The Internal Pressure Dependent Behaviour of a Cavitation Bubble in Refrigerants

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Abstract

The nonlinear oscillation of a bubble in compressible refrigerants subject to a periodic pulsating pressure is theoretically analyzed. For nonlinear oscillations of the gas bubble, the approximation that the pressure within the bubble follows a polytropic relation. It has several limitations and needs to be reconsidered. A new formulation of the dynamics of bubble oscillations in refrigerants is presented in which the internal pressure of the bubble is calculated numerically. Our results are compared with that obtained from polytropic formulation.

Several comparisons are given for results of the two formulations, which describe in some detail limitations of the polytropic formulation. The good agreement is found between the results of the polytropic formulation and the numerical method only when the oscillation amplitude is small.

Keywords: Cavitation, Ultrasound, Refrigerants, Bubble dynamics

التصرف المعتمد على الضغط الداخلي لفقاعة التفقع في موائع التثليج

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الخلاصة

تم أجراء تحليل نظري للتذبذب اللاخطي لفقاعة في موائع التثليج الانضغاطية المعرضة لضـغط نبضي متكرر. بالنسبة للتذبذبات اللاخطية لفقاعة غازية، فأن تقريب الضغط داخل الفقاعة يتبـع علاقـة بولتوربية. تمتلك هذه العلاقة عدة محددات وتحتاج إلى مراجعة. تم تقديم صيغة جديدة لحركـة تذبـذب

الفقاعة في موائع التثليج، اذ يحسب الضغط الداخلي للفقاعة بطـريقتين. الأولـى, باسـتعمال الصـيغة البولتروبية. بينما في الثانية فيحسب الضغط عددياً.

تم أعطاء عدة مقارنات بين نتائج الصيغتين، توضح هذه المقارنات بعـض محـددات الصـيغة البولتروبية. وجد تطابق بين نتائج الصيغة البولتروبية والعددية فقط عندما تكون سعة التذبذب صغيرة.

Introduction

The dynamics of a gas bubble in a liquid is strongly dependent on the pressure of the gas contained in it. In principle, this quantity must be determined from the solution of the conservation equations of continuum mechanics inside and outside the bubble joined together by suitable boundary conditions at the bubble interface. This task is very complicated and can only be carried out analytically for small- amplitude motion in which the equation can be linearized (see, for example, Refs. [1-3]). For large–amplitude motion, it is customary in the literature to make use of a polytropic relation of the form:

$$P = P_o\left(\frac{R}{R_o}\right)^{3K} \qquad \ldots(1)$$

Where P is the pressure of the gas in bubble, R is the bubble radius, K is the polytropic index, and the subscript zero indicates equilibrium values. This relation, with K=1, entered bubble-dynamics literature explicitly with the pioneering numerical studies of forced large-amplitude oscillations carried out by Noltingk and Neppiras [4] in the early 1950s. After that, it has been used by virtually every writer on the subject such as Flynn [5], Apfel [6], Lauterborn [7], and many authors [8, 9].

In spite of its appealing simplicity, the use of eq.(1) posses many problems. In the first place, the polytropic index can range in the interval from 1 (isothermal) to the ratio of specific heats γ (adiabatic), and appropriate criteria for the proper choice are available only for the small-amplitude linear case. Further, it is present unknown how realistic this relation is when the linear value of K is used in the nonlinear regime. Second, if P is given by eq.(1), Pdv (where, v is the volume of the bubble) is a perfect differential, and its integral over a cycle vanishes. As a consequence, using eq.(1) yields no energy loss associated with the heating and cooling of the gas. This fact is very unfortunate since it is known from the linear studies that this thermal damping is, in fact, the dominant form of energy absorption over a wide range of physical conditions. As a partial remedy, it was suggested in ref. [10] that the liquid viscosity could be artificially augmented by an amount in such a way that the correct damping would result in the linear case. This prescription has recently been put to an experimental test [11] and has been found to result in a large overestimate of the damping affecting the nonlinear oscillations in the region of the first nonlinear resonance. On the basis of these considerations, it must be concluded eq.(1) is not adequate for a precise theoretical analysis of bubble dynamics. Of particular concern is the application of eq.(1) to the study of the chaotic regime of forced oscillations [12], which is known to be strongly influenced by the details of energy dissipation.

In an attempt to go beyond eq.(1), Flynn [13] presented a mathematical formulation which reduced the exact set of partial differential equations expressing the conservation laws in the gas to a system of ordinary differential equations. This result was obtained at the price of a number of approximations the most notable of which, that of spatially uniform pressure distribution in the bubble. Flynn's formulation, however, is far from simple and, probably for this reason, has not been widely used.

The mathematical formulation to be presented here is more precise. For a prefect gas with spatially uniform pressure, the continuity and energy equations can be combined to obtain an exact expression for the velocity field in terms of the temperature gradients. In this way, the problem is reduced to a nonlinear partial differential equation for the temperature field and to an ordinary differential equation for the internal pressure. A numerical technique for the treatment of these equations is used, and several numerical results are included for the purpose of illustrating the method and demonstrating the limited accuracy of the polytropic approximation.

Mathematical Model

In this paper, we attempt to study the thermomechanical behaviour of the gas contained in a bubble in spherically symmetric motion. The diffusion of the gas in and out of the bubble has significant dynamical effects only at very low ambient pressures, when the small quantity of gas diffusing into the liquid is an appreciable fraction of the total amount of gas contained in the bubble [14]. This effect, however, can be ignored at higher pressures. A further consequence of diffusion manifests itself over time scales much longer than that associated with the typical oscillatory period of bubble motion. Accordingly, we shall disregard diffusion altogether and assume the bubble boundary to be impervious to the gas. The partial pressure of the liquid vapour in the bubble is assumed to be much smaller than the gas pressure, and the effect of the vapour present in the bubble is also disregarded.

The main approximations contained in the present article are: (a) the pressure is spatially uniform in the bubble; (b) the gas is perfect; (c) the bubble maintains a spherical shape; (d) the bubble wall temperature remains unperturbed; and (e) the effects of the vapour contained in the bubble are negligible. The first assumption requires the Mach number of bubble wall motion, calculated with respect to the gas speed of sound, to be small. The second, third, and fourth assumptions are also likely to break down in the conditions of extreme gas temperature and pressure prevailing near the end of violent collapses, they caused by large–amplitude acoustic driving or by the recovery of the ambient pressure typical of flow cavitation. The last two assumptions (d) and (e) enable us to disregard the energy equation in the liquid and the gas–vapour diffusion equation in the bubble. In summary, our formulations should be of value in a variety of situation involving forced motion in a relatively cold liquid, provided that the velocity of the bubble interface remains relatively small with respect to the speed of sound in the gas. We now assume that the gas can by adequately described by the perfect gas laws with constant specific heats. In this case, the following expression for the velocity field inside the bubble [15]

$$u(r,t)=\frac{1}{\gamma\,p}\left[(\gamma-1)k\frac{\partial T}{\partial r}-\frac{1}{3}r\dot{P}\right] \qquad \ldots(2)$$

with the aid of the velocity boundary condition

$$u(R,t)=\dot{R} \qquad \ldots(3)$$

Equation (2) can be turned into a differential equation for P by evaluating it at r=R:

$$\dot{P}=\frac{3}{R}\left[(\gamma-1)k\left.\frac{\partial T}{\partial r}\right|_{r=R}-\gamma\,P\,\dot{R}\right] \qquad \ldots(4)$$

In view, eq.(2) shows the velocity distribution in the bubble is increased linearly with distance from the center.

In this study, we choose to use the energy equation [16]

$$\frac{\gamma}{\gamma-1}\frac{P}{T}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}\right)-\dot{P}=\nabla\cdot(k\ \nabla T.).(5)$$

Where

$$\nabla\cdot(k\ \nabla T)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(k\,r^2\frac{\partial T}{\partial r}\right) \qquad \ldots(6)$$

This choice was suggested by the fact that the thermal conductivity k will be allowed to depend on the temperature. At the bubble wall, the correct boundary condition on the temperature field is continuity of temperature. When the effect of the vapour is not significant, we can use the condition

$$T(R,t)=T_\infty \qquad ...(7)$$

This approximation simplifies the problem considerably since it asserts that a consideration of the temperature field in the liquid in unnecessary.

During the motion of the bubble wall, the bas can undergo large variations in temperature, and the dependence of its thermal conductivity on this quantity cannot be neglected. A convenient way to account for this variation is to introduce a new variable

$$\beta = \int_{T_\infty}^{T} k(\theta)\, d\theta \qquad \ldots(8)$$

Furthermore, for numerical analysis, it is convenient to have a fixed rather than a moving boundary and we, therefore, let

$$\tau = \frac{r}{R(t)} \qquad \ldots (9)$$

With eqs. (2), (8) and (9), the energy equation (eq. (5)) takes the from

$$\frac{\partial\beta}{\partial t} + \frac{\gamma-1}{\gamma P R^2}\left[\frac{\partial\beta}{\partial\tau} - \left.\frac{\partial\beta}{\partial\tau}\right|_{\tau=1} \tau\right]\frac{\partial\beta}{\partial\tau} - D\dot{P} = \frac{D}{R^2}\nabla^2\beta \qquad \ldots(10)$$

Where

$$D(P,T) = \frac{\gamma-1}{\gamma}\frac{k(T)\, T}{P} \qquad \ldots(11)$$

is the appropriate form of the thermal diffusivity for a perfect gas. The boundary conditions, eq.(7), can be written, in term of β,

$$\beta(\tau = 1, t) = 0 \qquad \ldots(12)$$

When the compressibility of the liquid is not neglected, the motion of the bubble boundary (wall) is governed by the following equation [17].

$$R\ddot{R}\left[1 - \frac{2\dot{R}}{c} + \frac{23}{10}\frac{\dot{R}^2}{c^2}\right] + \frac{3}{2}\dot{R}\left[\dot{R} - \frac{4\dot{R}^2}{3c} + \frac{7\dot{R}^3}{5c^2}\right] + \frac{1}{\rho_\infty}\left[\begin{array}{l}(P_{L\infty} - P_{L2}) - \dfrac{R}{c}\dfrac{dP_{LR}}{dt} + \dfrac{1}{c^2} \\ \left\{\begin{array}{l} 2R\dot{R}\dfrac{dP_{LR}}{dt} + (P_{L\infty} - P_{LR}) \\ \left[\dfrac{1}{2}\dot{R}^2 + \dfrac{3}{2}\dfrac{(P_{L\infty} - P_{LR})}{\rho_\infty}\right]\end{array}\right\}\end{array}\right] = 0 \ldots(13)$$

Where $$P_{L2} = P_{LR} - \frac{4\mu}{3c^2 \rho_\infty} \frac{dP_{LR}}{dt} \qquad \ldots(14)$$

and P_{LR} is the liquid pressure on the external side of the bubble wall, which is related to the internal bubble pressure P by [18].

$$P = P_{LR} + \frac{2\sigma}{R} + \frac{4\mu \dot{R}}{R} \qquad \ldots(15)$$

The total liquid pressure ($P_{L\infty}$) is the sum of the static liquid pressure (P_∞) and a non constant ambient pressure such as sound field (P_s); it is given by [19].

$$P_s = \in P_\infty \sin(2\pi f\, t) \qquad \ldots(16)$$

Here, ∈ is the dimensionless sound amplitude.

In this study, we have approximated the dependence of k upon T by a linear function[20]:

$$k(T) = A\,T + B \qquad \ldots(17)$$

In this case, inversion of eq.(8) leads to the following relation between T and :

$$T = \frac{1}{A}\left\{ \left[k^2 (T_\infty) + 2A\beta \right]^{\frac{1}{2}} - B \right\} \qquad \ldots(18)$$

The numerical values of A and B in ref. [20] give a good fit to the thermal conductivity of air.

In this study, the natural frequency of a bubble can be expressed as follows [21]:

where $$f_o = \frac{1}{2\pi R_o}\left[E - \frac{F^2}{4} - \frac{F}{c}\left(\frac{3}{2}\, E - \frac{F^2}{2} \right) \right]^{\frac{1}{2}} \qquad \ldots(19)$$

$$E = \frac{1}{\rho_\infty}\left[(3\gamma - 1)\left(\frac{2\sigma}{R_o}\right) + 3\gamma\, P_\infty\right] \qquad \ldots(20)$$

and

$$F = \frac{4\mu}{\rho_\infty\, R_o} \qquad \ldots(21)$$

The maximum of the oscillation amplitude is defined by:

$$X = \frac{R_m - R_o}{R_o} \qquad \ldots(22)$$

where R_m is the maximum value attained by the radius during a steady oscillation. The dimensionless time is defined by:

$$\psi = 2\pi f\, t \qquad \ldots(23)$$

Results

Equation (13) must be solved simultaneously with the energy equation in the bubble, eq.(10), and the equation of the internal pressure, eq.(4).

We proceed now to discuss some numerical results for large-amplitude oscillations with the predictions based upon the polytropic approximation. These results are far from exhaustive and are presented here primarily to demonstrate the usefulness and range of applicability of the present formulation.

In this paper, we interested with a kind of refrigerants, R-134a, and air as the gas in the bubble. In the calculations, the following values of the physical parameters are used [22]

$P_\infty = 350\ \text{kPa}$, $T_\infty = 5.03\ ^{o}\text{C}$, $\gamma = 1.4$,

$\rho_\infty = 1278\ \text{kg/m}^3, \mu = 254.2854 \times 10^{-6}\ \text{Pa.s}$

and $\sigma = 0.00074\ \times 10^{-3}\ \text{N/m}$

In the following, when we refer to the present work, we indicate our formulation of bubble dynamics in which the internal pressure is calculated from eq.(4). The words polytropic model, on the other hand, will indicate use of the polytropic relation for the internal pressure (eq. (1)).

By showing in Figs. 1 and 2 a comparison between the frequency response curves calculated with the present work and the polytropic formulation for a sound–pressure amplitude $\in = 0.6$ and equilibrium bubble radii of 50 and 10 m, respectively. These graphs display for each value f/f_o of the ratio of the sound frequency. The dotted line is the polytropic model, and the solid line dentes results obtained from the present technique. The most striking difference between the two formulations consists in the location and the height of the peaks to the fundamental and higher resonances. The relative shift between these peaks in some cases results in considerable differences between the two oscillations, amplitudes at a given frequency.

Fig. 3 shows a comparison of the R (t) curves according to the present theory and to the polytropic formulation for $R_o = 50$ m. The polytropic model predicts a larger maximum radius. The collapse point is approached with a large velocity, and the minimum radius is correspondingly smaller. In turn, the large velocity and small radius give rise to an excessive energy dissipation presumably concentrated in a large burst near the point of minimum radius. The pressure history, shown in Fig. 4, is correspondingly much more peak for the polytropic case. The collapse point is approached with the minimum radius. At this point, the maximum pressure in the bubble is occurred.

In the case of smaller bubble, $R_o = 10$ m, the behaviour of which at $\in = 0.6$ is illustrated in Figs. 5 and 6. Now, R(t) curves are remarkably dissimilar with a large phase shift and a significant difference in the oscillation amplitude is occured. A comparison of the two curves shows that this is not due to a smaller overall damping coefficient, but to smaller oscillation amplitude and velocity. The internal pressure for the case with $\in = 0.6$ is shown in Fig. 6. Since no energy loss associated with the pressure in the bubble calculated by polytropic formulation, it is the polytropic peak that is higher.

The good agreement of Figs. 7 and 8 is found because the oscillation amplitude is relatively small, but also, very importantly, because the bubble is driven far from resonances.

Conclusions

It has been that a new technique of bubble dynamic based upon an evaluation of the internal pressure within the bubble can give considerably different predictions of behaviour when compared with the standard treatment in which the internal pressure is approximated by polytropic relation. The radius–time curves predicted by the two models can differ very markedly. The differences are especially in the higher frequency regions.

A general conclusion that may be drawn from the above comparison is that the polytropic model is unreliable especially at large pulsation amplitude. The good agreement is found between the polytropic model and the present model only when the oscillation amplitude is relatively small.

Nomenclature

c: Sound speed in the liquid (m/s).

D: Thermal diffusivity (m^2/s)

f: Sound field frequency (Hz)

f_o: Bubble natural frequency (Hz)

k: Thermal conductivity (W/m.k)

P: Gas pressure in the bubble (Pa)

$\dot{P}$: Derivative of the bubble pressure ($\frac{dP}{dt}$)

P_{LR}: Liquid pressure at bubble wall (Pa)

P_o: Static pressure in the bubble (Pa)

P_s: Sound field pressure (Pa)

P_∞: Static liquid pressure (Pa)

$P_{L\infty}$: Sum of the static liquid pressure and the sound field pressure (Pa)

r: Radial distance (m)

R: Bubble radius (m)

$\dot{R}$: Bubble wall velocity ($\frac{dR}{dt}$) (m/s)

R_o: Equilibrium bubble radius (m)

R_m: Maximum bubble radius (m)

t: Time (s)

T: Temperature (K)

T_∞: Static liquid temperature (K)

X: Maximum oscillation amplitude

γ: Ratio of specific heats of gas

μ: Liquid viscosity ($N.s/m^2$)

ρ_∞: Liquid density (kg/m^3)

σ: Liquid surface tension (N/m)

$\in$: Dimensionless acoustic pressure amplitude

ψ: Dimensionless time

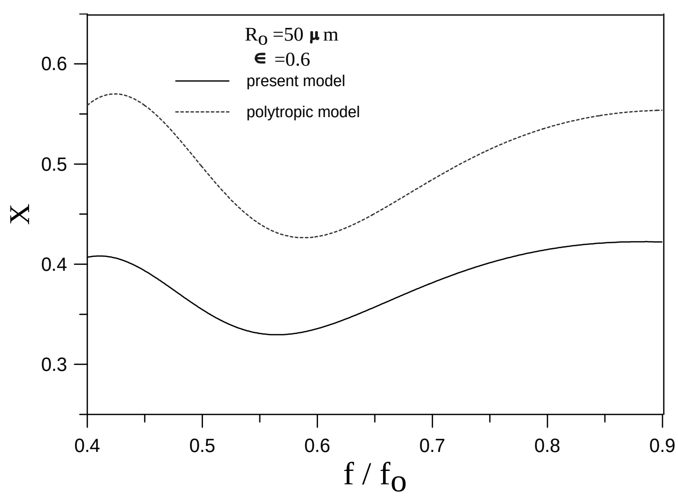

Fig. 1 The normalized amplitude of the radial oscillations as a function of the driving sound frequency (dimensionless)

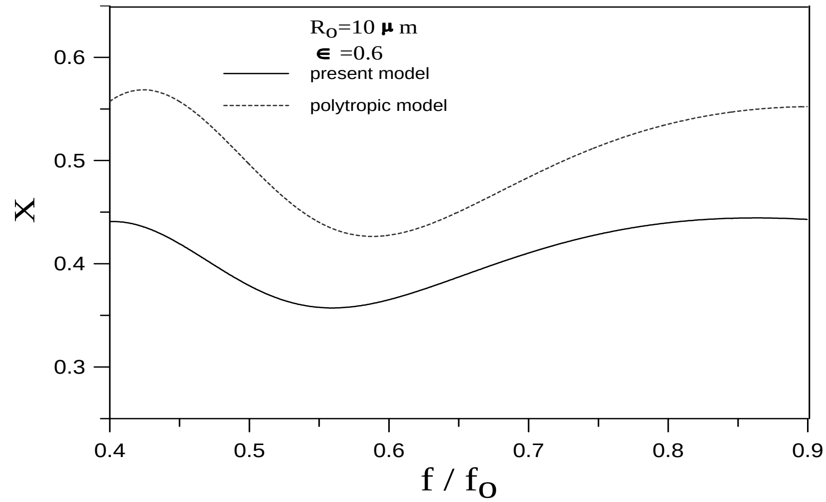

Fig. 2 The normalized amplitude of the radial oscillations as a function of the driving sound frequency (dimensionless)

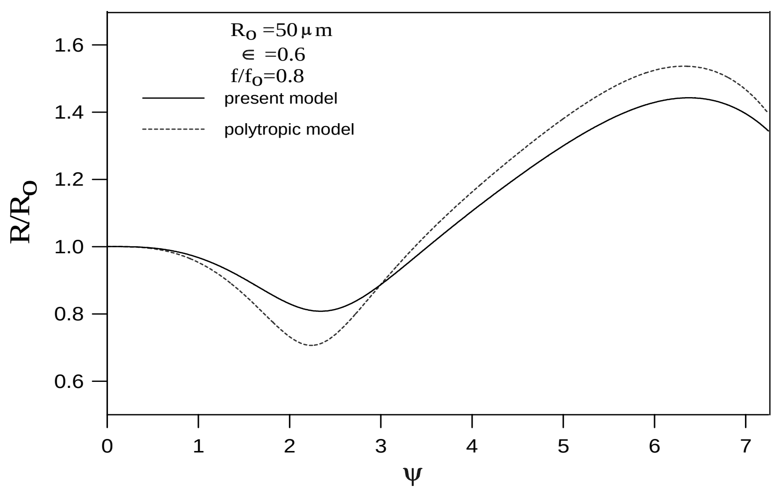

Fig. 3 The normalized radius-time curves for the steady oscillations

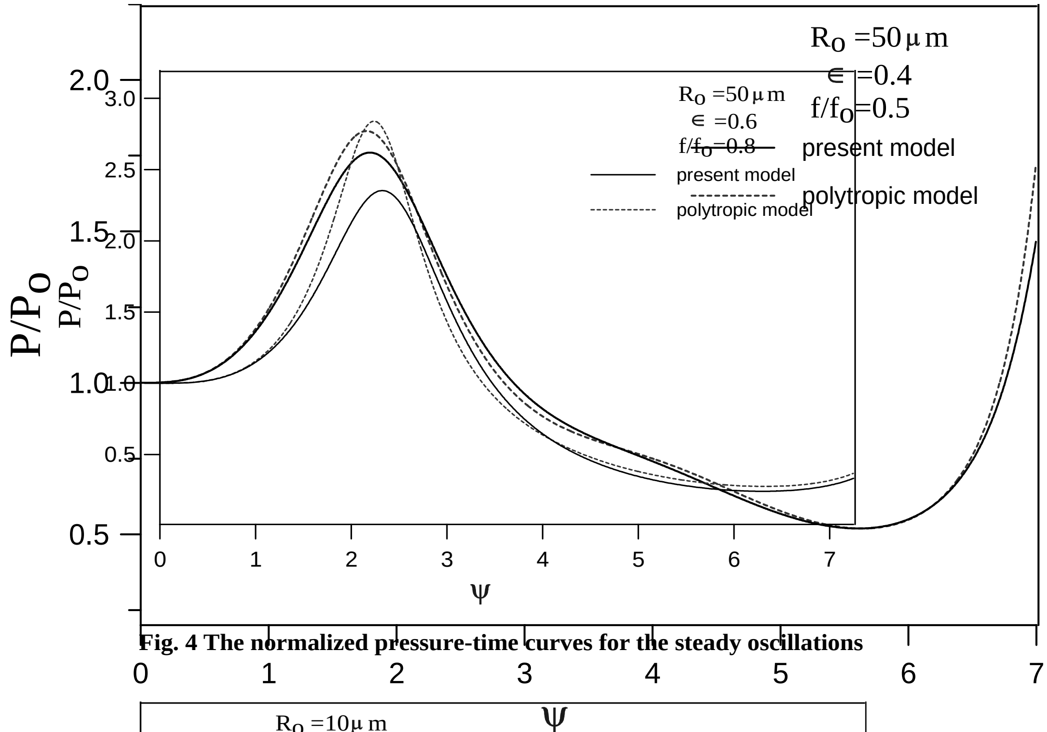

Fig. 4 The normalized pressure-time curves for the steady oscillations

Fig. 8 The normalized pressure-time curves for the steady oscillations

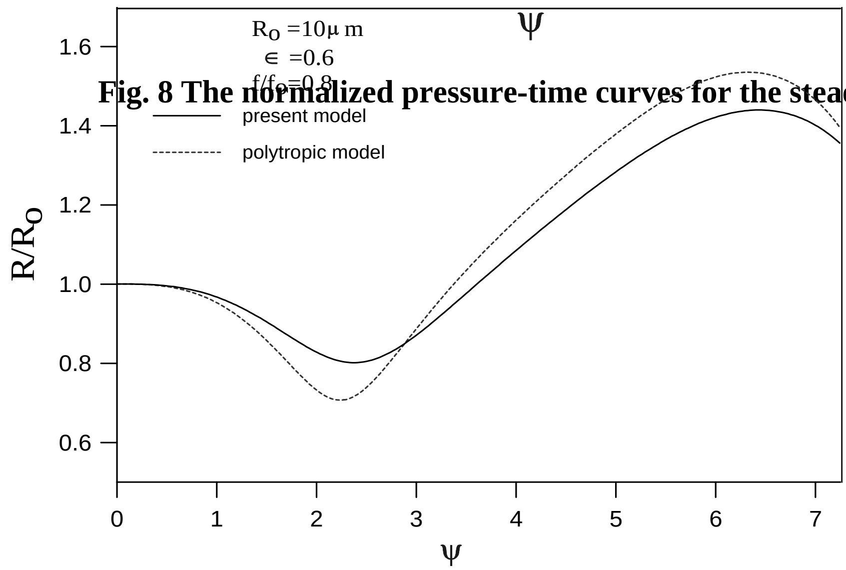

Fig. 5 The normalized radius-time curves for the steady oscillations

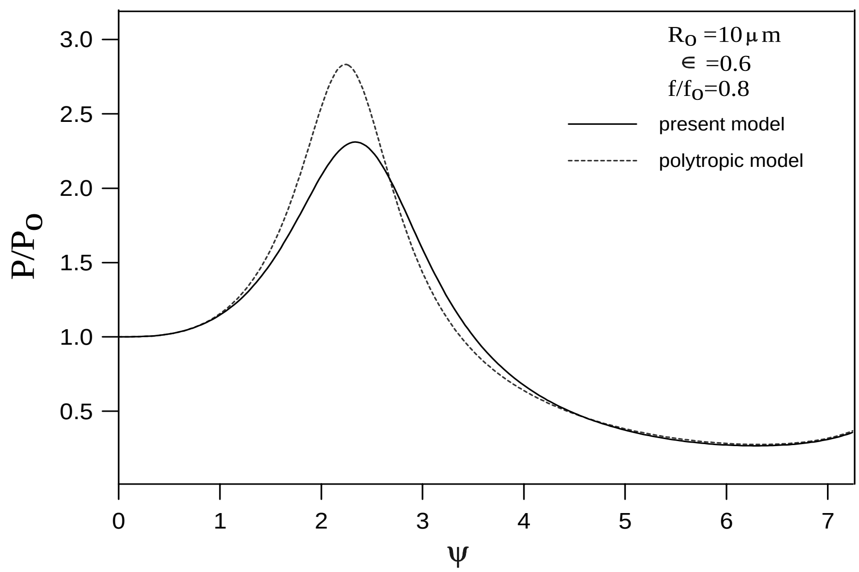

Fig. 6 The normalized pressure-time curves for the steady oscillations

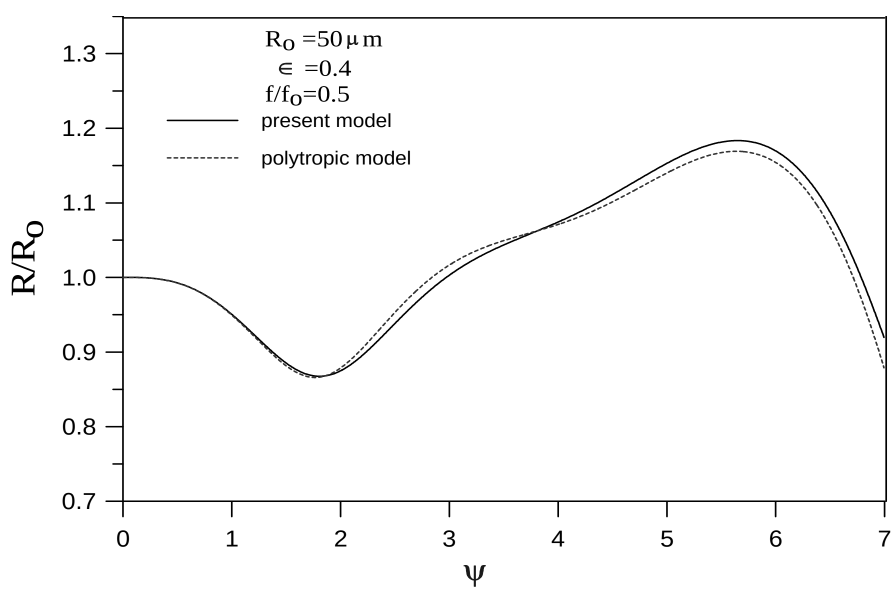

Fig. 7 The normalized radius-time curves for the steady oscillations

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