Almost semi -strongly θ-continuous function on Bitopological spaces

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Abstract:
[Abd El-monsef and Hanafy] and [Takashi Noiri , 1994] studied several of almost semi strongly θ-continuous in topological space , in this paper it is shown that results similar to these in bitopological space.

الخلاصة:
هذا البحث يتنـاول موضـوع almost semi strongly θ-continuous function فـي الفضـاءات ثنائيـة التبولـوجي . بعض النظريات والخصائص لهذه الدالةفي الفضاء التبولوجي تمت دراستها في هذا الفضاء.

1- Introduction

[noiri and Yang , 1984] introduced the concept of almost strongly θ-continuous function in topological space (X,t_1) and [Abd El-Monsef , 1990] gave the concept of semi strongly θ-continuous function .[Takashi Noiri , Esref Hatir and Saziye Yuksel , 1994] introduced the notion of almost semi-strongly θ-continuity which is generalized of both almost strongly θ-continuous function and *semi-strongly θ-continuous* function. the concept of semi strongly θ-continuous function and almost-semi strongly θ-continuous function all these concepts will be define in this paper and also we shall investigate all these concepts and relation ship between them but in bitopological space (X,t_1,t_2).

let (X,t_1,t_2) and let A be a subset of X the *closure* and the *interior* of A are relative to t_i, i=1or2 is denoted by $cl_{ti}(A)$ and $int_{ti}(A)$ respectively. A subset A of X is said to be regular-open set(resp. regular closed set) if $int_{ti}(cl_{ti}(A))=A$ (resp. $cl_{ti}(int_{ti}(A))=A$) and it is said to be semi open set (resp. semi closed set) if $A \subseteq cl_{ti}(int_{ti}(A))$ (resp. $int_{ti}(cl_{ti}(A))\subseteq A$) . the family of all semi open set of X is denoted by SO(X). the family of all semi-open set of X containing apoint x is denoted by SO(X,x), appoint x of X is said to be in the θ-semi-clouser of A if $A \subseteq cl_{ti}(U)$ for each $U \in SO_{ti}(X,x)$ and it is denoted by $\theta\text{-}scl_{ti}(A)$, i=1or2. a subset A of X is said to be θ-semi closed set if $\theta scl_{ti}(A)=A$ and the complement of θ-semi closed set is called θ-semi open set .

θ-semi interior of A is defined by the union of all θ-semi open sets contained in A and it is denoted by $\theta\text{-}sint_{ti}(A)$ and for all these definitions i=1or2.

Definition(1-1): A subset A of a bitopological space (X,t_1,t_2) is said o be:

1- semi regular set if it is both semi open set and semi closed set .

2-θ-g-closed set if $cl_\theta(A) \subseteq U$, U is t_i-open set ,i=1or2.

Definition(1-2): The bitopological space (X,t_1,t_2) is said to be :

1- extremely disconnected if the closure of every ti-open set of X is open set in X.

2- S*-regular space if for each $U\in SO(X)$ and each $x\in X$ there exist regular closed set F such that $x \in F \subseteq U$.

3- (θ-s)Hausdorff space if for each $x,y\in X$ there exist $U,V\in SO(X)$ such that $x\in U$,$y\in V$ and$cl_{ti}(U) \cap cl_{ti}(V)=\emptyset$,i=1or2.

4- almost regular space if for any regular closed set F of X and $x\in X-F$ there exist ti-open set U,V of X such that $x\in U$,$F\cap V=\emptyset$ and $U\cap V=\emptyset$.

Remark(1-3): If the bitopological space (X,t_1,t_2) is regular space then a subset A of X is θ-g-closed set if and only if A is g-closed set.

The following diagram give us the relation between these sets:

θ- closed set →→→θ-generalized closed set
↓ ↓
Regular closed set → closed set→→ generalized closed set
↓
Semi closed set →semi generalized closed set
↓
Generalized semi closed set

Definition(1-4): A function f from a bitopological space (X,t_1,t_2) into a bitopological space (Y,p_1,p_2) is said to be
1- *weakly continuous* function if for each x∈X and each pi-open set V of f(x) there exist ti-open se U containing x in X such that $f(U) \subseteq cl_{ti}(V)$, i=1or2.
2-*generalized –continuous* function if for x∈X each x∈X and each pi-generalized -open set V of f(x) there exist ti-open set U containing x in X such that $f(U) \subseteq V$, i=1or2.
3- *θ-generalized continuous* function if each x∈X and each pi-open set V of f(x) there exist ti-θ-generalized-open set U containing x in X such that $f(U) \subseteq V$, i=1or2.
Definition(1-5): A function f: $(X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ is said to be
strongly θ-continuous (resp. *almost strongly θ-continuous* , *almost continuous, θ-continuous)* if for each x∈X and each pi-open set V in Y containing f(x) there exist ti-open set U in X containing x such that $f(cl_{ti}(U)) \subseteq V$ [resp. $f(cl_{ti}(U)) \subseteq int_{pi}(cl_{pi}(V))$, $f(U) \subseteq int_{pi}(cl_{pi}(V)))$, $f(cl_{ti}(U)) \subset cl_{pi}(V)$], i=1or2

Theorem(1-6): let f: $(X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ be onto strongly θ-continuous and let $g:(Y,p_1,p_2)\rightarrow (Z,w_1,w_2)$ be θ-continuous such that Z is extremely disconnected then gof is is strongly θ-continuous.
Proof: let x∈X and V is wi-open set in Z containing y=f(x), since g is θ-continuous there exist pi-open set U in Y containing y such that $g(cl_{pi}(U)) \subseteq cl_{wi}(V)$. now since f is strongly θ-continuous there exist ti-open set H containing x such that $f(cl_{ti}(H)) \subseteq U$ then $g(f(cl_{ti}(H)) \subseteq g(U) \subseteq g(cl_{pi}(U)) \subseteq cl_{wi}(V)=V$ from that gof is strongly θ-continuous.

2- the main results
Theorem(2-1): : let f: $(X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ be onto θ-continuous and let $g:(Y,p_1,p_2)\rightarrow (Z,w_1,w_2)$ be strongly θ-continuous then gof is strongly θ-continuous.
Proof: let x∈X and V is wi-open set in Z containing y=f(x), since g is strongly θ-continuous there exist pi-open set U in Y containing y such that $g(cl_{pi}(U))\subseteq V$. now since f is θ-continuous there exist ti-open set H containing x such that $f(cl_{ti}(H)) \subseteq cl_{pi}(U)$ then $g(f(cl_{ti}(H)) \subseteq g(cl_{pi}(U)) \subseteq V$ from that we get gof is strongly θ-continuous.

Theorem (2-2): let f: $(X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ be onto θ-continuous and let $g:(Y,p_1,p_2)\rightarrow (Z,w_1,w_2)$ be θ-continuous then gof is is θ-continuous.
Proof: let x∈X and V be wi-open set in Z containing y=f(x), since g is θ-continuous there exist pi-open set U in Y containing y such that $g(cl_{pi}(U))\subseteq cl_{wi}(V)$. now since f is θ-continuous there exists ti-open set H containing x such that $f(cl_{ti}(H)) \subseteq cl_{pi}(U)$ then $g(f(cl_{ti}(H)) \subseteq g(cl_{pi}(U)) \subseteq cl_{wi}(V)$ from that gof is θ-continuous.
Definition(2-3) :A function f: $(X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ is said to be *semi-strongly θ-continuous(resp. almost quasi-continuous*) if for each x∈X and each pi-open set V in Y containing f(x) there exist ti-semi open set U in X containing x such that $f(cl_{ti}(U)) \subseteq V$ (resp. $f(U) \subseteq int_{pi}(cl_{pi}(V))$) , i=1or2

Definition(2-4): A function f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ is said to be almost semi-strongly θ-continuous if for each $x\in X$ and each pi-open set V in Y containing f(x) there exist ti-semi open set U in X containing x such that $f(cl_{ti}(U))\subseteq int_{pi}(cl_{pi}(V))$, i=1or2.
For easy we show the diagram:

θ-generalized continuous→generalized continuous→generalized semi continuous
↑
Strongly θ-continuous → → almost strongly θ-continuous →almost continuous
↓
Semi-strongly θ-continuous→almost-semi-strongly θ-continuous→almost-quasi

continuous

Remark(2-5): Non of implication in the diagram is revisable as we show in the following examples:

Example(2-6): let X={1,2,3,4} andt_1={X ,{3},{1,2},{1,2,3}} , t_2={X, {1}} and Y={k,l,m} p_1={Y,{m},{l},{m,l}}, p_2={Y,{l}} , define f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ as follow f(1)=f(2)=k , f(3)=f(4)=m then f is almost semi strongly θ-continuous but it is not almost strongly θ-continuous.

Example(2-7): let X be the set of real number and t be the co-countable topology for X , let Y={a,b} and p={Y,{a}} define f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ as follow f(x)=a if x is rational number and f(x) =b if x is irrational number then f is almost strongly θ-continuous but it is not semi strongly θ-continuous.

Example(2-8): let X={a,b,c,d} andt_1={X, ∅} , t_2={X, {c},{a,b},{a,b,c}} and Y={x,y,z} p_1={Y,{x},{y},{x,y}}= p_2 , define f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ as follow f(a)=f(b)=y , f(c)=f(d)=z then f is almost continuous but it is not almost semi strongly θ-continuous.

Theorem(2-9) : if f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ is weakly continuous function and Y is almost regular space then f is almost strongly θ-continuous.

Proof: let $x\in X$ and is pi-regular open set containing f(x).there exists pi-regular open set Wof Y such that $f(x)\in W\subset cl_{pi}(W)\subset V$ .since f is weakly continuous function , there exist ti-open set G containing x such that $f(G)\subset cl_{pi}(W)\subset V$. there for f is almost continuous .now let $U=f^{-1}(W)$ then U is ti-open set containing x and
$cl_{ti}(U)\subset f^{-1}(cl_{pi}(W))\subset V$ and then f is almost strongly θ-continuous.

Theorem(2-10): let $(X,t_1,t_2)$and (Y,p_1,p_2) are two bitopological spaces and
f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ be a function then the following are equivalent :

1- f is almost semi strongly θ-continuous
2- $f^{-1}(W)$ is θ-semi open set in X for every pi-regular open set W of Y.
3- $f^{-1}(G)$ is θ-semi closed set in X for every pi-regular closed set G of Y.
4- for each $x\in X$ and regular open set W containing f(x) there exist pi-semi open set U in X containing x such that $f(cl_{ti}(U))\subseteq W$.

Proof: (1)→(2) , let W is pi- regular open set , since f is almost semi strongly θ-continuous then there exist ti-semi open set U in X such that $f(cl_{ti}(U))\subseteq W$ then f is semi strongly θ-continuous and then $f^{-1}(W)$ is θ-semi open set in X .

(2)→(3) , let G be pi-regular closed set in Y then Y-G is regular open set and by (2)
$f^{-1}(Y-G)=X-f^{-1}(G)$ is θ-semi open set in X and then $f^{-1}(G)$ is θ-semi closed set in X.

(3)→(4) , let $x\in X$ and W is regular open set containing f(x) then Y-W is regular closed set and by (3) $f^{-1}(Y-W)=X-f^{-1}(W)$ is θ-semi closed set and $f^{-1}(W)$ is θ-semi open set containing x . let $f^{-1}(W)=U$ then $f(U)\subseteq W$ and there for $f(cl_{ti}(U))\subseteq W$.

(4)→(1) , by (4) and since every regular open set is pi-open set the result exist.

Theorem(2-11): let f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ be a function then the following properties hold:

1- if X is extremely disconnected and f is almost semi strongly θ-continuous then f is almost strongly θ-continuous.
2- if Y is semi regular space and f is almost semi strongly θ-continuous then f is semi strongly θ-continuous.

3- if X is s*-regular space and f is almost quasi continuous then f is almost semi strongly θ-continuous.

Proof: (1) , since f is almost semi strongly θ-continuous then for each $x \in X$ and each pi-open set C containing f(x) there exist ti-semi open set U containing x such that $f(cl_{ti}(U) \subseteq int_{pi}(cl_{pi}(C))$.

(2) . the proof exist by the condition of the space Y is semi regular space where V is semi open and semi closed.

(3) since f is almost quasi continuous then foe each $x \in X$ and pi-open set V containing f(x) there exist ti-open set U containing x such that $f(U) \subseteq int_{pi}(cl_{pi}(V))$, since X is S*regular space $f(cl_{ti}(U)) \subseteq int_{pi}(cl_{pi}(V))$ and then f is almost semi strongly θ-continuous.

Theorem(2-12): If $f: (X,t_1,t_2) \rightarrow (Y,p_1,p_2)$ is semi strongly θ-continuous and $g:(Y,p_1,p_2) \rightarrow (Z,w_1,w_2)$ is almost-continuous function then gof is almost-semi strongly θ-continuous.

Proof: the proof is omitted

Theorem(2-13): if $f: (X,t_1,t_2) \rightarrow (Y,p_1,p_2)$ is onto semi strongly θ-continuous and $g:(Y,p_1,p_2) \rightarrow (Z,w_1,w_2)$ is continuous then gof is semi strongly θ-continuous.

Proof: let V be wi-open set and $y \in Y$, $g(y) \in V$ then there exist pi-open set U in Y containing y such that $g(U) \subseteq V$, since f is semi strongly θ-continuous and U is pi-open set in Y and y=f(x) ,there exist semi open set H such hat $f(cl_{ti}(H)) \subseteq U$ then $g(f(cl_{ti}(H)) \subseteq g(U) \subseteq V$, i=1or2 hence gof is semi strongly θ-continuous .

Theorem(2-14): let $f: (X,t_1,t_2) \rightarrow (Y,p_1,p_2)$ be an almost θ-continuous such that X is extremely disconnected and Y is semi regular space then f is generalized continuous function.

Proof: let V is pi-open set , since Y is semi regular space then V is semi open set and semi closed set and since f is almost θ-continuous there exist ti-open set U in X such that $f^{-1}(int_{pi}(cl_{pi}(V)) \subseteq cl_{ti}(U)=U$ and also $int_{pi}(cl_{pi}(V))=V$ since X is extremely disconnected and V is regular closed set then $f^{-1}(V)$ is ti-closed set in X and then f is generalized closed set.

Theorem(2-15): let $f: (X,t_1,t_2) \rightarrow (Y,p_1,p_2)$ be an almost continuous such that Y is semi regular space then f is generalized continuous function.

Proof: let V is pi-close set in Y since Y is semi regular space then V is both semi open and semi closed set then $V = int_{pi}(cl_{pi}(V))$ and since f is almost continuous then $f^{-1}(V)$ is ti-closed set in X and then it is generalized closed set .

Theorem(2-16): let $f: (X,t_1,t_2) \rightarrow (Y,p_1,p_2)$ be an semi strongly θ-continuous such that X is semi regular space and extremely disconnected space then f is generalized continuous function.

Proof: let V is pi-closed set in Y and $x \in X$ such that $f(x) \in V$ then Y-V is pi-open set such that $f(x) \notin Y-V$ since f is semi strongly θ-continuous and X is regular space there exist ti-open set U such that $f(U) \subseteq int_{ti}(cl_{ti}(Y-V))=Y- int_{ti}(cl_{ti}(V)$ then $f^{-1}(Y-V)=X-f^{-1}(V)$ is ti-open set and then $f^{-1}(V)$ is ti-closed set therefore $f^{-1}(V)$ is generalized closed set.

Theorem(2-17):a mapping $f:(X,t_1,t_2) \rightarrow (Y,p_1,p_2)$ is generalized continuous function such that X is regular space if and only if f is θ-generalized continuous .Proof: since X is regular space then V is pi-closed set in Y if and only if it is θ-generalized closed set then the result exist.

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