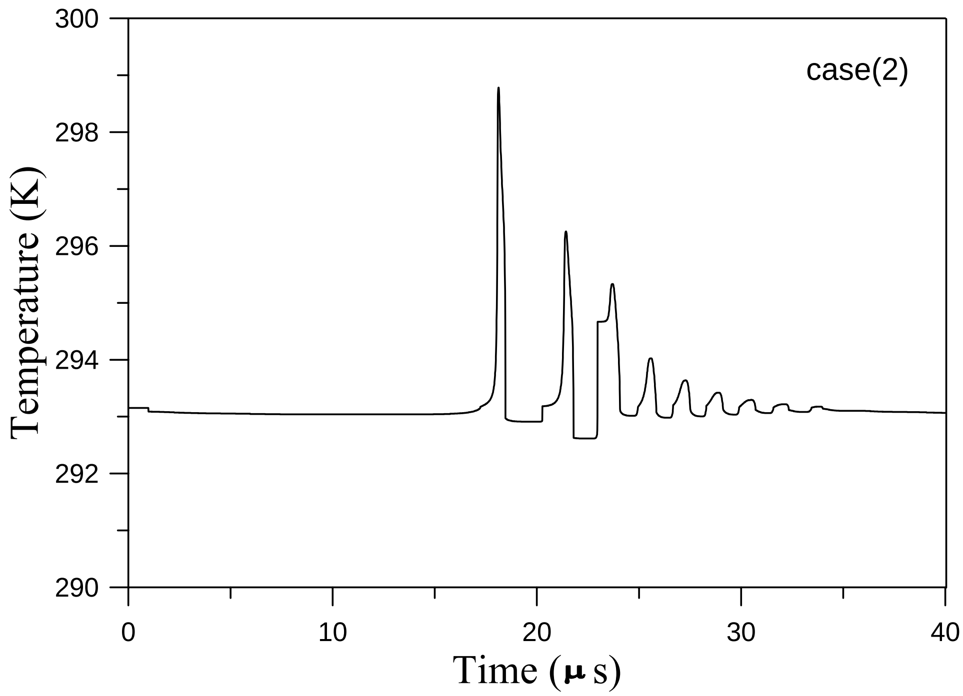

Fig. 9. The liquid temperature at the bubble wall (T_{Lb}) as a function of time.

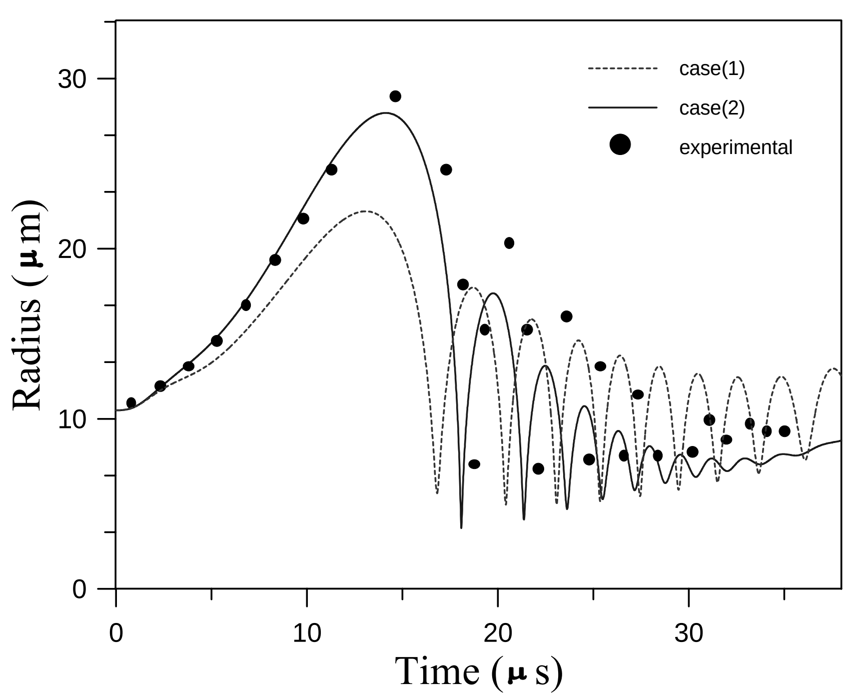

Fig. 10. Comparison between the calculated results and the experimental data[17] of radius-time curve for acoustic cycle.

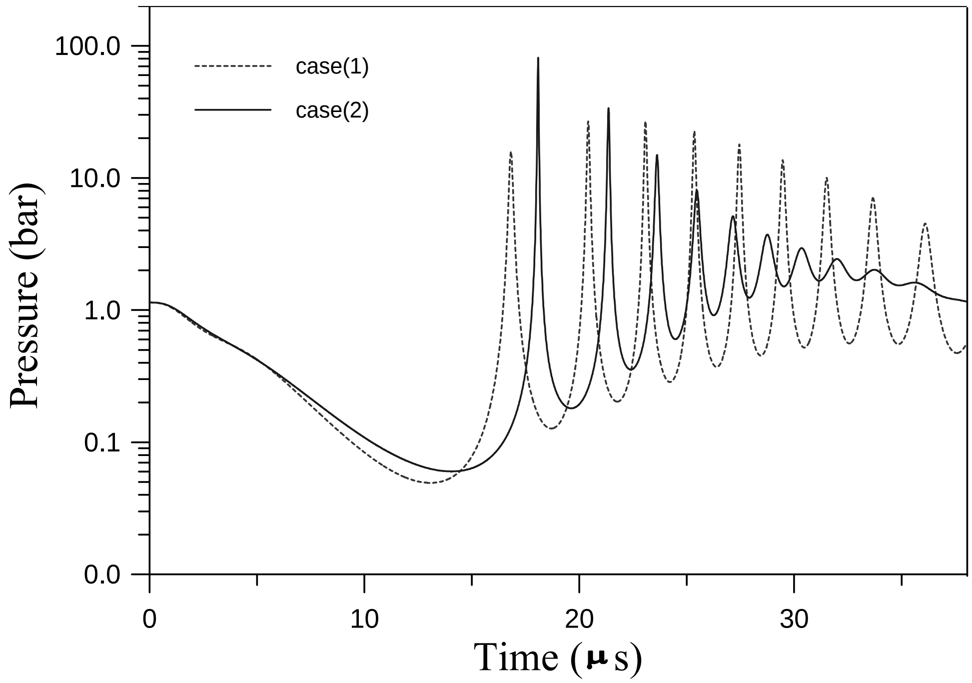

Fig. 11. The pressure inside the bubble (P_g) as a function of time with logarithmic vertical axis for two cases.

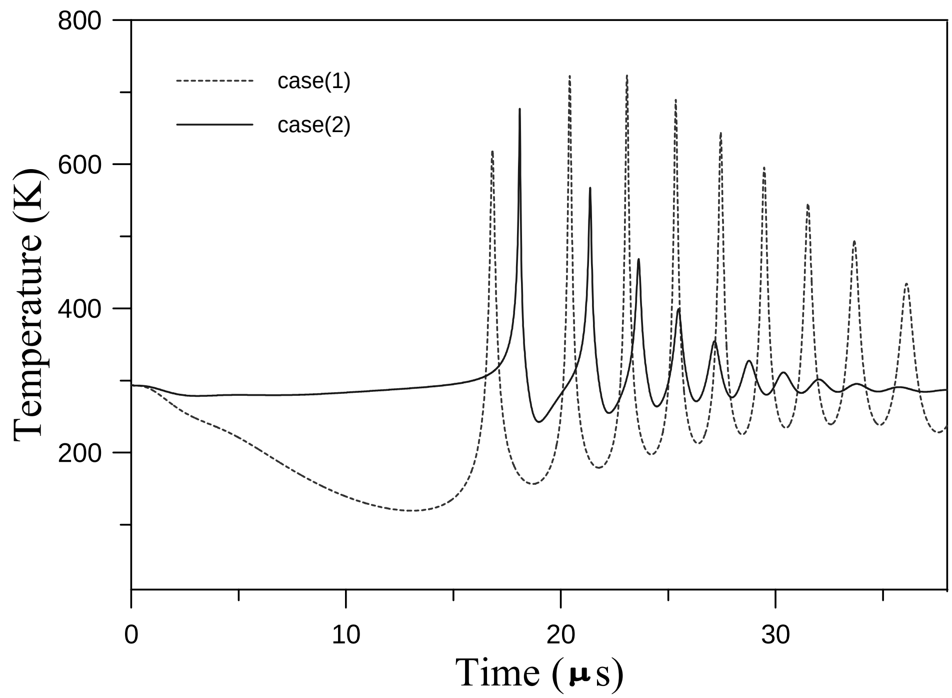

Fig. 12. The temperature at the bubble center as a function of time.