Some Factors Affecting the Natural Frequency of a Bubble Oscillating in liquids

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Abstract

Using Keller–Kolodner equation which includes the important physical properties, an equation of the natural frequency of a gas-field bubble oscillating in a liquid is derived. The components of the damping (the liquid compressibility, viscosity and surface tension) are included in this solution. But the gas diffusion and thermal conduction are neglected. The solution of Minneart is shown to be special case of this solution. The effects of the damping components of a bubble in water are examined. The effect of the ratio of specific heats of gas in the bubble is also examined. Comparisons with Minneart's solution are also presented and give good agreement.

Keywords: Cavitation, Natural Frequency, Bubble dynamics

بعض العوامل المؤثرة على التردد الطبيعي لفقاعة متذبذبة في السوائل

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الخلاصة

بأستخدام معادلة Keller-Kolodner التي تتضمن الخواص الفيزياوية المهمـة، تـم أشـتقاق معادلة التردد الطبيعي لفقاعة مملوؤة بالغاز ومتذبذة في سائل. يتضـمن هـذا الحـل مركبـات المخمـد (أنضغاطية السائل واللزوجة والشد السطحي), لكن تم أهمال أنتشارية الغاز والتوصيل الحراري. نلاحـظ أن حل (Minneart) يمكن استنتاجه كحالة خاصة لهذا الحل. تم اختبار تأثيرات مركبات المخمد لفقاعة في سائل ونسبة الحرارات النوعية في الفقاعة. قورنت النتائج مع طريقه (Minneart) حيـث أظهـرت تطابقاً جيداً.

Introduction

Studies on the natural frequency of a bubble in a liquid have been regarded by many investigators as a problem related to the behavior of cavitation bubbles, such as can commonly occur in a water turbine, pump, or other piece of hydraulic machinery, and many papers on these problems have been published [1-5].

An accurate theory of the natural frequency of bubbles that are as small (may be called minute bubbles) should take into account the effect of the surface tension, the viscosity, and in addition, the compressibility of the liquid. Among the existing theories of the natural frequency of a bubble, some consider the effect of surface tension [1, 6] alone, others the effects of surface tension and viscosity [7, 8]. In that papers, the effects of liquids compressibility were neglected.

In this paper, the natural frequency of a bubble vibrating in a viscous compressible liquid is derived, including the effect of surface tension but neglecting the effect of heat conduction. Therefore, the effects of compressibility, surface tension, viscosity, and ratio of specific heats of gas on the natural frequency are determined as a function of the static radius.

Theoretical Analysis

As shown in Fig. A, it is assumed that there is a spherical bubble whose static radius is R_o, at point O in a viscous compressible liquid, and that the bubble vibrates radially as shown by the dotted line. The effects of gas diffusion, gravity, heat conduction and evaporation-condensation are neglected. It is also assumed that the gas inside the bubble is adiabatic. In this section, the standard approach to the analysis of the pulsations a gas bubble is to assume that the pressure within the bubble follows a polytropic relation.

The main approximation contained in the present article is: the bubble contains only gas (*i.e.* the effect of the vapour contained in the bubble is negligible).

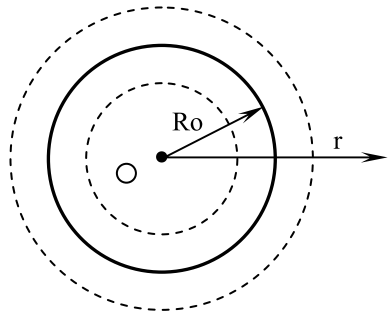

Fig. A Vibration of a bubble

For the motion of a spherical bubble, the following equation of Keller-Kolodner is used [9]:

$$R\frac{d^2R}{dt^2}\left(1-\frac{1}{c}\frac{dR}{dt}\right)+\frac{3}{2}\left(\frac{dR}{dt}\right)^2\left[1-\frac{1}{3c}\frac{dR}{dt}\right]=\left(1+\frac{1}{c}\frac{dR}{dt}\right)H+\frac{R}{c}\frac{dH}{dt} \quad ...(1)$$

where

$$H=\frac{1}{\rho}\left(P_w-P_o\right) \quad \ldots(2)$$

Assuming that the gas inside the bubble behaves adiabatically as the bubble oscillates; the pressure in the liquid at the bubble wall, P_w is given as follows [10].

$$P_w=P_e\left(\frac{R_o}{R}\right)^{3\gamma}-\frac{2\sigma}{R}-\frac{4\mu}{R}\frac{dR}{dt} \quad ...(3)$$

where,

$$P_e=P_o+\frac{2\sigma}{R_o} \quad \ldots(4)$$

Substituting eq.(2), eq.(3) and eq.(4) into eq.(1), have

$$R\frac{d^2R}{dt^2}\left[1-\frac{1}{c}\frac{dR}{dt}\right]+\frac{3}{2}\left(\frac{dR}{dt}\right)^2\left[1-\frac{1}{3c}\frac{dR}{dt}\right]=\frac{P_o}{P}(X-1)\left[1+\frac{1}{c}\frac{dR}{dt}\right]+\frac{1}{\rho c}Z \quad ...(5)$$

where,

$$X=\frac{P_e}{P_o}\left(\frac{R_o}{R}\right)^{3\gamma}-\frac{2\sigma}{P_oR}-\frac{4\mu}{P_oR}\frac{dR}{dt} \quad \ldots(6)$$

and

$$Z = -3\gamma \frac{P_e}{P_o}\left(\frac{R_o}{R}\right)^{3\gamma}\frac{dR}{dt} + \frac{2\sigma}{P_o R}\frac{dR}{dt} - \frac{4\mu}{P_o}\left[\frac{d^2R}{dt^2} - \frac{1}{R}\left(\frac{dR}{dt}\right)^2\right] \quad \ldots(7)$$

The initial conditions of eq.(5) are at t = 0

$$R = R_0 \quad \ldots(8)$$

$$\frac{dR}{dt} = 0 \quad \ldots(9)$$

Here, let us introduce the following dimensionless parameters:

$$y = \frac{R}{R_o} \quad \ldots(10)$$

$$\tau = \frac{t\sqrt{M}}{R_o} \quad \ldots(11)$$

where

$$M = \frac{P_o}{\rho} \quad \ldots(12)$$

Next, for the purpose of nondimensional expression of the basic eq.(5), the following quantities are introduced,

$$m = \frac{P_e}{P_o} \quad \ldots(13)$$

$$S = \frac{2\sigma}{P_o R_o} \quad \ldots(14)$$

and

$$Q = \frac{4\mu}{R_o\sqrt{\rho P_o}} \quad \ldots(15)$$

Then, eq.(5) can be reduced to:

$$y\frac{d^2y}{d\tau^2}\left[1 - \frac{\sqrt{M}}{c}\right] + \frac{3}{2}\left(\frac{dy}{d\tau}\right)^2\left[1 - \frac{\sqrt{M}}{3c}\frac{dy}{d\tau}\right] = (X_o - 1)\left[1 + \frac{\sqrt{M}}{c}\frac{dy}{d\tau}\right] + \frac{Z_o}{c} \quad \ldots(16)$$

where,

$$X_o = m\left(\frac{1}{y}\right)^{3\gamma} - \frac{S}{y} - \frac{Q}{y}\frac{dy}{d\tau} \qquad ...(17)$$

and

$$Z_o = \sqrt{M}\left[\begin{array}{c} -3\gamma m\left(\frac{1}{y}\right)^{3\gamma} + \frac{S}{y} - \\ Q\,\frac{\frac{d^2y}{d\tau^2}}{\frac{dy}{d\tau}} + \frac{Q}{y}\frac{dy}{d\tau} \end{array}\right]\frac{dy}{d\tau} \qquad \ldots(18)$$

The linear solutions of eq.(16) can be obtained by supposing that a bubble oscillates with very small amplitude. Thus, we put

$$y = 1 + \in \qquad ...\ (19)$$

where $(\in << 1)$.

Substituting eq.(19) into eq.(16), and neglecting the higher order terms of $\in$, $0(\in^2)$, have

$$\frac{d^2\in}{d\tau^2} + \left[\frac{S}{1+\in} - \frac{m}{(1+\in)^{3\gamma}} + 1 + \frac{Q}{1+\in}\frac{d\in}{d\tau}\right] \ldots(20)$$
$$\left(1 + \frac{\sqrt{M}}{c}\frac{d\in}{d\tau}\right) - \frac{\sqrt{M}}{c}$$
$$\left[\begin{array}{l} \frac{S}{1+\in} - \frac{3\gamma m}{(1+\in)^{3\gamma}} + 1 - Q\,\frac{\frac{d^2\in}{d\tau^2}}{\frac{d\in}{d\tau}} + \\ \frac{Q}{1+\in}\frac{d\in}{d\tau} - 1 \end{array}\right]\frac{d\in}{d\tau} = 0$$

With the aid of the following equation (see ref. [3]).

$$\frac{A}{1+\in} - \frac{B}{(1+\in)^{3\gamma}} + 1 = [(3\gamma - 1)A + 3\gamma]\in \qquad ...(21)$$

Therefore, eq.(20) becomes:

$$\left[1+\frac{\sqrt{M}}{c}Q\right]\frac{d^2\in}{d\tau^2}+\left[Q+\frac{\sqrt{M}}{c}\right]\frac{d\in}{d\tau}+\left[(3\gamma-1)S+3\gamma\right]\in=0 \qquad ...(22)$$

Equation (22) is the equation for the vibration of a bubble in a liquid.

Then, the circular frequency of the bubble is the imaginary part of the root of the quadratic eq.(22). Namely,

$$\omega=\frac{\sqrt{4\left[1+\frac{\sqrt{M}}{c}Q\right]\left[(3\gamma-1)S+3\gamma\right]-\left[Q+\frac{\sqrt{M}}{c}\right]^2}}{2\left[1+\frac{\sqrt{M}}{c}Q\right]} \qquad ...(23)$$

If we put as

$$\omega=\omega_n R_o\sqrt{\frac{\rho}{P_o}}=\frac{\omega_n R_o}{\sqrt{M}} \qquad ...(24)$$

Hence, the bubble natural frequency f_n is given by

$$f_n=\frac{\omega_n}{2\pi} \qquad …(25)$$

Therefore,

$$f_n=\frac{\omega}{\frac{2\pi R_o}{\sqrt{M}}} \qquad …(26)$$

Substituting eq.(23) into eq.(26), have

$$f_n=\frac{\sqrt{4\left[1+\frac{\sqrt{M}}{c}Q\right]\left[(3\gamma-1)S+3\gamma\right]-\left[Q+\frac{\sqrt{M}}{c}\right]^2}}{4\pi R_o\left[\frac{1}{\sqrt{M}}+\frac{Q}{c}\right]} \qquad …(27)$$

Here, neglecting the effects of the compressibility $\left(\frac{\sqrt{M}}{c} << 1\right)$, surface tension $(\sigma = 0)$ and viscosity $(\mu = 0)$, we have

$$f_n = \frac{1}{2\pi R_o}\sqrt{\frac{3\gamma P_o}{\rho}} \qquad \text{... (28)}$$

This equation is identical with the one given by Minneart as is reviewed in ref. [11].

Results

In this section, as an example of numerical calculations for the bubble natural frequency, comparisons of the present solution with Minneart's solution are made. The effects of liquid temperature, surface tension, viscosity, and ratio of specific heats for gas are also determined.

The value used for the pressure P_o in the liquid at infinity from bubble is 101.3 kPa. The liquid and gas used in the calculation are water and air respectively. The physical properties of water show in Table 1 [12].

Table 1 Properties of water at 101.3 kPa

T (°C)	ρ (kg/ m^3)	σ (mN/m)	μ (mPa.s)
10	999.7	74.22	1.007
50	988	67.95	1.008

The effect of water temperature. As the liquid temperature changes, various physical values of the bubble, liquid density, surface tension, and viscosity will changes as shown in Table 1. So, even if the size of the bubble is the same, the value of the natural frequency (f_n) changes. Hence the calculated values of f_n in each case of water temperature 10 and 50 °C is shown in Fig. 1 and Fig. 2, respectively. From the figures, it can be seen that the value of f_n becomes slightly larger as the water temperature rises.

The effect of surface tension. The comparison of the calculated results with and without the effect of surface tension (σ) is shown in Fig. 3 and Fig. 4 for water temperature 10 and 50 °C, respectively. In these figures the case of considering surface tension is shown by the solid line and the case of neglecting it is shown by the dotted line. It can be seen from this figure that the effect of the surface tension (σ) appears noticeably at R_o< 0.01mm and that the value of f_n becomes somewhat larger.

The effect of viscosity. In Fig. 5, the comparison of the calculated results with and without the effect of viscosity is shown. It is evident that the effect of viscosity is slightly negligible.

The effect of specific heat ratio. In Fig. 6, the comparison of calculated results in the case that the specific heat ratio of gas (air) in the bubble (γ) is 1.4 with those in the case that it is 1.2 is shown. It can be seen from this figure that the effect of the specific heat ratio is slight.

Comparison with Minneart's solution. The results of comparisons with Minneart's solution are shown in Fig. 7 and Fig. 8 for water temperature (T_∞) 10 °C and 50 °C, respectively. In these Figures the solid lines are the present work and the dotted lines are Minneart's solutions. Minneart's solution doesn't include the consideration of compressibility, viscosity and surface tension.

From these figures, it can be seen that these effects scarcely appear for R_o>0.01mm. The present solution shows larger value than those of Minneart as R_o becomes smaller.

Conclusions

The results obtained in the present study may be summarized as follows:

1- The theoretical equation of the natural frequency in a viscous compressible liquid (including the effect of surface tension) is given by eq.(25). But, gas diffusion and heat conduction are neglected.

2- Minneart's equation can be obtained as a special case from our equation, *i.e.*, eq.(28).

3- As an example of calculated value, f_n-R_o curves are drawn in both cases of considering the effects of compressibility, viscosity and surface tension (partly or all) and the case of neglecting them. From this, the following facts can be concluded: (a) the effects of compressibility, viscosity, and water temperature scarcely appeared for R_o>0.01mm. (b) The effect of surface tension appears remarkable within the range of R_o<0.01mm. (c) The effects of water viscosity and ratio of specific heats of gas (air) in the bubble are slight.

Nomenclature

c: Sound speed in the liquid (m/s).

f_n: Bubble natural frequency (Hz)

H: Enthalpy at bubble wall (J/kg)

P_e: Static pressure in the bubble (Pa)

P_o: Pressure in the liquid at infinity (Pa)

P_w: Pressure at bubble wall (Pa)

R: Bubble radius (m)

R_o: Equilibrium bubble radius (m)

t: Time (s)

T_o: Static liquid temperature (K)

γ: Ratio of specific heats of gas

μ: Liquid viscosity ($N.s/m^2$)

ρ: Liquid density (kg/m^3)

σ: Liquid surface tension (N/m)

ω: Angular frequency (rad/s)

ω_n : Normalized angular frequency (1/s)

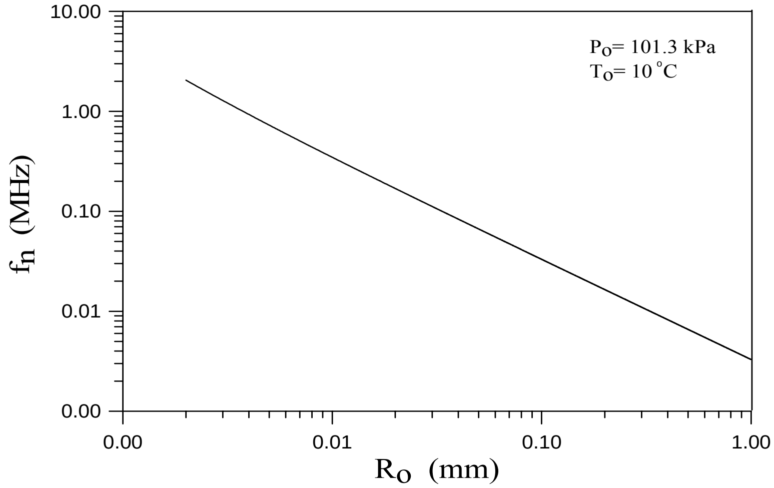

Fig. 1 The natural frequency as a function of the bubble static radius

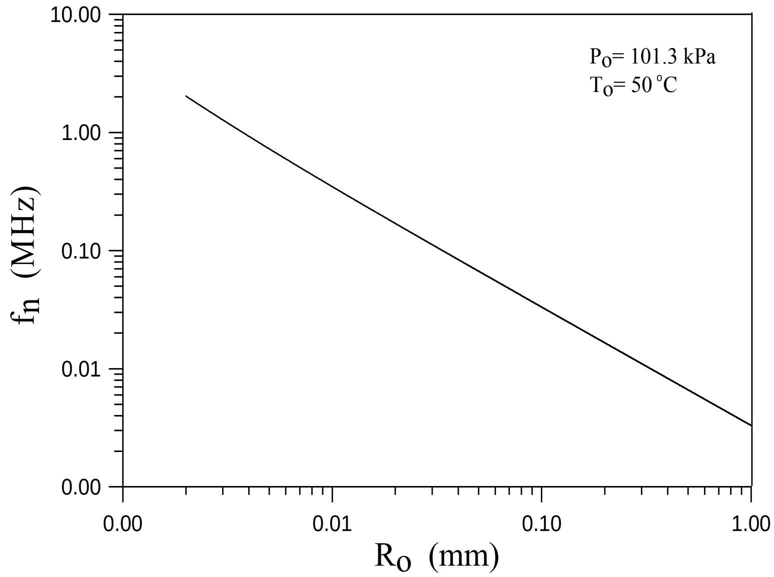

Fig. 2 The natural frequency as a function of the bubble static radius

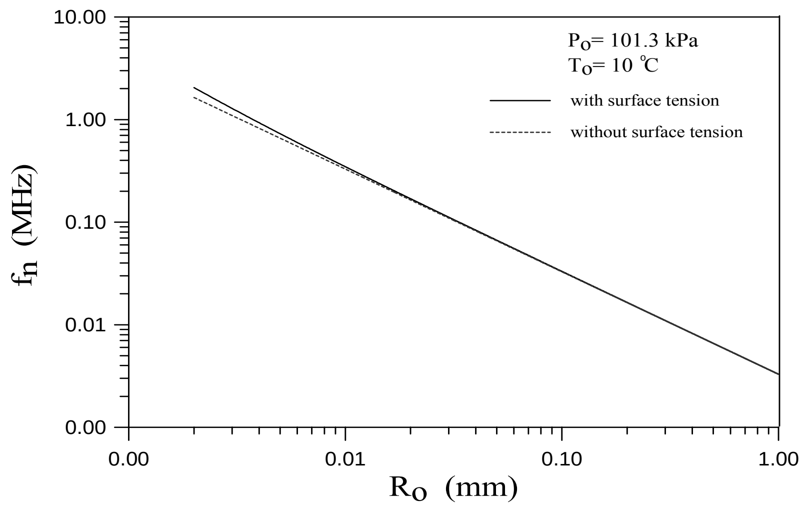

Fig. 3 Effect of surface tension on th relation between the natural frequency and the bubble static radius

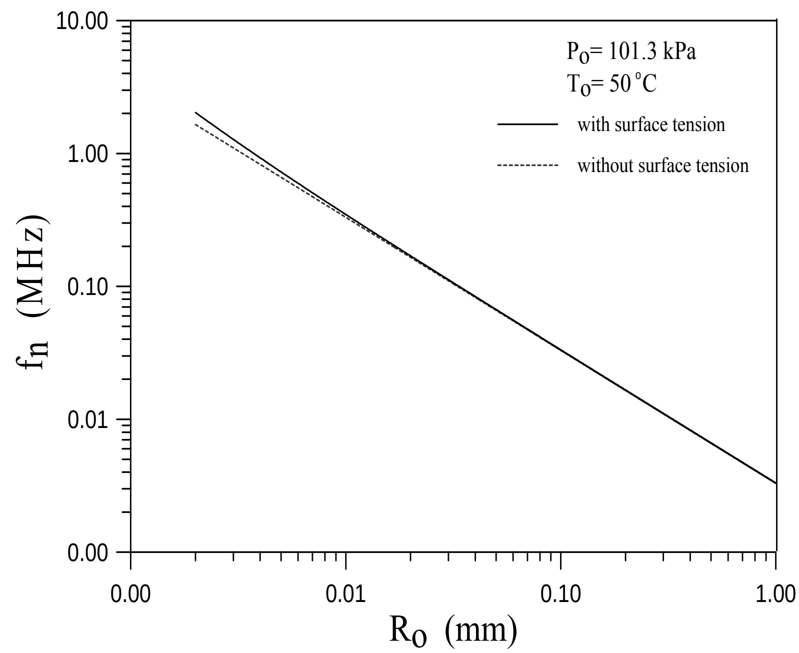

Fig. 4 Effect of surface tension on th relation between the natural frequency and the bubble static radius

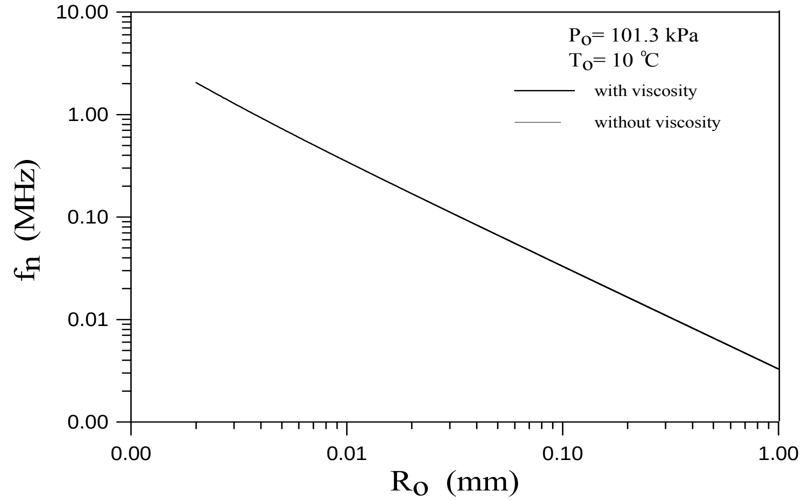

Fig. 5 Effect of viscosity on th relation between the natural frequency and the bubble static radius

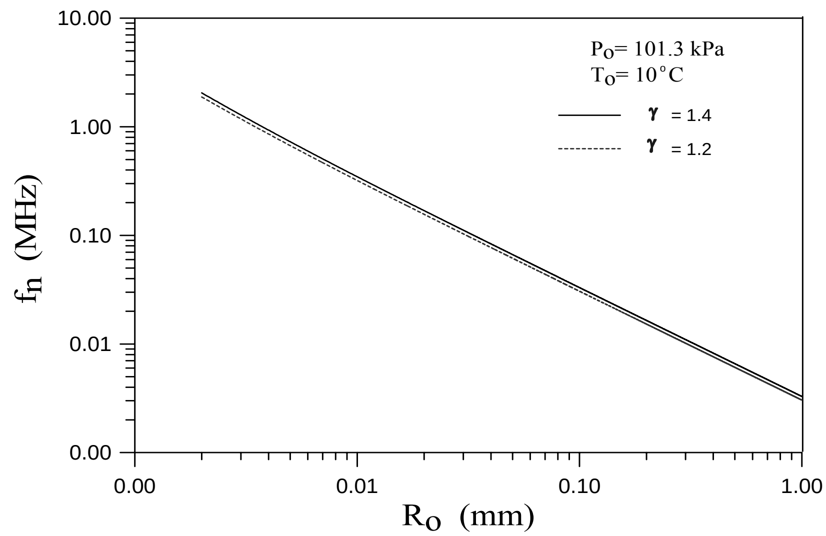

Fig. 6 Effect of specific heat ratio on th relation between the natural frequency and the bubble static radius

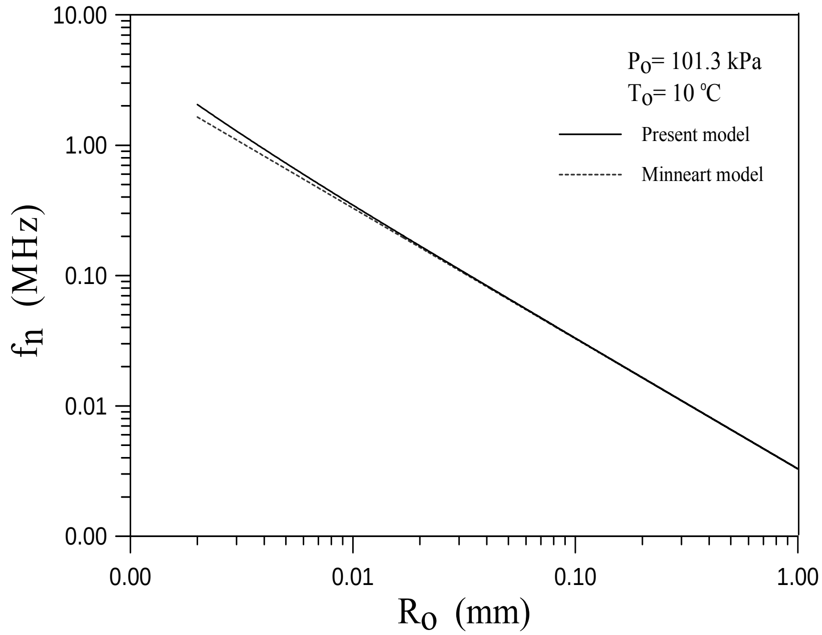

Fig. 7 Relation between the natural frequency and the bubble static radius comparison with an existing model

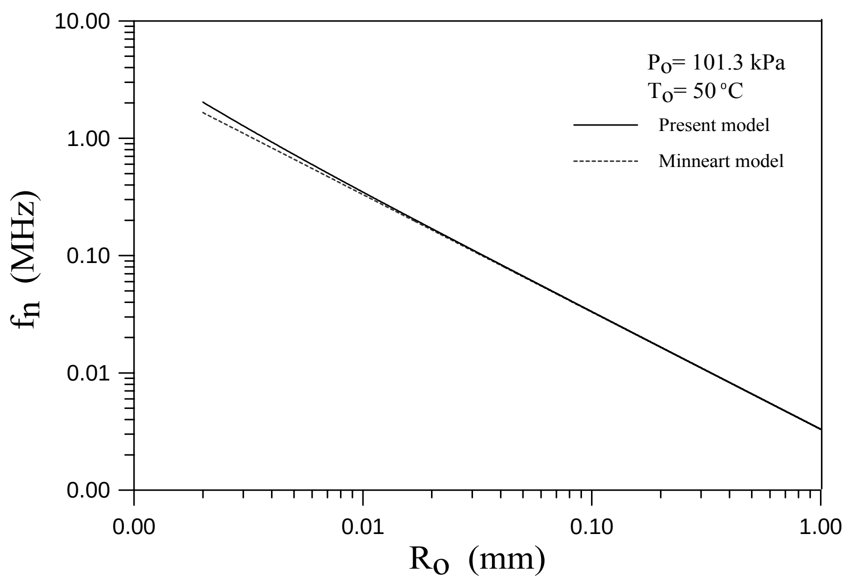

Fig. 8 Relation between the natural frequency and the bubble static radius comparison with an existing model

References

A. Prosperetti, L. A. Crum and K. W. Commander, Nonlinear bubble dynamics. Vol. 83, No. 2, 1988.

A. Shima, T. Tsujino and H. Nanjo, Nonlinear oscillations of gas bubble in viscoelastic fluids. Ultrasonics, Vol. 24, 1986.

A. Shima, The natural frequencies of two spherical bubble oscillating in water. J. of Basic Engineering, Sep. , 1971.

E. A. Neppiras and B. E. Noltingk, Cavitation produced by ultrasonics, Theoretical conditions for the onset of cavitation. Proceedings of the physical society, Vol. 64, 1951.

F. D. Smith, On the destructive mechanical effects of the gas bubble liberated by the passage of intense sound through a liquid. Philosophical Magazine, Vol. 19, 1935.

G. Houghton, Theory of bubble pulsation and cavitation. J. Acoust. Soc. Am., Vol. 35, 1963.

M. Strasberg, The pulsation frequency of nonspherical gas bubble in liquids. J. of Acoust. Soc. of Am., Vol. 25, 1953.

S. Sochard, A. M. Wilhelm and H. Delmas, Gas-vapour bubble dynamics and homogeneous sonochemistry. Chem. Eng. , 51, 1997.

T. Fujiwara A. Shima, Nonlinear oscillations of bubbles in compressible hydraulic oils. J. Acoust. Soc. Am. 68 (5), 1980.

T. G. Leighton, The acoustic bubble. Academic Press, N. Y., 1994.

T. Hirose and M. Okuyama, Mechanics on the cavitation produced by ultrasonics (II) conditions for cavitation and stationary cavitation. J. Acoust Soc. of Japan, Vol. 13, 1957.

T. Niederdrank and B. Wiesand, The temperature dependent behavior of a cavitation bubble field. Acoustica, 84, 1998.