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On α-Open Sets In Topological Spaces

حول المجموعات الالفا مفتوحة في الفضاءات التوبولوجية

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Abstract:
In this paper we present a new notion by using α-open sets, which is The α-dense-in-itself set in topological spaces, and We prove some theorems that are related by the new definition .

الخلاصه:
في هذا البحث قدمنا مفهوم جديد باستخدام المجموعة آلالفا مفتوحة , وهذا المفهوم هو المجموعة الالفا كثيفة بنفسها في الفضاءات التوبولوجية و أثبتنا بعض النظريات المتعلقة بهذا التعريف الجديد .

1. INTRODUCTION

Before we present the α-dense-in-itself we give some definitions and remarks .

Definition(1-1)[3] : A subset A of a topological space (X,τ) is called α-open if A ⊂(interior(closure(interior(A)))) that is $A \subset [\left(\overline{A^0}\right)]^{\circ}$

Definition (1-2)[3]: The complement of α-open set is called α-closed set.

The family of all α-open sets of X is denoted by *α(X)* .

Example (1-3): let *X={a,b,c,d }* , with *τ={ Ø,X,{a},{b,c},{a,d},{a,b,c}}* is a topology on X. the complement of *τ is{X, Ø,{b,c,d},{a,d} {b,c},{d}}*, consider the subset A={a,b,c},then *A* is α-open since

$$A^o = \bigcup\{\{a\},\{b,c\},\{a,b,c\}\} = A = X^o = X$$

$$\therefore \left[\overline{(\{a,b,c\})^o}\right]^o = X, \because A \subset X, A \subset \left[\overline{(\{a,b,c\})^o}\right]^o$$

implies $A \subset [\left(\overline{A^0}\right)]^{\circ}$.

Definition(1-4)[3]: let *(X,τ)* be a topological space and $A \subset X$ then *A* is called α-neighborhood of a point *x* in *X* ,if there exist α-open set *U* in *X* such that $x \in U \subset A$.

Remark (1-5) : Every open set in *(X,τ)* is α-open . Its proof is immediately from definition (1-1)

Definition (1-6)[3] : A point $x \in X$ is said to be *α-limit point* of $A \subset X$ iff $U \in \alpha(X)$ implies $A \cap (U-\{x\}) \neq \emptyset$.

Definition(1-7)[3] : The set of all α-limit points of $A \subset X$ is called the *α-derived set* of *A* and is denoted by *Dα(A).*

Remark (1-8) : Since every open set in *(X,τ)* is α-open set, so every α-limit point of $A \subseteq X$ is Limit point of *A* . That is $D\alpha(X) \subseteq A'$, where A' is the derived set of *A* .

Definition (1-9)[3] :The union of all α-open sets contained in A is called the *α-interior of A* and denoted by *α-int (A)* .

Definition (1-10)[3]: The intersection of all α-closed sets containing A is called the *α-closure of A* and denoted by *αcl (A)* .

2. α-DENSE- IN- ITSELF

Definition (2-1) : A subset A of a topological space (X,τ) is called *α-dense- in-itself* if $A \subseteq D\alpha(A)$ that is every points of A is α-limit point of A .

Example (2-2):let $X=\{a,b,c,d,e\}$, with $\tau=\{ \emptyset,X,\{b\},\{d,e\},\{b,d,e\},\{a,c,d,e\}\}$is a topology on X .consider the subset $A=\{a,c\}$,then a is α-limit point of A since the α-nhds of a are $\{a,c,d,e\}$ and X each of which contains a point of A other than a , also c is α-limit point of A since the α-nhds of c are $\{a,c,d,e\}$ and X each of which contains a point of A other than c .hence A is α-dense- in-itself .

proposition (2-3) : Every α-dense -in-itself set is dense - in – itself .

Proof : let A be α-dense-in-itself set that is (every point in A is α-limit point of A) ,since every α-limit point is a limit point then each point of A is a limit point then A is dense-in-itself .

proposition (2-4): If A is α-dense -in -itself set then $\alpha cl\,(A)$ is α-dense-in-itself.

Proof : By theorem (let A be a subset of a topological space (X,τ) then $\alpha cl\,(A) = A \cup D\alpha(A)$ [2],[3]) since A is α-dense -in -itself that is (every point of A is α- limit point of A) then $A \cup D\alpha(A) = A$ hence $\alpha cl\,(A) = A$ then $\alpha cl\,(A)$ is α-dense-in-itself .

Theorem (2-5) : The union of any family of α-dense-in-itself sets is α-dense - in-itself .

Proof : let $\{A_i\}$, $i \in I$, be a family of α-dense-in-itself sets . so $A_i \subseteq D\alpha(A_i)\ \forall i \in I$, Let $p \in \cup A_i$ then $p \in A_i$ .for some $i \in I$.

Hence for each α-pen set U with $p \in U$, $A_i \cap (U-\{p\}) \neq \emptyset$.

Thus $(\cup A_i) \cap (U-\{p\}) \neq \emptyset$, hence $p \in (D\alpha(UA_i))$ therefore $\cup A_i \subseteq (D\alpha(\cup A_i))$; hence $\cup A_i$ is α-denes-in-itsef .

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