Study of the scattering and Annihilation of low energy positron with noble atom

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Abstract

Although the electron and positron scattering experiments have been studied for long time, the construction of the scattering potential is still under the studies and it is one of the most important matter which influence the parameters calculations such as the phase shift (δ_0), scattering length (A_o) , cross section (σ) , elastic and inelastic where they play as a major role in evaluating the electron- positron annihilation process. In the present calculation we model the positron target system by a complex interaction potential v(r), it consist from static potential $V_{st}(r)$, the correlation polarization potential $V_{sp}(r)$ and the absorption $V_{abs}(r)$. Using the above mentioned potentials, values of scattering length, phase shift, cross section is calculated and used to work out the annihilation parameters. The calculation suggests that the features of the model potentials can produce reliable total cross section for positron atom scattering at a certain impact energies. The annihilation parameter (Z_{eff}) is one of the essential factors in it. Two different formula have been applied for low positron energy interact with atomic and ionic system where as the most recent one of (Z_{eff}) is used and it's distinguished on that of the atom is that it can calculate the annihilation parameter for P and D wave in addition to the S- wave. Throughout (Z_{eff}) we evaluated the annihilation parameters ($\sigma_a, \lambda_a, \tau$) and electron density dependents. A good comparison of the factors have been made. We have borrowed some explicit and implicit well known formula in evaluations the inelastic cross section of positron with free atom. In general the comparison our data with the other is good for a certain channels and so well for some other channels.

دراسة تشتة وافناء البوزترون ذو الطاقة الواطئة من ذرات النبيلة

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الخلاصة

على الرغم من ان استطارة الالكترون والبوزترون قد تمتدراستها ولفترة طويلة من الزمن الا ان عملية تركيب جهد الاستطارة واحدة من اهم الامور المؤثرة في معاملات استطارة المحسوبة, حيث ان كلا من ازاحة الطور (δ_0) , طول الاستطارة (A_0) والمقاطع العرضية وغير المرنة تعمل كقاعدة اساسية في تحديد عملية فناء البروتون والالكترون. في حسابتنا الحالية قمنا بتركيب نموذج جهد التفاعل المعقد والذي يتكون من الجهد الاستاتيكي, جهد الاستقطاب التراكبي وجهد الامتصاص وباستخدام الجهود المذكورة اعلاه فان قيم كل من طول الاستطارة, ازاحة الطور والمقاطع العرضية قد تم حسابها واستخدتمها في ايجاد معاملات الفناء. ان الحسابات تقترح ان الخصائص لنماذج الجهود هذه يمكن ان تعطي مقاطع عرضية كلية مقبولة لحالة استطارة بوزترون – ذرة عند قيم طاقات سقوط معينة. ان لمعامل الفناء والذي يمثل بدوره المعامل الرئيسي في عملية الفناء هذه صيغتين طبقت على تفاعل البوزترون الواطئ الطاقة مع منظومة الذرة والايون والاحدث فيها مع الذرة والذي يميزه من انه يمكن حساب معملات الفناء لموجات D,P بالاضافة الى S. من خلال هذا المعامل فاننا نستطيع حساب معاملات اخرى مثل المقطع العرضي للفناء, ومعدل الفناء والذي يعتمد على الكثافة الالكترونية ومن خلال حساب معدل الفناء فاننا نستطيع حساب زمن العمر. عملت مقارنة جيدة لهذه الحسابات وقمنا بالاستعاضة ببعض الصيغ الضمنية والصريحة والمعروفة بشكل جيد في حساب المقاطع العرضية غير المرنة لحالة استطارة البروتونات من الذرة الحرة. بصورة عامة كانت المقارنات بين نتائجنا المستحصلة وتلك الخاصة بالحسابات الاخرى جيدة لبعض القنوات وجيدة جداً لقنوات اخرى.

Introduction

one of the most important events happen when positron interact with atom is the annihilation process of postronium (e^++e^-) atom which is proble structure formed during the interaction . The condition for positron binding to a neutral atom is dependent on the ionization potential I of the parent atom. If (I) is greater than the binding energy of positronium (i-e E(Ps)) = 0.25 Hatree= 6.8ev) then the energy of the positron- atom complex must be less then the ground state. Energy of the atom for the system to be stable against dissociation into (e^+ + atom)- left side of fig (1). The positron affinity, which is defined as the energy gained by the positron when attached to the atom, is in this case equal to the binding energy (ε)of the positron atom complex. However; if (I) in less than 0.25 Hartree, then the condition for positron binding require that the (Ps + A^+) system be stable against dissociation. This occurs when the positron affinity of the atom exceeds (0.25- I)$^-$ see the right side of fig (1). For atom with I< 0.25 Hartree, the positron affinity does not

equal to the binding energy in the present calculation we have contracted our model of the interaction (effective) potential from the static , correlation , polarization and absorption potentials . the hasrtree-fock wave function was used to obtaine the static potential (1) , where as for the polarization potential we used the formula given by D.D.Raid et al (2) . also the quasi free model of the abortion potential given by Reid wadehra et al. the non –relativistic partial wave method is applied to study the positron interaction . Althought the calculation was concentrated on the elastic and anelastic cross section extended to higher energy impact . but a low energy positron needed to creat the annihilation using the above mentioned potential values of the scattering length phase shifts . croo section is calculated and use to work out the annihilation parameters . the calculation suggest that the features of the model potential can produced reliable total cross section for positron scattering at a certain impact energies . The three different types of potentials have been added together to study the annihilation phenomena [4-5]. We have used two different formulas of annihilation parameters (Z_{eff}) proposed by G.F. Gribakin et al [6] and J. Mitory el at [7] depending in their general formula on phase shift which in turn have been calculated using the partial wave method. We noticed our data of (Z_{eff}) (P.W.1) a greed with the calculation of Mceachram el at [8] and the enhanced, and the pole (Z_{eff}) approximation of Mitroy and Ivanov where as , our data of (Z_{eff}) (P.W.2), some what showed a convenience with modified pole (Z_{eff}) data for Mitroy and Ivanov for all the noble system. Through out (Z_{eff}) we evaluated the annihilation cross section (σ_a), the electron density dependents, the annihilation rate (λ_a) and the time required for positron to annihilate with one of the target electron, the life time (τ) which it represent the inverse of the annihilation rate [9]. A good comparison of the factor have been made between theoretical and experimental data, we notice the life time for positron interacting with inert atoms is large compared with the life time of the alkali and other atoms. Energetic positron may have a fair possibility of the atomic electron to form an electron positron bound state, the postronium (Ps) atom. In the present work we have used a formula of the Ps- formation cross section (σ_{PS}) depend on the effective number of electron

(Z_{eff}) proposed by Gribakin and Ludlow [10]. There has been an increasing interest in investigating ionization phenomena by positron impact . Most of the system, which have been used to calculate and measure the ionization cross section were the hydrogen and noble atom. A comparison our data with the theoretical and experimental have been made where our calculation --- generally in fair agreement with recent calculation and those available of experiments.

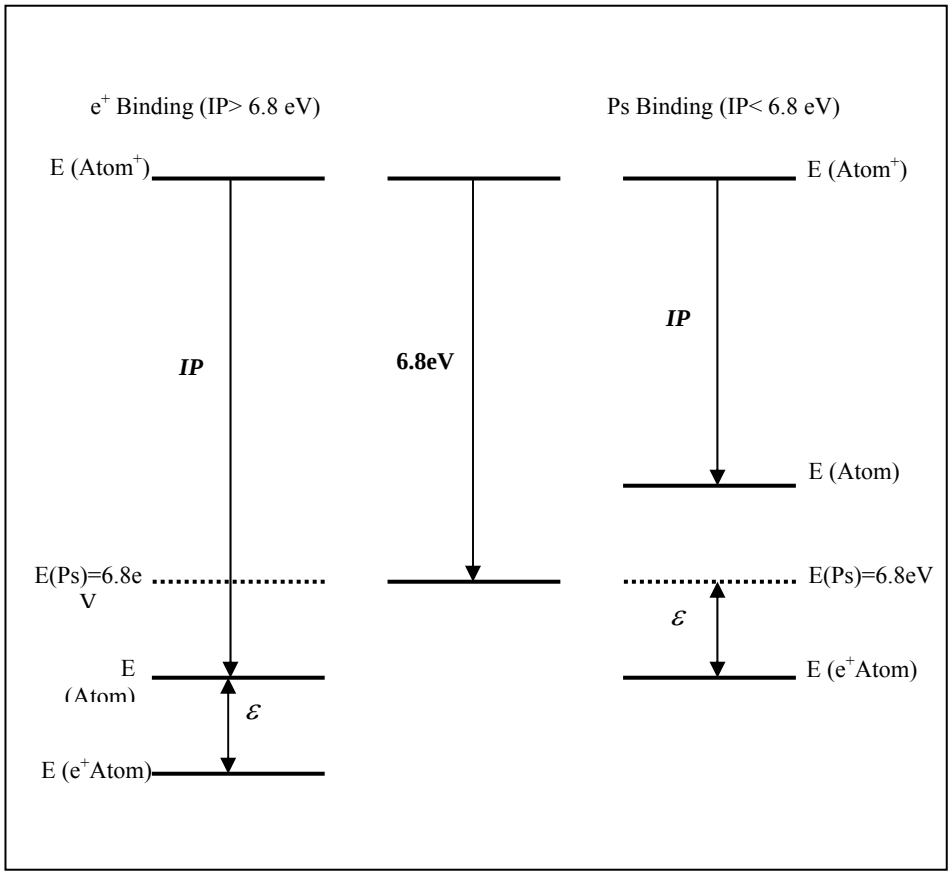


Figure (1): Energy level Diagram for the two conditions for e^+-Atom binding depending on whether the ionization potential of the parent atom is greater than or less than the binding energy of

Mathematical Description

The construction of the model is given by the following relation [4-5]

$$V(r) = V_{st}(r) + V_{cp}(r) + iV_{abs}(r) \quad \text{…….. (1)}$$

Where the static potential is determined by radial part of the electron charge density of the target atom $\rho(r)$ which is obtained using Hartree- Fock wave function of Clementi and Roetti et al[1]

$$V_{st}(r) = \frac{Z}{r} - 4\pi \int \frac{\rho(r)}{r} r' dr' \quad \text{……(2)}$$

Where z is atomic no of the target atom

r is the great of r (position of positron) and r' (position of the electron)

In fig (2, 3)we show the static and effective (optical) potential. The polarization is given by D.D. Reid et al

$$V_{cp}(r) = \frac{\alpha_d r^2}{2(r^2 + d^2)^3} \quad \text{……..(3)}$$

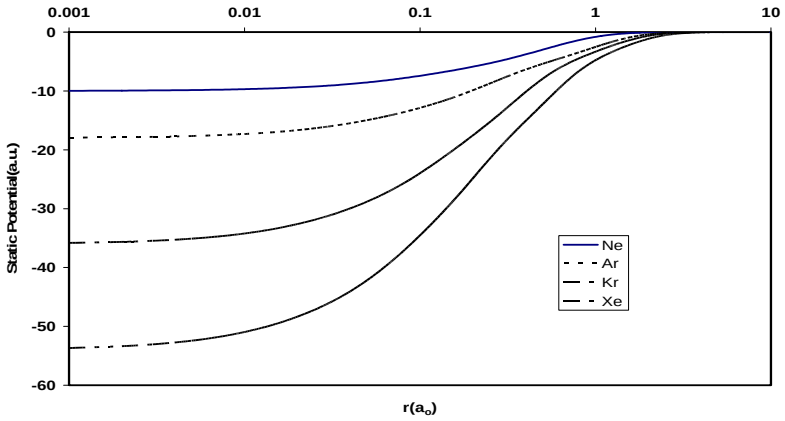
Where α_d is the dipole polaizability of the core and d is cut off parameter .

Reid and Wadehra et al have derived a formula of the absorption potential for positron- atom scattering.

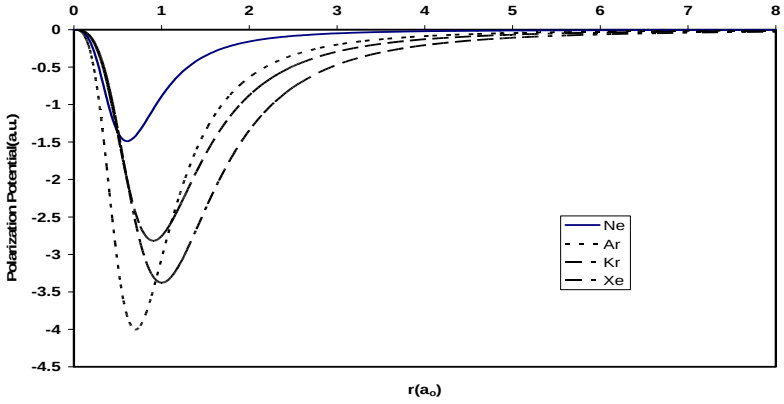
$$V_{abs} = -\frac{1}{2} \rho \overline{\sigma_b} v \quad \text{……..(4)}$$

Where v is the local speed of projectile and σ'_b is the average cross section for the binary cell is on between the position and the target electrons .

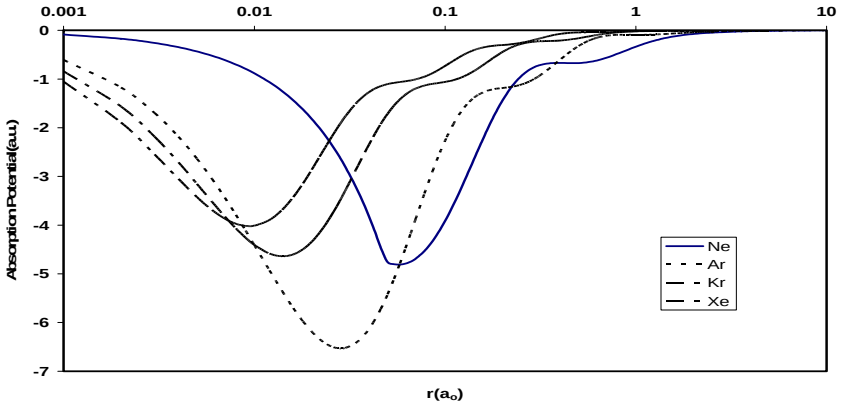
In figure (4) is shown the absorption potential with distance (r) and distance density parameter (r_s). For convenience Reid and Wadehra et al used the following notation to derive the average binary cross section they define



Figure(2):present the static potential of four nobel systems



Figure(3):present the polarization potential of four nobel systems



Figure(4):present the absorption potential of four nobel systems

As we mentioned earlier in our analysis we have used the partial wave method, and this lead to write down the following relations

The total cross- section is given by:

$$\sigma_{total}(k) = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell(k) \qquad \text{……..(5)}$$

Also, the momentum transfer cross section can be expressed as

$$\sigma_M = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(\ell+1)\sin^2(\delta_\ell(k)-\delta_{\ell+1}(k)) \qquad \text{…… (6)}$$

Where δ_l (k) is the phase shift

The general expression for the effective number of electron (Z_{eff}) which is independent of density and characterized the position –atom system can be written as [10,11].

$$Z_{\text{eff}} = \int\sum_{i=1}^{z}\delta(r-r_i)\left|\psi_k(r_1-r_z\right|^2 dr_1 - -dr_z dr \qquad \text{……..(7)}$$

Where (z) is the number of the target electrons, (r_1) and (r) are the coordinate of the electron and positron respectively , $\psi_k(r_1 \ldots r_z, r)$ is the total wave function of the system, and $\delta(r-r_i)$ is the kroneck or dealt function Equation (7) describes the scattering of the positron with initial momentum (k) from atomic target in the ground state (ϕ_K) and it's normalized as:

$$\psi_K(r_1 \ldots r_z, r) \approx \varphi_0(r_1 \ldots r_z)e^{ikr} \qquad r >> R_a \qquad \text{……..(8)}$$

Where (R_a) is the radius of the target, and not the cut- off radius (r_0).

The scattering wave function is determined by the positron interaction with the charge distribution of the ground target and electron- positron correlation interaction. If we denote the corresponding wave function as $\psi_k^{(0)}$

The direct annihilation can be expressed as [7,12]

$$Z_{eff}\left\langle\psi_k^{0)}\left|\sum_{i=1}^{z}\delta(r-r_i)\right|\psi_k^{(0)}\right\rangle \qquad \text{……..(9)}$$

Such that the wave function contain contribution of the incoming and scattered positron waves.

$$\psi_k^{(0)}(r_1 - -r_z, r) \approx \varphi_0(r_1 - -r_z)\left[e^{ikr} + f(\Omega)\frac{e^{ikr}}{r}\right] \quad \text{.....(10)}$$

Where $f(\Omega)$ is the scattering amplitude. Because of the repulsion from the atomic nuclei, the low-energy positron does not penetrate deeply inside the atomic system. Accordingly, the positron annihilation mostly done with outer valence electron, where the electron and positron densities overlap after substitution of expression (10) in the (9) one obtain.

$$Z_{eff} = \int \rho(r)[e^{ikr} + f(\Omega)\frac{e^{ikr}}{r}]x[e^{-ikr} + f(\Omega)\frac{e^{-ikr}}{r}]\, r^2 dr d\Omega \quad \text{.....(11)}$$

Where ρ(r) = $\varphi_0\left|\sum_{i=1}^{z}\delta(r - r_i)\right|\varphi_0 >$ is the electron densities in the ground state of the system

$$Z_{eff}^{(dir)} = \int \rho(r)[1 + f^{\circ}(\Omega)e^{ikr}\frac{e^{-ikr}}{r} + f(\Omega)e^{-ikr}\frac{e^{-ikr}}{r} + \frac{f(\Omega)f^{\circ}(\Omega)}{r^2}]r^2 dr d\Omega$$
$$= \int \rho(r)[r^2 + f^{\circ}(\Omega)re^{ikr}e^{-ikr} + f(\Omega)re^{-ikr}e^{ikr} + f^{\circ}(\Omega)f(\Omega)]dr d\Omega$$
$$\text{.....(12)}$$

The electron density drops quickly outside the atom and the positron density decrease rapidly inside the atom. Therefore; the integration in equation (12) should be taken over a relatively thin shell of thickness (δR_a) enclosing the atomic system for small positron momentum ($KR_a < 1$), Equ. (12) yield.

$$Z_{eff}^{(dir)} = 4\pi\rho_e \delta R_a (R_a^2 + \frac{\sigma_{el}}{4\pi} + 2R_a R_e f_0) \quad \text{.....(13)}$$

Where (ρ_e) is the electron density in the annihilation range, (σ_{el}) is the elastic cross section,

$\sigma_{el} = \int |f(\Omega)|^2 d\Omega$. and f_0 the spherical Symmetric part of the scattering amplitude

$f_0 = (4\pi^{-1})\int f(\Omega)d\Omega$. for positron interaction with an atom, the latter is simply equal to the S-wave scattering amplitude it's real part is expressed

interms of the S-wave phase shift (δ_0) as $R_e f_o = \sin 2\delta / 2k$. The S- wave gives a dominate contribution to the cross section (σ_{el}) at low energies [13] for $k \longrightarrow o$

$\sigma_{el} = 4\pi A_0^2$ *as* $f(\Omega = -A_0$ where the factors, $\rho_e, \delta R_a R_a$ are free parameter that are fixed for each atom by comparison with experiment or abinito calculation [6]. Equation (21) allows one to analysis the typical features of (Z_{eff}) due to direct annihilation. The factor ($4\pi\rho_e \delta R_a = F$). Equ. (21) can be estimated using the electron density at the origin of Ps (1s), Pe ~Ps= $\frac{1}{8\pi}$ [7] and $\delta R_a \sim 1$ which yield (F=0.5). therefore; unless σ_{el} is much greater than the geometric size of the target, direct annihilation given Z_{eff} ~1-10. [13] when scattering cross section is large , the annihilation rate is greating enhanced. If the energy close to zero, ($\varepsilon_0 = \pm \frac{k^2}{2}, k = A_0^{-1}$) , $|k| << R_a^{-1}$ .The S- wave scattering amplitude is given by [14].

$$f_0 = \frac{1}{k + iK} \qquad \text{.....(14)}$$

and the cross section peaks strongly at small momentum $\sigma_{el} = \frac{4\pi}{k^2 + K^2}$, it's magnitude being much , greater than the geometric size of the target. In this case the 2nd term in the brackets in equ.(14) dominates, and $Z_{eff}^{(dir)}$ shows a similar peak [14].

$$Z_{eff}^{(dir)} \cong F \frac{1}{k^2 + K^2} \qquad \text{.....(15)}$$

Gribakin et al in his formula equ. (9) developed a theoretical frame work that could be used to explain the wide range of phenomena associated with positron annihilation or atom.

The number of calculation of the annihilation rate (λ_a) for positron atom scattering is few [17,18] also there have been a few calculation of the annihilation cross section [19,20] .

Most of the information on (Z_{eff}), comes from the experimental measurement in positron trap. This set up ensures that the positron are fully thermalized and the annihilation take place in binary collision [13]. The life time of the positron two- quantum decay is given essentially, by the integral of

the square of the electron- positron wave function evaluated where the electron and positron coordinates are equal, [21] and it can be expressed as, [22]

$$\tau = \frac{1}{\lambda_a} \qquad \ldots\ldots\ldots\ldots(16)$$

Only few investigation performed to study the life time for inert atoms [17,18] and alkali,[21] Experimentally measurement of life time of positron in noble gases, has indicated a dependence on temperature, Miller et al [23], Canter and Roelling [24].

The studies of annihilation problem for positron interacting with ions has been done recently by Novikov. et al [25]. By deriving an analytical expression for (Z_{eff}) of the following equ.

$$Z_{eff}^{(\ell)} = w(\eta)[a_\ell(v) + b_\ell(v)\frac{k^2}{(Z-1)^2} + O(k)^4] \qquad \ldots\ldots\ldots(17)$$

Where $\eta = \frac{Z-1}{k}, v = \frac{Z-1}{Z}, w(\eta)\frac{2\pi\eta}{e^{2\pi\eta}-1}$, for $\ell = 0,1,2$

The partial wave be s, p, d. some typical value of $\boldsymbol{a}_\ell$ and $\boldsymbol{b}_\ell$ are listed in appendix. A.

The formula of $\boldsymbol{Z}_{\boldsymbol{eff}}^{\ell}$ in equ….. (17) distinguished on that of the atom is that it can calculate the annihilation parameter for p and d- waves, in addition to the s- wave.

The low energy positron scattering and annihilation from atomic target is very difficult to describe using theoretical such as variation technique [26] or close coupling approach [27]. The difficultly lies in accurately describing both the polarization of the atom by the incident positron and virtual positronium (Ps) formation when the target has more valance electron. This led to the application of more approximate method such as the polarization orbital method [7], our method which uses an adiabatic treatment of the positron.

The formula of the absorption cross section can be expressed in term of ($\boldsymbol{\delta}_\ell$) and for all partial waves as [28]

$$\sigma_{abs}(k) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|\delta_\ell|^2) \qquad \ldots\ldots\ldots\ldots(18)$$

Where $\delta_\ell = e^{2i\delta_\ell}$ $,\delta_\ell = \alpha_i + i\beta_\ell$

Where α_i and β_ℓ one real and also define [29] where as the M- shell lionization cross section that we have used it for calculation, were derived from a semi empirical formula of lotz [29]

$$\sigma_{ion} = \frac{2.77\, a_0^2 n_s}{(I_B / Ryd)^2} x \frac{\ell_n (E / I_\beta)}{(E / I_\beta)} \qquad \text{.....(19)}$$

Where (Ryd) is the Rydberg energy, (a_0) is the Bother radius, and (n_5) is the number of electron per subshell.

Result and Discussion

As far as the polarization potential concern. Table (1) give the polarizablity, cut off parameter and $\Delta = E_{ion} - 6.8W$. The energy gap between the target state energy and the final energy of the originally bound target electron, we noticed that the more Z electron, no of the atom the more polarizability and less energy gap. Where in table (2) the calculation of the scattering length involves the collision problem in the zero- energy limit. The rate of the polarization interaction becomes very critical for the calculation of scattering length. In the positron case, the zero- energy scattering is even more difficult due to strong polarization and correlation effects and the cancellation between attractive polarization and repulsion static potentials. In this energy region, only the S- wave (ℓ=o) is significant, and therefore higher- order partial waves can be neglected in this part of the calculation [30]. In table (2) we present our result of (A_0) for atoms and the available value of other researchers. The fact that the scattering length are negative means that the attraction due to the polarization of the atom by the positron field effectively overcomes the repulsion experienced by the positron in the static atomic potentials. Several theoretical techniques are known- which provides the positron- atom scattering cross- section fairly accurately at either lower or higher impact energy region. The investigation of positron scattering from noble gases at intermediate energies was very few. From the experimental side there are measurements made by Detroit group [31,32] whereas, for theoretical

side there is a calculation made by Reid and Wadehra which were the only calculation available for that energy range concerning the total inelastic momentum transfer cross section, there are no theoretical or experimental investigation available up to this tme, as far as we know. In figure (5) we show our result of total and momentum transfer cross section for positron scattering from noble gases (He, Ne, Ar, Kr & Xe) there were no investigation available for (He , Ne) atoms to compare our data with it as far as a comparison of our calculation with other worker is concerned , we see that the present total cross section for (Ar, Kr, & Xe) atoms are in fine agreement with the calculation of Ried and Wadehra [33] as well as with the experimental data of kauppila et al, the agreement was good specially for the energy range (400- 800)ev where it's very clear that the effect of the absorption potential doing well in that region. In figure (6), we present our results of the (DCS) for noble gas atom (Ar, Kr and Xe) at the incident energies (200, 300, 400) ev. No theoretical calculation or experimental measurements are available at intermediate energies of the (DCS) for positron scattering from noble system at this time to make a comparison with our data.

In the present work (P.W) we apply two formulas of the annihilation parameter (Z_{eff}). The 1st one represent in Equ. (13) where there are many factors effect in the behavior of (Z_{eff}), the three terms in the brackets are due the incoming positron plan wave, the scattered wave, and the interference term, respectively, where as the term outside the brackets will be consider as a fitting parameters.

Even if the cross section ($\sigma_{e\ell}$), is zero or very small, the annihilation parameter (Zeff) is non- zero. It magnitude is determined by the effective annihilation radius (R_a) and the factors ($\rho_e, \delta R_a$), also the s- wave phase shift (δ_0) is one of the important factors, which influence in that formula. That means (Z_{eff}) depends much more critically on the actual form of the total wave function, and hence is likely to be more affected by the approximation that had been made. The 2nd formula of (Z_{eff})represented in Equ(15) where the fitting parameter in thisequation is constant (F=0.5), and it depend on the scattering length and the positron momentum only. Table (3) give the information about

the factors, which influence in the two formula for (Z_{eff}) of the system under study. Our results of the annihilation parameters (Z_{eff}) for noble atoms was compared with Mecachram et al data,

Table (1) polarizability , cut-off parameter , and $\Delta = E_{ion} - 6.8eV$ noble atoms

Atom	$\alpha_d(\alpha_0^3)$	$d(\alpha_0)(P.W.)$	$\Delta(eV)$	Note
He	1.383[(a)]	0.551	17.78	(a) refers to Ref.[51]
Ne	2.663[(a)]	0.6074	14.76	
Ar	11.05[(a)]	0.70165	8.96	
Kr	16.74[(a)]	0.90535	7.2	
Xe	27.29[(a)]	1.00292	5.33	

Table (2) the present work scattering length compared with the data of other references.

Atom	$A_0(\alpha_0)P.W.$	$A_0(\alpha_0)$		Note
He	-0.5	-0.575[(a)]		(a) refers to Ref [52]
Ne	-0.55	-0.614[(b)]	-0.542[(e)]	(b) refers to Ref [53]
Ar	-4.2	-5.3[(c)]	-4.11[(e)]	(c) refers to Ref [54]
Kr	-8.6	-10.37[(d)]	-7.69[(e)]	(d) refers to Ref [55]
Xe	-22.84	-45.3[(d)]	-20.3[(e)]	(e) refers to Ref [56]

Table (3) the values of the scattering length (A_0) (P. W.) , radius of the target (R_a) , and the fitting parameter $4\pi\rho_e\delta R_a$ for noble atom

Atom	$A_0(a.u)P.W.$	$R_a(a.u.)$	$4\pi\rho_e\delta R_0(a.u.)$	Note
He	-0.5	3.9[(a)]	0.21[(a)]	(a) refers to Ref [57]
Ne	-0.55	5.0[(a)]	0.23[(a)]	
Ar	-4.2	4.3[(a)]	0.42[(a)]	
Kr	-8.6	4.2[(a)]	0.41[(a)]	
Xe	-22.84	4.2[(a)]	0.41[(a)]	

where they apply the polarized orbital method in their calculation and shown in figure (7). We notice a convenience agreement. In figure (8) we present our predicted values of the annihilation cross section (σ_a) for the two of three γ - rays and the annihilation rate (λ_a) for noble gases. Yet there are no theoretical or experimental investigation available to indicate to these parameter as a function to the positron momentum (k), to make a comparison with it.

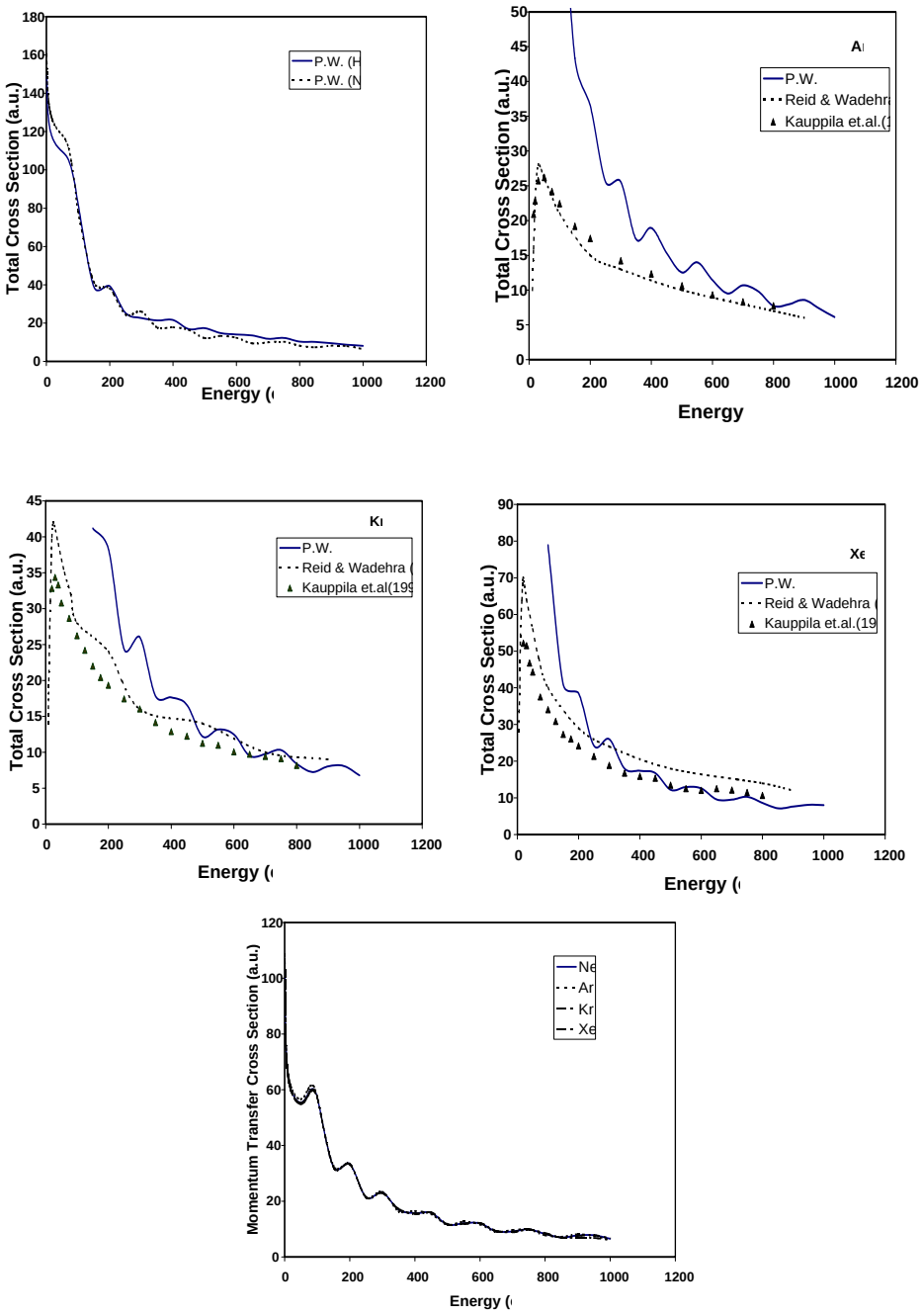


Figure (5): The Total and momentum transfer cross sections for noble atoms.

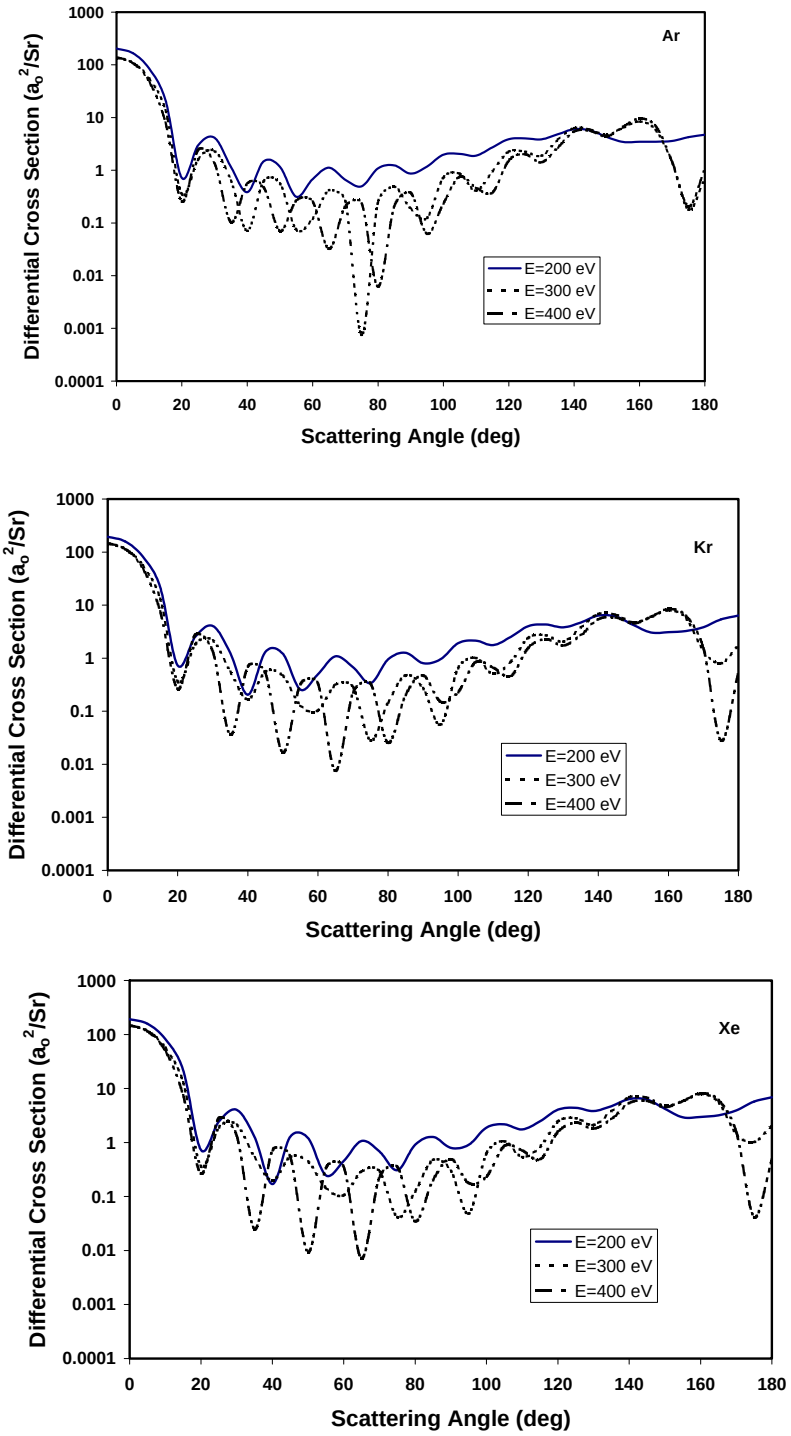


Figure (6): The Differential Cross Sections for (Ar, Kr, & Xe) atoms.

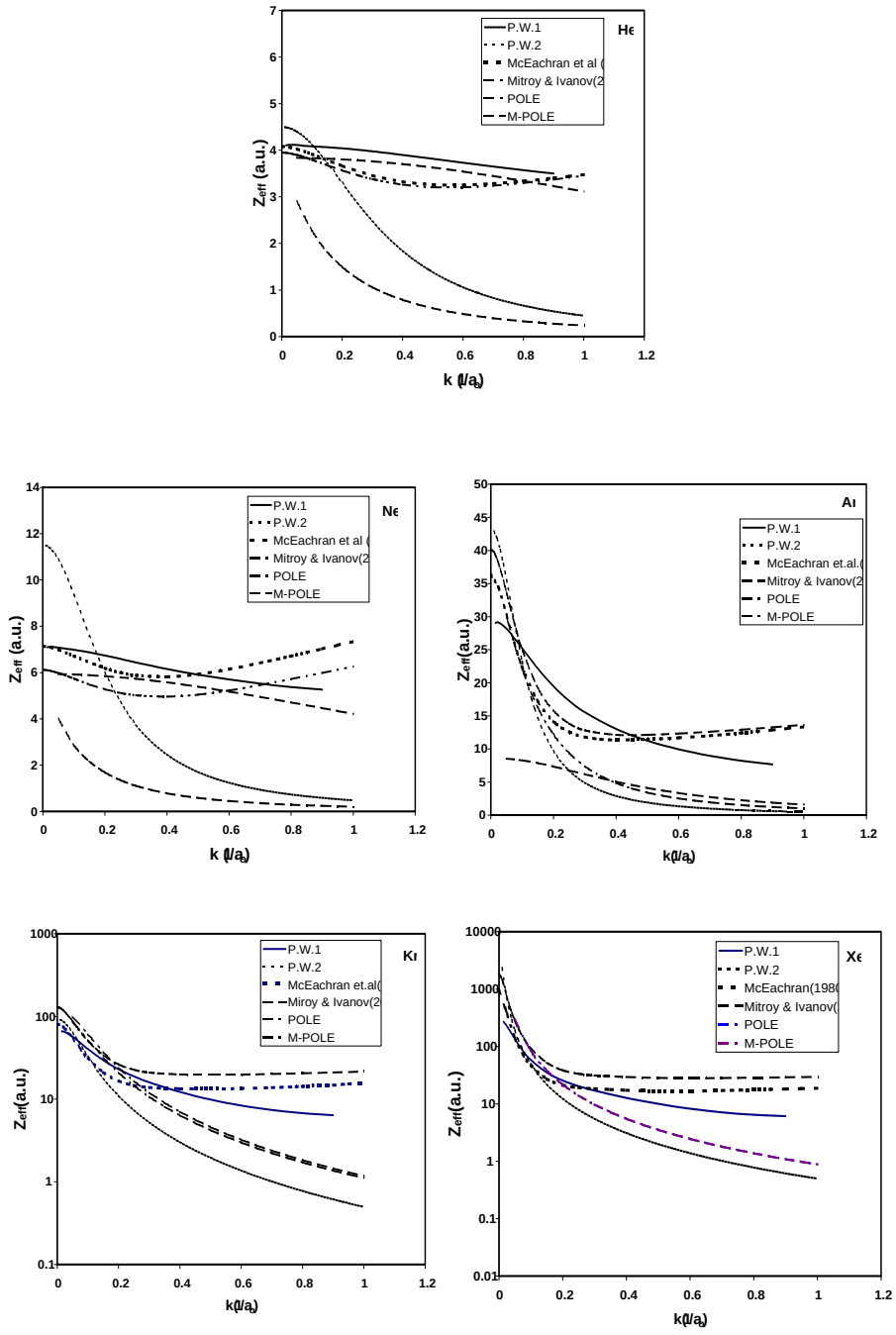
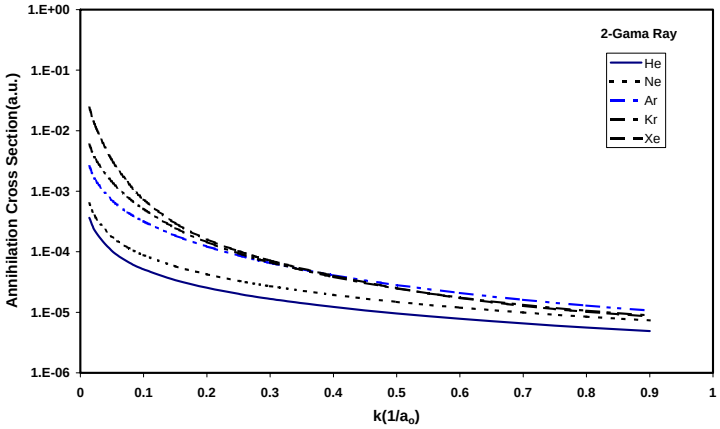
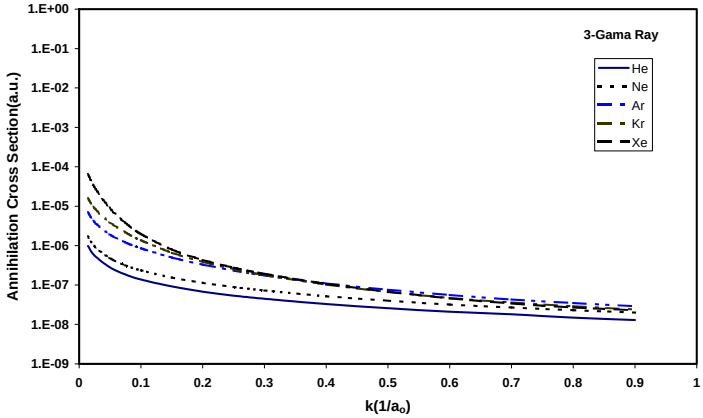
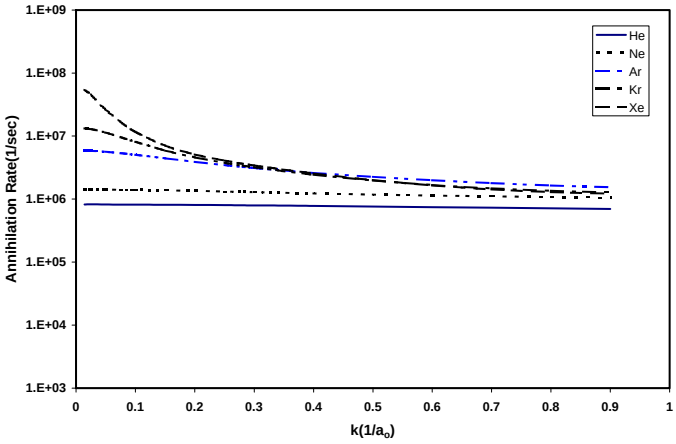


Figure (7): The annihilation parameter (Z_{eff}) for noble-atoms







for noble-atoms (λ_a) & (σ_a) Figure (8): The annihilation parameters

The shape for all these curves of (Z_{eff}), (σ_a) & (λ_a) look very similar i-e quite flat at large energies but rising very steeply at zero energy approached . At small positron energies (Z_{eff}) can be very different from (z), first, there is a strong repulsion between the positron and the nucleus , which prevents the positron from penetrate deep into the atom, as a result, most of the annihilation events involve electron of the valance and near- valance sub- shells making (Z_{eff}) smaller. On the other hand, outside the target the positron motion is affected by an attractive long- range polarization potential. This lead to increase of the positron density nears the target and enhance (Z_{eff}). At large energies would be equal to the total number of electrons in the target (z). In table, (4) we present our predication for noble, alkali and other atom, the life time, compared with the calculation of other researchers where we notice the life time for positron interacting with inert atom is large compared with the life time of the alkali and other atoms, we attribute, this behavior to the stability of the inert atoms, which consider as closed- shell system. We also noticed that the life time of sub- closed shell like. (Be, Mg, Zn, Ld) is larger than that of alkaliy, which in turn represent an open- shell system for the same reason.

The in elastic cross section for positron- atom scattering is presented in figure (9) the absorption cross sections with wave number (k), for positrons scatters from inert gases is compared with the calculation of Mitroy and lvanov, we a achieved an interesting agreement between them.

Table (4) the lifetime for a noble atom .

Atom	$\tau(ns)(P.W.)$	$\tau(ns)$	Note
He	1218.41	$1300^{(a)}$	(a) refers to Ref [58]
Ne	705.40	$500\text{-}900^{(b)}$	(b) refers to Ref [59]
Ar	383.67	$350 \pm 30^{(b)}$	(c) refers to Ref [60]
Kr	313.302	$325 \pm 6^{(c)}$	
Xe	195.21	$178 \pm 3^{(c)}$	

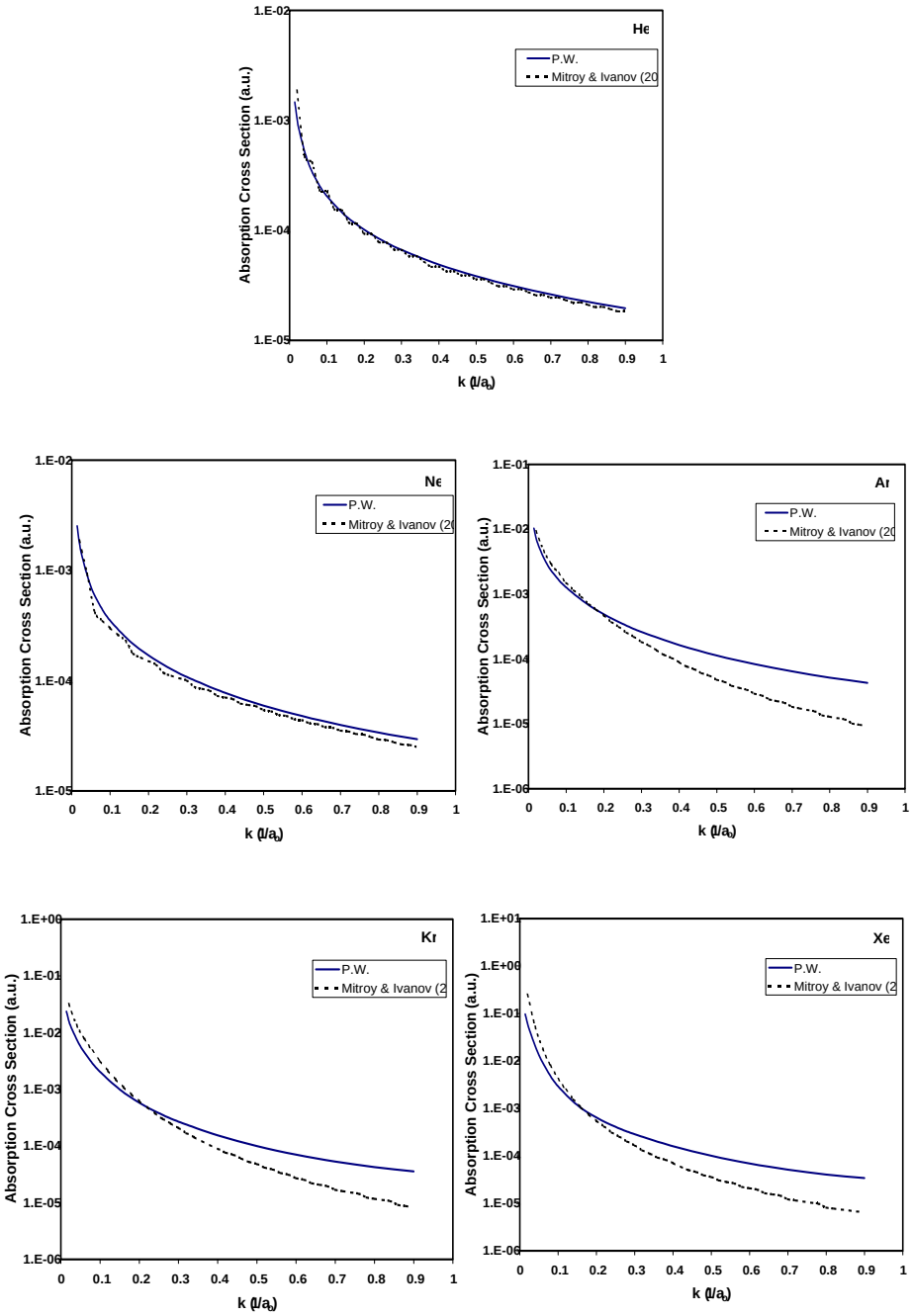


Figure (9): The absorption cross sections for noble-atoms

Every Ps- threshold energy can be evaluated by the subtraction of the ionization energy of the atom from the amount (17.78, 14.76, 8.96, 7.2 and 5.33) ev for (He, Ne, Ar, Kr and Xe), respectively. In figure (10) we presented our results of the Ps- formation cross section for positron scatters from noble atoms, where, for He- atom we compared our data with the calculation of Khan and Ghosh [34] and compel et al [35], and the measurement of Chariton et al [36] and Overtone et al [37] . For Ne- atom we compare our data with the calculation of Mcalinder and Walter [38], and the experimental data of Chariton et al. Diana et al [39], and laricchia et al [40]. For (Ar, Kr, Xe) we made a comparison between our results and the theoretical investigation and MeAlindent of Walter et al, and the experimental measurement of Charlton et al, stem et al and Larricchia et al. we notice from figure (10)that these are good a greement between our results and the experimental measurements at low energies (E< 30ev), after this value of the incident energy the agreement become less. Any way the behavior of our Ps- formation cross- section for all noble atoms is smooth and increase with energy.

In the present work, the values of the binding energy (I_B) that we have used it in calculating the ionization cross section (σ_{ion}) for hydrogen and noble atom under investigation was 13.6, 24.98, 23.14, 16.08, 14.26, and 12.44 for H, He, Ne, Ar, Ke, and Xe respectively. In figure (11) we present our results of the ionization cross section for positron scatters from above mentioned atoms. A comparison have been made with experimental and theoretical calculation of many workers. [41,42,43,44,45,46,47,48,49] some difference, are noticeable at high energies in H- atom, the same was for He- atom but the agreement was very well with the theoretical calculations concerning Ne- atom, the cross section of ionization of us showed an interesting a agreement with both the theoretical and experimental data that compared with. The situation for Ar-atom was similar for those of H-atom. The situation for Ar-atom was similar for those of H- atom, where our (σ_{ion}) agreed with calculation and measurements at low energies, except it was below the mentioned data. When the energy began to increase. Finally the ionization cross section for Kr and Xe atoms showed a magnificent agreement with the theoretical of experimental data, which we had compared it with. It worth to mention that the formula of (σ_{ion}) in Equ.(17) was proposed to be worked for the M-shell system only, where as, as we noticed from the present results it was valid for other shell for the systems that we had work on.

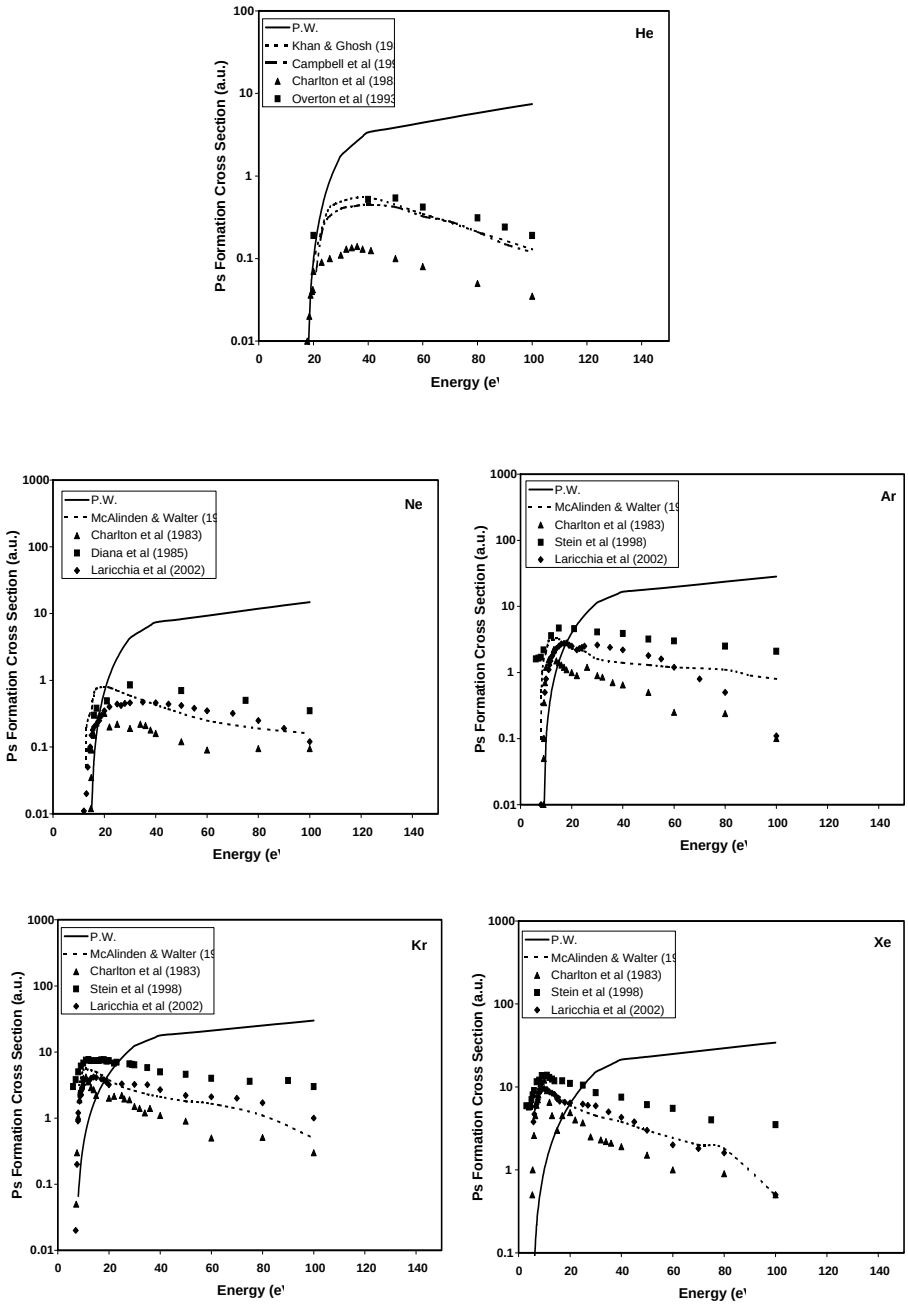


Figure (10): The Ps-formation cross sections for noble-atoms

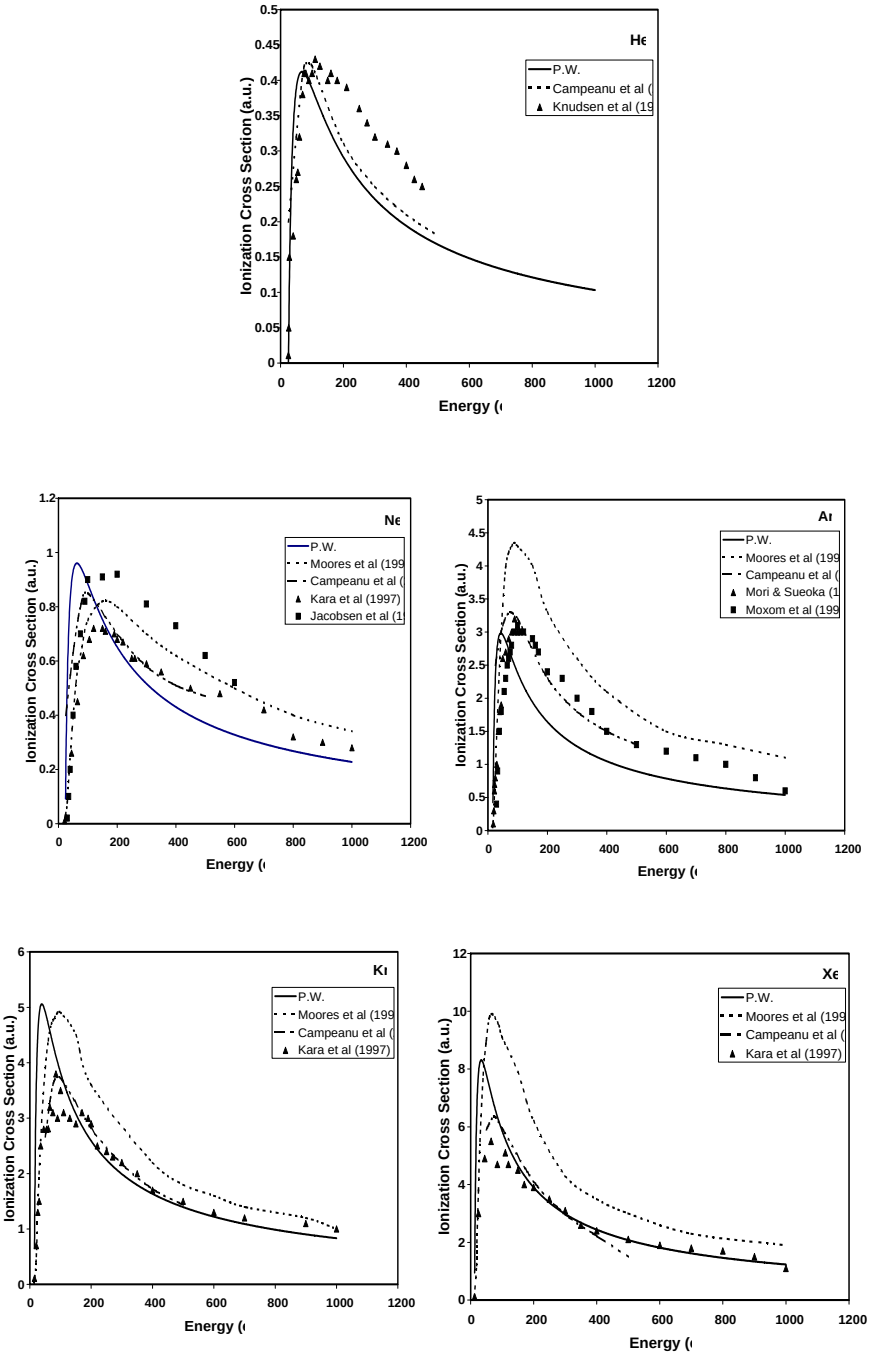


Figure (11): The ionization cross sections for (He, Ne, Ar, Kr, Xe)-atoms

Appendix

The interesting feature of this analyas is that the S, P, and D waves have terms in power series eaqansion of Z_{eff}^{ℓ} is the same order of k^2 . However , the S-ware coefficient $a_0(v)$ is larger than P-wave coefficient $a_1(v)$, which is in turn larger than the D-wave coefficient $a_2(v)$.

	v	a_0	a_1	a_2	b_0	b_1	b_2
3	0.666	8.8109	2.6115	1.257	-7.2369	0.08679	0.4870
5	0.800	14.0408	5.4829	0.367	-18.206	-2.6695	1.1924
9	0.888	19.2296	8.7363	0.7046	-32.593	-7.9946	1.9504

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