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Two-Photon interference with thermally pumped three-level atomic system

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Abstract

The system of atomic three –level cascade emission has been investigated as a potential source for two–photon interference experiments. The theory of a two-photon interference experiment with thermally pumped three-level atomic system acting as a source of the two-photon state is formulated. General results for mean number of photon coincidences in the two output arms of a beam-splitter in two-photon interference experiment are obtained. The results generalized previous results of other workers for the same system. It is found that such a system exhibits clear signs of two-photon interference effects for small atomic pumping rate. The results obtained show interference effects that cannot be accounted for by the classical theory of light. They form a contribution to the series of quantum optical effects studied before.

Keywords. Photon-interference; Two-Photon quantum state.

1. Introduction

The effect of two-Photon interference [12] has been extensively studied during the last few years. The effect can be used to affirm basic fundamental theorems of quantum theory and to distinguish classical and quantum laws of light [11].It has been used to obtain and study special quantum states that have very promising applications in quantum communication and computation [3,8]. The effect has also been used for spectroscopic studies [6]. It has been employed to register the length of photon wave packet on afemtosecond time sale [5].Almost all the experiments on two-photon interference have so far used the process of spontaneous parametric down conversion as a source for the required two-photon state [13,5].The correlated Pair of photons obtained in spontaneous emission of atomic three-level cascade emission has been examined as a potential source for two-photon interference [2]. In their study the atomic system was excited thermally and the results were limited to special values of pumping rate and detection time .In these cases they showed clear evidence of interference.

In the present study [1], we generalize the results of Fearn and Loudon (1989) by attempting a general investigation of the three-level atomic cascade emission as a potential source for the two-photon interference experiment. In the following section we outline the basic theory of the experiment and in sec. (3) we present our results and discuss them.

2. Theoretical Model

Consider a three- level cascade atomic system that is thermally pumped to the upper level at a rate R and relaxing in a ladder fashion resulting in the emission of two photons(figure(1)).The system is described by the following rate equations for the atomic density matrix ρ [10];

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= 2\gamma_1\rho_{11} - R \\
\frac{\partial}{\partial t}\rho_{22} &= 2\gamma_2\rho_{33} - 2\gamma_1\rho_{22} \\
\frac{\partial}{\partial t}\rho_{33} &= R - 2\gamma_2\rho_{33} \\
\frac{\partial}{\partial t}\rho_{21} &= -\gamma_1\rho_{21} \\
\frac{\partial}{\partial t}\rho_{32} &= -(\gamma_1+\gamma_2)\rho_{32}
\end{aligned} \qquad \dots (1)$$

Figure (2) shows a typical scheme for a two-photon interference experiment with a beam-splitter. The theory of the experiment is too complicated and involves the reliance on a number of theorems and relations [1]. In the following we give a very brief outline of the theory with the aim of only defining the different quantities involved. The mean number of photon coincidences in detector (3) and between the two detectors (3) and (4), during a total detection time T, are respectively given by;

$$\left\langle m_3^2(T) \right\rangle = \eta_3^2 \int_0^T dt \int_0^T dt' \ \Gamma^{(2)}(\vec{r}_3 t, \vec{r}_3 t'; \vec{r}_3 t', \vec{r}_3 t) \qquad \dots (2)$$

$$\langle m_3(T)m_4(T)\rangle = \eta_3\eta_4 \int_0^T dt \int_0^T dt' \; \Gamma^{(2)}(\vec{r}_3 t, \vec{r}_4 t'; \vec{r}_4 t', \vec{r}_3 t) \quad ..(3)$$

Where the function in the integrand is the degree of the second-order coherence of the detected light, defined by;

$$\Gamma^{(2)}(\vec{r},t,\vec{r},t';\vec{r},t',\vec{r},t) = \langle \hat{E}^-(\vec{r},t)\hat{E}^-(\vec{r},t)\,\hat{T}\,\hat{E}^+(\vec{r},t')\hat{E}^+(\vec{r},t)\rangle \quad ..(4)$$

In these equations, $\hat{E}^+$ and $\hat{E}^-$ are respectively the annihilation and creation operators of the detected light field, η_i is the efficiency parameter of the detector, $\hat{T}$ is the time-ordering operator and the angle brackets refer to quantum expectation values. The output field operators, $\hat{E}(\vec{r}_3 t)$ and $\hat{E}(\vec{r}_4 t)$,in these expressions can be related to those of the field in the input arms, $\hat{E}(\vec{r}_1 t)$ and $\hat{E}(\vec{r}_2 t)$, through the reflection (r) and transmission (t) coefficients of the beam-splitter using the quantum theorem of the beam-splitter [4] and these, in turn, can be expressed in terms of the atomic transition operators $\hat{\pi}$ and $\hat{\pi}^+$ as follows[9];

$$\hat{E}^+(\vec{r}t) = E_0(\vec{r})\hat{\pi}(t-\frac{r}{c})$$
$$\hat{E}^-(\vec{r}t) = E_0(\vec{r})\hat{\pi}^+(t-\frac{r}{c}) \quad ...(5)$$

Where $E_0(\vec{r})$ is a scalar function that describes the spatial distribution of the field. The resulting expectation values should at this step be treated carefully taking all possible arrangements, indicated by the time-ordering operator $\hat{T}$, into account. At this stage the resulting multi-time expectation values can be related to the solution of (1) first through the relations between the atomic density matrix elements and the atomic transition operators [9] and then through the use of the quantum regression theorem [7]. The resulting complicated expressions should now be subjected to the double integral procedure, indicated in (2) and (3), taking into account the correct different areas in the plane of the two variables t and t' .In the next section we summarize the final results of this complicated and lengthy procedure.

3. Results and Discussion

Define the following quantities:

$$\begin{aligned}
&\gamma_3 = \gamma_1 + \gamma_2 &&, \gamma_4 = \gamma_1 - \gamma_2 \\
&\gamma_5 = 2\gamma_1 + \gamma_2 &&, \gamma_6 = 2\gamma_1 - \gamma_2 \\
&\Delta = \omega_1 - \omega_2 &&, \Delta_1^2 = \gamma_2^2 + \Delta^2 \\
&\Delta_2^2 = \gamma_5^2 + \Delta^2 &&, \Delta_3^2 = \gamma_6^2 + \Delta^2 \qquad ...(6) \\
&\Delta_4^2 = \gamma_5^2 - \Delta^2 &&, \Delta_5^2 = \gamma_6^2 - \Delta^2, \Delta_6^2 = \gamma_2^2 - \Delta^2 \\
&\delta\tau = \frac{r_2}{c} - \frac{r_1}{c} &&, \quad \tau = |\delta\tau|
\end{aligned}$$

$$A = E_{01}^4(\vec{r}_1)\frac{R^2}{4\gamma_1^2}\left\{ T^2 - \frac{\gamma_3 T}{\gamma_1\gamma_2} + \frac{\gamma_2}{2\gamma_1^2\gamma_4}\left(e^{-2\gamma_1 T} - 1\right) - \frac{\gamma_1}{2\gamma_2^2\gamma_4}\left(e^{-2\gamma_2 T} - 1\right) \right\} \quad .... (7)$$

$$B = E_{01}^2(\vec{r}_1)E_{02}^2(\vec{r}_2)\frac{R^2}{4\gamma_1\gamma_2}\left\{ T^2 + \frac{T}{R} - \frac{\gamma_5 T}{2\gamma_1\gamma_2} + \frac{1}{2\gamma_1 R}\left(1 + \frac{R\gamma_2}{2\gamma_1\gamma_4}\right)e^{-2\gamma_1\tau}\left(e^{-2\gamma_2 T} - 1\right) - \right.$$
$$\left. - \frac{\gamma_1}{4\gamma_2^2\gamma_4}e^{-2\gamma_2\tau}\left(e^{-2\gamma_2 T} - 1\right) - \frac{1}{4\gamma_2^2}\left(e^{-2\gamma_2 T} - 1\right) \right\} \quad , \quad (\delta\tau > 0) \quad ...... (8)$$

$$= E_{01}^2(\vec{r}_1)E_{02}^2(\vec{r}_2)\frac{R^2}{4\gamma_1\gamma_2}\left\{ T^2 + \frac{T}{R} - \frac{\gamma_5 T}{2\gamma_1\gamma_2} + \frac{1}{2\gamma_1 R}\left(1 + \frac{R\gamma_2}{2\gamma_1\gamma_4}\right)\left(e^{-2\gamma_2 T} - 1\right) - \right.$$
$$\left. - \frac{e^{-2\gamma_2 T}}{4\gamma_2^2}\left(e^{-2\gamma_2 T} - 1\right) - \frac{\gamma_1}{4\gamma_2^2\gamma_4}\left(e^{-2\gamma_2 T} - 1\right) \right\} \quad , \quad (\delta\tau < 0) \quad ...... (9)$$

$$C = E_{02}^{4}(\vec{r}_2)\frac{R^2}{4\gamma_2^2}\left\{ T^2 - \frac{T}{\gamma_2} - \frac{1}{2\gamma_2^2}\left(e^{-2\gamma_2 T} - 1\right) \right\} \qquad \ldots\ldots (10)$$

$$\begin{aligned} D = E_{01}^{2}(\vec{r}_1)E_{02}^{2}(\vec{r}_2)\frac{R^2}{2\gamma_1\gamma_2}\Bigg\{ & \frac{\gamma_5 T}{\Delta_2^2} - \frac{\Delta_4^2}{\Delta_2^4} - \frac{\gamma_6 T}{\Delta_3^2}e^{-2\gamma_2\tau} + \frac{\Delta_5^2}{\Delta_3^4}e^{-2\gamma_2\tau} + \Bigg(\frac{\Delta_4^2\cos\Delta\tau - 2\gamma_5\Delta\sin\Delta\tau}{\Delta_2^4} - \\ & - \frac{\gamma_5 T\cos\Delta\tau - \Delta T\sin\Delta\tau}{\Delta_2^2} - \frac{\Delta_5^2\cos\Delta\tau - 2\gamma_6\Delta\sin\Delta\tau}{\Delta_3^4} + \\ & + \frac{\gamma_6 T\cos\Delta\tau - \Delta T\sin\Delta\tau}{\Delta_3^2}\Bigg)e^{-\gamma_5\tau} \Bigg\} \qquad , \quad (\delta\tau < 0) \ldots (11) \end{aligned}$$

$$\begin{aligned} = E_{01}^{2}(\vec{r}_1)E_{02}^{2}(\vec{r}_2)\frac{R}{\gamma_2}\Bigg\{ & \frac{R}{2\gamma_1}\Bigg\{ \frac{\gamma_5 T}{\Delta_2^2} - \frac{\Delta_4^2}{\Delta_2^4} + \Bigg(\frac{\Delta_4^2\cos\Delta\tau - 2\gamma_5\Delta\sin\Delta\tau}{\Delta_2^4} - \\ & - \frac{\gamma_5 T\cos\Delta\tau - \Delta T\sin\Delta\tau}{\Delta_2^2}\Bigg)e^{-\gamma_5\tau}\Bigg\} + \left(1 + \frac{R\gamma_2}{2\gamma_1\gamma_4}\right)e^{-2\gamma_1\tau}\Bigg\{ \frac{\gamma_2 T}{\Delta_1^2} - \frac{\Delta_6^2}{\Delta_1^4} + \\ & + \Bigg[\frac{\Delta_6^2\cos\Delta\tau - 2\gamma_2\Delta\sin\Delta\tau}{\Delta_1^4} - \frac{\gamma_2 T\cos\Delta\tau - \Delta T\sin\Delta\tau}{\Delta_1^2}\Bigg]e^{-\gamma_2\tau}\Bigg\} - \\ & - \frac{R}{2\gamma_4}e^{-2\gamma_2\tau}\Bigg\{ \frac{\gamma_6 T}{\Delta_3^2} - \frac{\Delta_5^2}{\Delta_3^4} + \Bigg[\frac{\Delta_5^2\cos\Delta\tau - 2\gamma_6\Delta\sin\Delta\tau}{\Delta_3^4} - \qquad \ldots\ldots (12) \\ & - \frac{\gamma_6 T\cos\Delta\tau - \Delta T\sin\Delta\tau}{\Delta_3^2}\Bigg]e^{-\gamma_6\tau}\Bigg\} \qquad , \quad (\delta\tau > 0) \end{aligned}$$

then, for the entrance state where the two photons are entered through the two different arms, the mean number of photon coincidences within detector (3) and between detectors (3) and (4) are, respectively given by;

$$\left\langle m_3^2(T)\right\rangle = \eta_3^2\left\{ |r|^4 A + |t|^4 C + 2|rt|^2 (B + D) \right\} \qquad \ldots (13)$$

$$\left\langle m_3(T)m_4(T)\right\rangle = \eta_3\eta_4\left\{ |rt|^2 (A + C - 2D) + (|r|^4 + |t|^4)B \right\} \qquad \ldots (14)$$

In the above expressions, ω_2 and ω_1 are the frequencies of the two cascade photons.

To obtain some insight into these complicated results, one has to change some special cases of them into graphs. Too many cases have been changed [1], and we reproduce here few of them (figures (3)-(6)). In all these figures, the quantity drawn is always ($\gamma_1^2/\eta_3\eta_4 E_{01}^{2}(\vec{r}_1)E_{02}^{2}(\vec{r}_2)$) times the mean number of photon coincidences against γ_1 times the path difference ($\delta\tau$).All figures are drowning for a (50/50) beam-splitter (that is to say that $|r|^2 = |t|^2$ and thus $|r|^2 + |t|^2 = 1$, and the beam-splitter is loss-less). Figures ((3)-(5)) represent the mean number of photon coincidences between the two output arms. The broken lines stand for entrance cases through the same input arm. The behavior in this case is that of classical independent particles showing no sign of interference. For the case where the two photons pass through the two different input arms, the behavior is completely different (the continuous lines). Interference effects clearly show up for some positive range of the path difference ($\delta\tau$). This range is the right range for the path difference that helps the two correlated photons to coincide on the beam splitter. The visibility of the interference effect, of course, depends on the path difference ($\delta\tau$) and the degree of correlation between the two photons. It is well known about this atomic system that the two emitted photons are strongly correlated for small values of pumping rate

R [10]. Changing the atomic relaxation rates (γ_1 and γ_2) will change the mean coincidence number of photons but has no drastic effect on the visibility of the interference effect. Introducing detuning between the two emitted photons will bring some modulation into the interference effect and somehow extends its range. Figure (6) represents the mean number of photon coincidences within the same output arm (3). They clearly show interference effects complementary to those in figures ((3)-(5)).

The results in (13) and (14) are general results and are valid for all values of the experimental parameters. In the special experimental cases in which $(R << \gamma_1, \gamma_2)$ and $(T >> \frac{1}{\gamma_1}, \frac{1}{\gamma_2}, \delta\tau)$ under the approximation in which $E_{01}^2(\vec{r}_1)\rho_{22}(\infty) = E_{02}^2(\vec{r}_2)\rho_{33}(\infty) = f$, the result in (14) reduces to the following form

$$\langle m_3(T)m_4(T)\rangle = \eta_3\eta_4 fT[\, fT + (\frac{f}{R})[|\boldsymbol{r}|^4 + |\boldsymbol{t}|^4 - 8|\boldsymbol{rt}|^2 \gamma_1 e^{-2\gamma_1\tau} * \\ * \frac{\gamma_2 - \{\gamma_2 \cos\Delta\tau - \Delta \sin\Delta\tau\} e^{-\gamma_2\tau}}{\gamma_2^2 + \Delta^2}]] \qquad (\delta\tau > 0) \qquad \text{……….(15)}$$

when in addition $(\Delta = 0)$, this result yields

$$\langle m_3(T)m_4(T)\rangle = \eta_3\eta_4 fT[\, fT + (\frac{f}{R})[|\boldsymbol{r}|^4 + |\boldsymbol{t}|^4 - 8|\boldsymbol{rt}|^2 (\frac{\gamma_1}{\gamma_2}) e^{-2\gamma_1\tau}(1 - e^{-\gamma_2\tau})]] \quad (\delta\tau > 0) \quad \text{…(16)}$$

Equation (15) is the main result in the study of Fearn & Loudon (1989) and the plotted quantity in their study is that in equation (16) after dropping its first term.

The terms in (13) and (14) that depend on (T^2) account for coincidences of two photons in different atomic emissions. Thus, such terms have a negative effect on the visibility of the produced interference effect. As mentioned, Fearn & Loudon (1989) have dropped such terms in their graphs. Our shown graphs represent the full expressions in (13) and (14), and yet the visibility of the interference effect is still comparable to that found in the special case of Fearn & Loudon (1989). Such visibility can now be adjusted using the full ranges of all the experimental parameters involved.

In summary, although the visibility of the interference effect is limited, light produced by a thermally excited three- level atomic cascade emission clearly exhibits signs of a two-photon interference for some wide ranges of the system parameters.

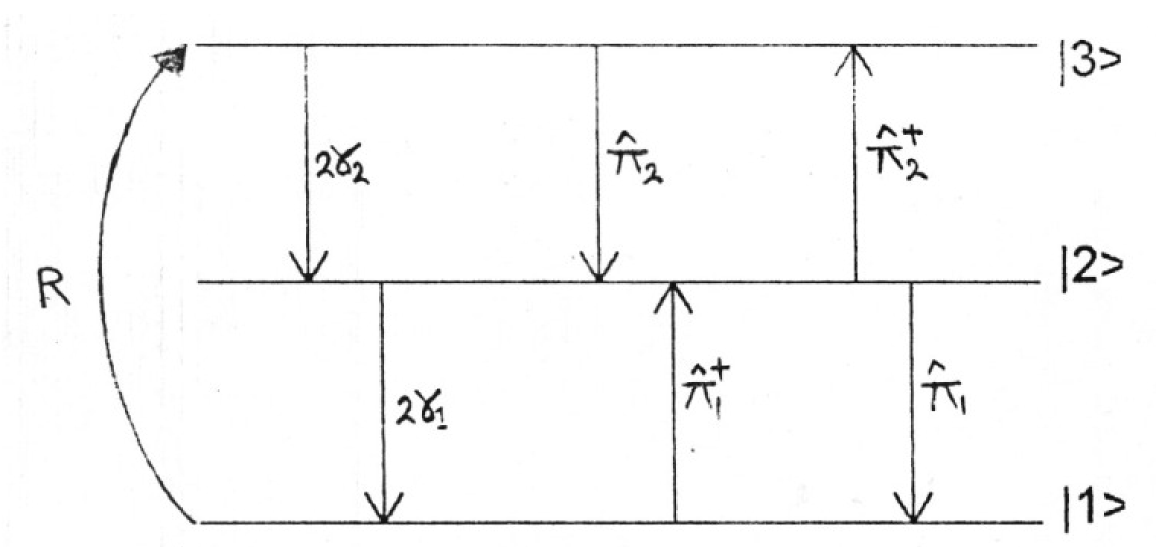

Fig.(1); Schematic diagram of the three-level atomic system showing pumping rate, relaxation rates and raising and lowering operators.

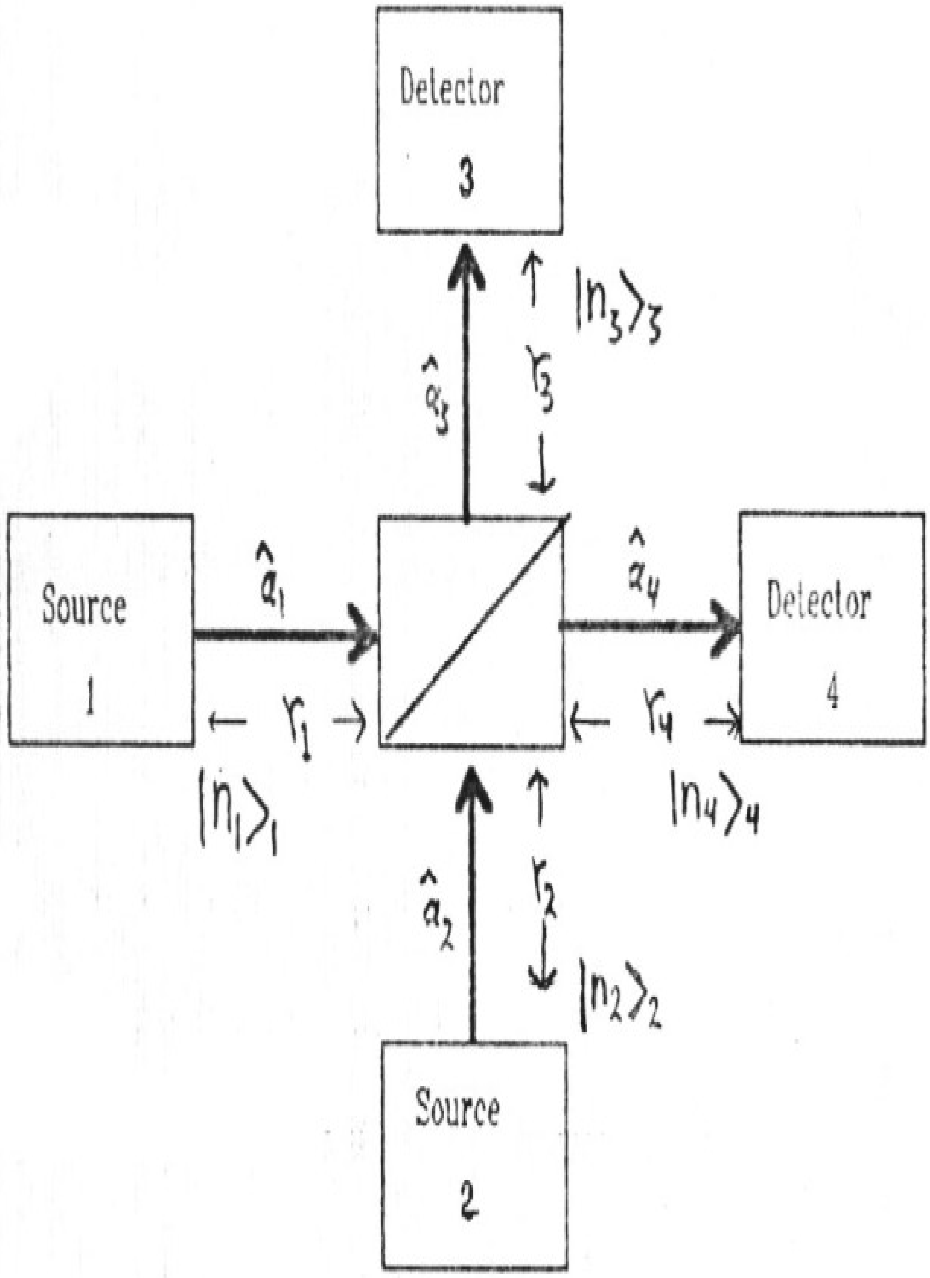

Fig. (2); Schematic diagram of the two-photon interference experiment with abeam-splitter.

$$Q \equiv \left(\gamma_1 / \eta_3\eta_4 E_{01}^2(\vec{r}_1) E_{02}^2(\vec{r}_2)\right)\left\langle m_3(T) m_4(T)\right\rangle$$

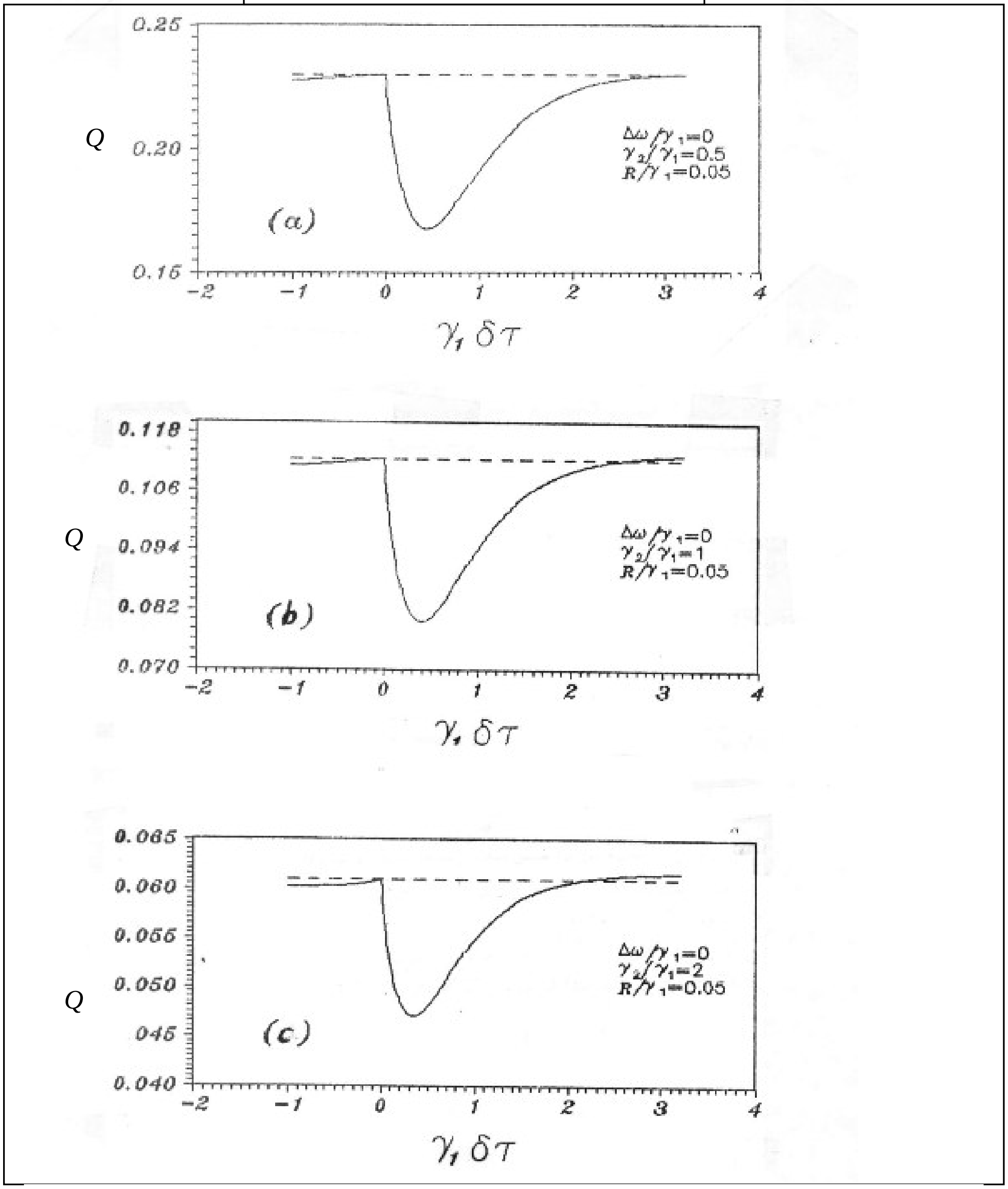

Fig. (3); Graphs for the quantity ($\gamma_1^2/\eta_3\eta_4 E_{01}^2(\vec{r}_1) E_{02}^2(\vec{r}_2)$) $\langle m_3(T)m_4(T)\rangle$ against $\gamma_1\delta\tau$ for different experimental values. The broken line stands for the same quantity for entrance cases through the same input arm.

$$Q \equiv \left(\gamma_1 / \eta_3 \eta_4 E_{01}^2(\vec{r}_1) E_{02}^2(\vec{r}_2)\right)\left\langle m_3(T) m_4(T)\right\rangle$$

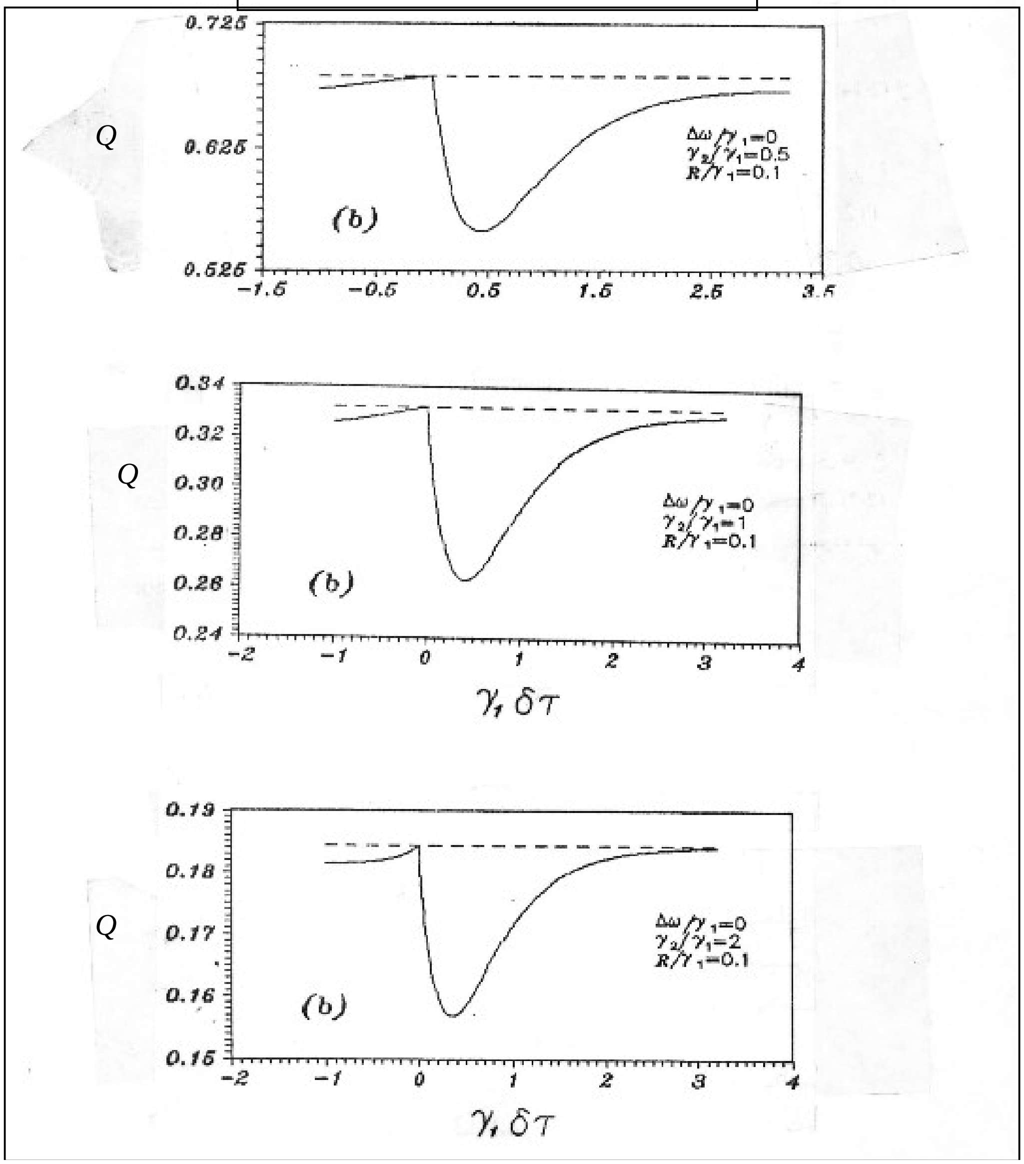

Fig. (4); As in Fig. (3).

Q

$$Q \equiv \left(\gamma_1 / \eta_3 \eta_4 E_{01}^2(\vec{r}_1) E_{02}^2(\vec{r}_2)\right)\langle m_3(T) m_4(T)\rangle$$

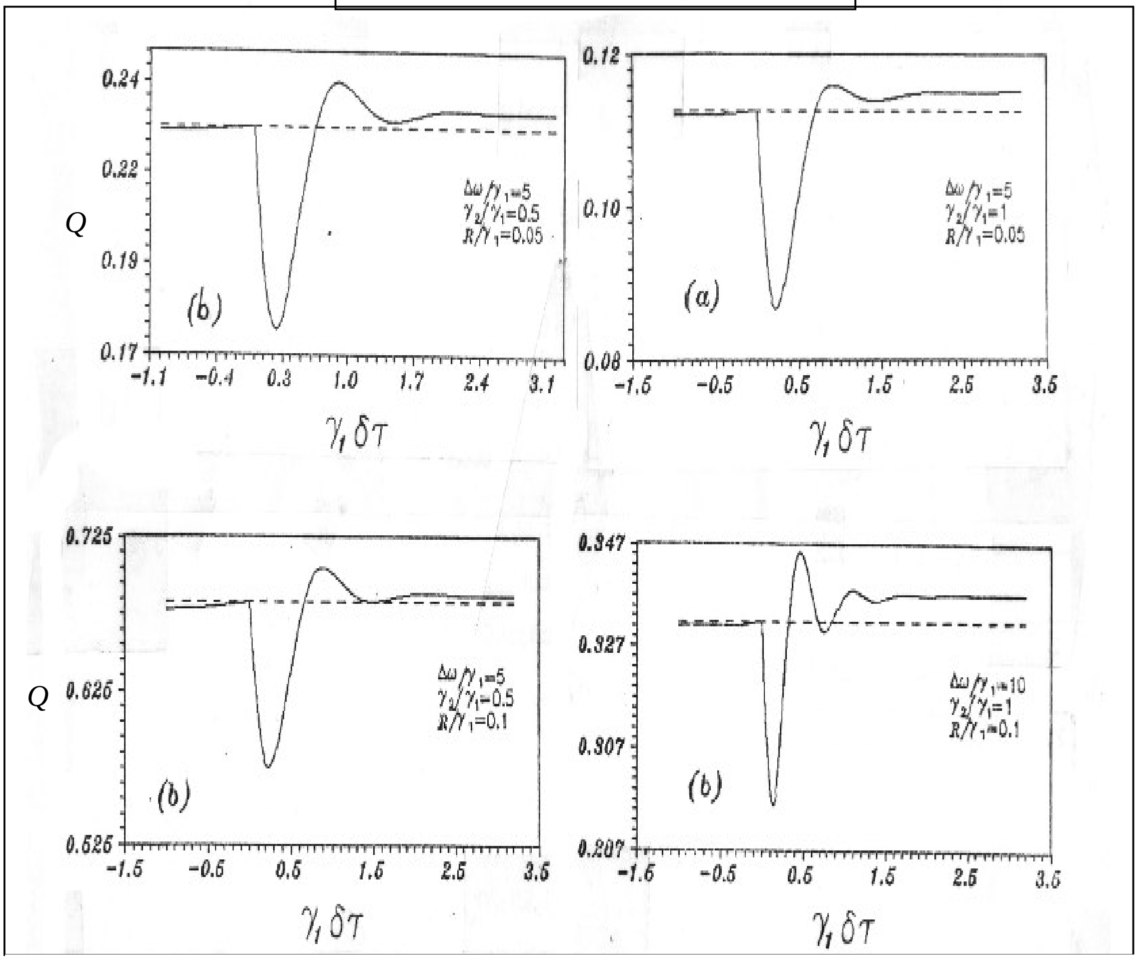

Fig. (5); As in Fig. (3).

$$Q1 \equiv \left(\gamma_1 / \eta_3 \eta_4 E_{01}^2(\vec{r}_1) E_{02}^2(\vec{r}_2)\right)\left\langle m_3^2(T)\right\rangle$$

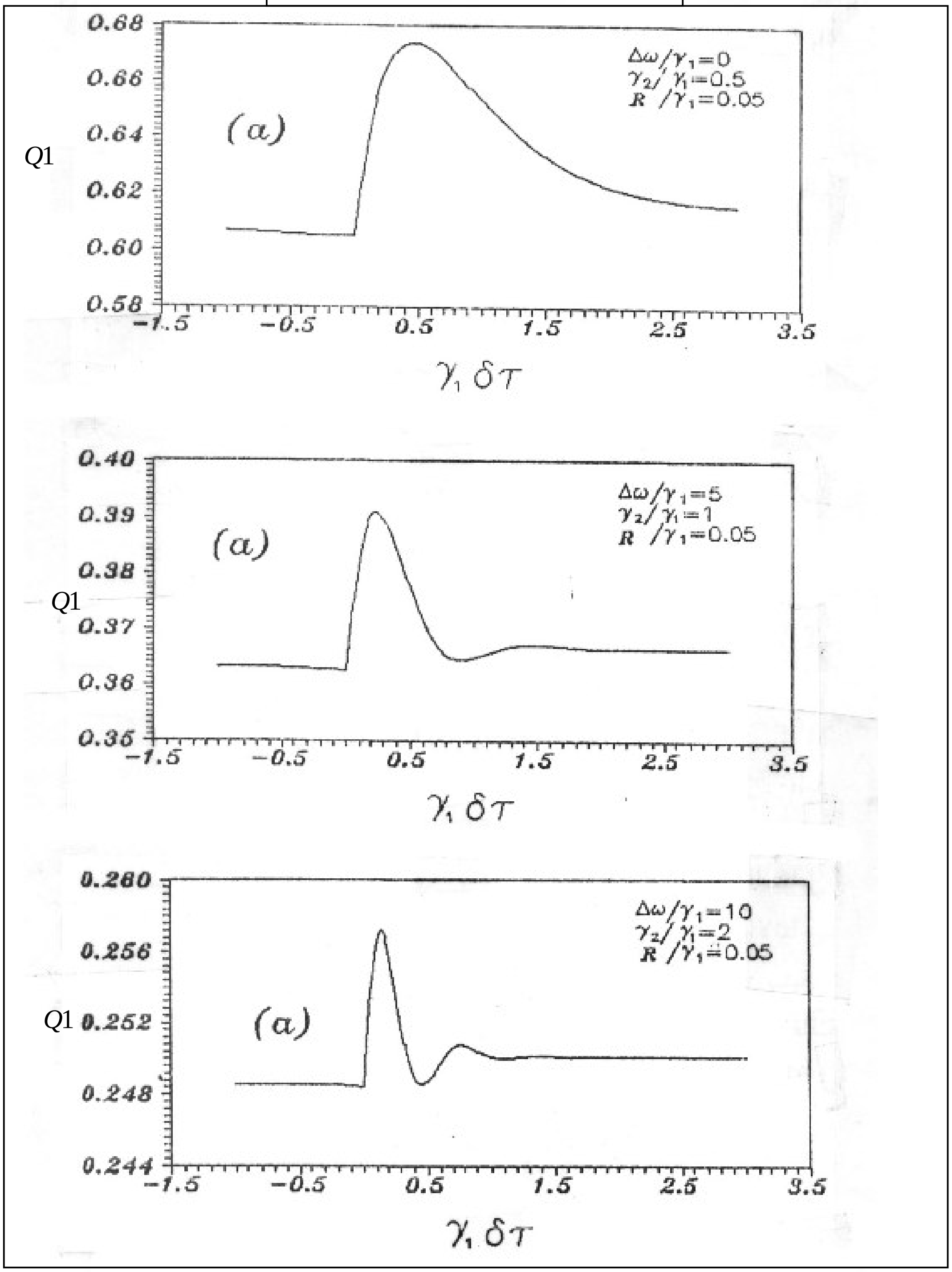

Fig. (6); Graphs for the quantity ($\gamma_1^2/\eta_3\eta_4 E_{01}^2(\vec{r}_1)E_{02}^2(\vec{r}_2)$) $\left\langle m_3^2(T)\right\rangle$ against $\gamma_1\delta\tau$ for different experimental values.

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تداخل- الفوتونين باستعمال نظام ذري ثلاثي- المستوي يهيج حراريا

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المستخلص

تم في هذه الدراسه اختبار النظام الذري ثلاثي- المستويات متعاقب الانبعاث كمصدر فعال لتجارب تداخل-الفوتونين. تم صياغة نظرية لتجربة تداخل-الفوتونين باستعمال نظام ذري ثلاثي-المستويات مضخ حرارياً كمصدر لحالة الفوتونين. تم الحصول على نتائج عامة لمعدلات توافق الفوتونات في ذراعي الاخراج لمجزئ الشعاع الضوئي في تجربة تداخل-الفوتونين. النتائج تشكل تعميماً لنتائج سابقة لنفس النظام. لقد وجد أن النظام يظهر اشارات تداخل واضحة لقيم صغيرة من معدل الضخ. النتائج المستحصلة تظهر تأثيرات تداخل لا يمكن تفسيرها بالاعتماد على النظرية الكلاسيكية للضوء, وهي تشكل مساهمة في سلسلة التأثيرات البصرية الكمية المدروسة سابقاً.