Two – photon interference with coherently-driven three-level atomic system

T. Al-Khoza'I and A. Al-Hilfy
Department of physics,College of Education,
University of Basrah,Basrah, Iraq.

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Abstract

Two-photon interference experiments have historically been associated with two-photon state produced in the process of spontaneous parametric down conversion. We investigate the entangled two-photon state obtained from a three-level cascade atomic system driven by two coherent optical fields as an input state in a two photon interference experiment with a beam-splitter. For weak-coupling scheme and particular choices of the atomic parameters, interference effects have been shown with higher visibility than those obtained with the same atomic system with thermal excitation considered before.

Keywords: Photon interference, two-photon quantum state.

1. Introduction

Studies that aim to produce and investigate the properties of some special quantum optical states have gathered momentum during the last few years due to an increasing of possible valuable practical applications. Thus, the quantum state of two entangled photons [16] has proved to be of great importance for fundamental theoretical reasons[5] and for potential practical applications in quantum cryptography and quantum computing [8], spectroscopy [6], metrology [8], and measurement of group delay times [18] Two-photon quantum states can be investigated through two-photon interference experiments [13]. The first experimental observations of two-photon interference in a beam-splitter were reported in 1985s. [10]and [16]. Ever since, almost all the two-photon interference experiments have relied on two-photon states produced in the process of spontaneous parametric down-conversion [11] It is thus of interest to investigate other sources for the two entangled photons. Fearn and Loudon (1989) have considered a thermally excited three-level cascade atomic system. [2]has considered the same system. They both showed signs of two-photon interference, but with a limited visibility.

In this work [4], we consider a two-photon interference experiment with a beam-splitter using a three-level cascade atomic system driven by two coherent optical fields as a source for the two entangled photons. The next section contains the theoretical model of the experiment. In section (3) we summarize the analytical results and discuss them in section (4).

2. Theoretical Model

Due to the limited space, we only give here a brief outline of the way to formulate the results of the experiment. The formalism employs different theorems and a more detailed account can be found in [4].We consider a two-photon interference experiment with a beam-splitter (fig.1).The two-photon state is entered either through the same input arm (1 or 2) or through the two different input arms (1&2). The mean number of photon coincidences during a detection time T at the same detector (3 or 4) or between the two detectors (3 & 4) is registered. These numbers can respectively be expressed as;

$$\left\langle m_3^2(T) \right\rangle = \eta_3^2 \int_0^T dt \int_0^T dt' \; \Gamma^{(2)}(\vec{r}_3 t, \vec{r}_3 t'; \vec{r}_3 t', \vec{r}_3 t) \quad \text{.... (1)}$$

$$\left\langle m_3(T)\, m_4(T) \right\rangle = \eta_3 \eta_4 \int_0^T dt \int_0^T dt' \; \Gamma^{(2)}(\vec{r}_3 t, \vec{r}_4 t'; \vec{r}_4 t', \vec{r}_3 \quad \text{... (2)}$$

where the function in the integrand is the degree of second-order coherence of the detected field,

$$\Gamma^{(2)}(\vec{r}_i t, \vec{r}_j t'; \vec{r}_j t', \vec{r}_i t) = \left\langle \hat{E}^-(\vec{r}_i t) \hat{E}^-(\vec{r}_j t') \; \hat{T} \;\; \hat{E}^+(\vec{r}_j t') \hat{E}^+(\vec{r}_i t) \right\rangle \quad \text{..........(3)}$$

With $\hat{E}^{-}$ and $\hat{E}^{+}$ are respectively the annihilation and creation operators, $\hat{T}$ is time-ordering operator and η_i is a constant that contains the efficiency parameter of the detector i. These field operators are related to the emitted field operators through the beam-splitter transformation relations and these latter operators can be related to the atomic operators. Thus, the expectation values in (3) can finally be expressed in terms of the atomic density matrix elements. For the atomic density matrix, (fig.2)shows a schematic diagram of the coupled atom-exciting fields system. For such a system, the equations of motion of the reduced atomic density matrix (Optical Bloch equations) are [1], [3]

$$\frac{\partial}{\partial t}\rho_{11} = ig_1\alpha_1\rho_{12} - ig_1^*\alpha_1^*\rho_{21} + 2\gamma_{12}\rho_{22}$$

$$\frac{\partial}{\partial t}\rho_{22} = ig_1^*\alpha_1^*\rho_{21} + ig_2\alpha_2\rho_{23} - ig_1\alpha_1\rho_{12} - ig_2^*\alpha_2^*\rho_{32} - - 2\gamma_{12}\rho_{22} + 2\gamma_{23}\rho_{33}$$

$$\frac{\partial}{\partial t}\rho_{12} = -(\gamma_{12} - i\Delta_1)\rho_{12} + ig_1^*\alpha_1^*\rho_{11} + ig_2\alpha_2\rho_{13} - ig_1^*\alpha_1^*\rho_{22}.$$

$$\frac{\partial}{\partial t}\rho_{13} = -(\gamma_{23} - i\Delta_3)\rho_{13} + ig_2^*\alpha_2^*\rho_{12} - ig_1^*\alpha_1^*\rho_{23}.$$

$$\frac{\partial}{\partial t}\rho_{23} = -(\gamma_{13} - i\Delta_2)\rho_{23} + ig_2^*\alpha_2^*\rho_{22} - ig_1\alpha_1\rho_{13} - ig_2^*\alpha_2^*\rho_{33}. \qquad \ldots\ldots\ldots(4)$$

with the other four elements $\rho_{21}, \rho_{31}, \rho_{32}$ and ρ_{33} are respectively determined by the conjugates ρ_{12}, ρ_{13}, ρ_{23} and by

$$\rho_{33} = 1 - (\rho_{11} + \rho_{22}) \qquad \ldots.(5)$$

The relaxation parameter γ_{13} is defined by

$$\gamma_{13} = \gamma_{12} + \gamma_{23}\ , \qquad \ldots.(6)$$

α_1 and α_2 are the complex amplitudes of the two coherent driving fields, and g_1 and g_2 are coupling constants that contain the electric-dipole matrix elements of the two atomic transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively.

Accordingly, if the general solutions of (4) are expressed in the form

$$\rho_{ij}(t') = \sum_{kl} \alpha_{kl}^{(ij)}\ (t' - t)\rho_{kl}(t)\ ,\quad t' > t \qquad \ldots(7)$$

then, the degrees of second-order coherence in (1) and (2) can be expressed in terms of the functions $\alpha_{kl}^{(ij)}$. At this stage, the resulting complicated expressions should be subjected to a very careful double integration procedure taking into account the respective areas of integration. The final results are summarized in the next section.

3. Results & Discussion

The system of equations (4) has no simple explicit analytic general solution. Hence, one has to look for a solution under some special conditions. As long as the effect under study, that is the effect of two-photon interference, is essentially a quantum effect [14] and such effects are mostly manifested in atomic systems that are weakly-driven [12], we accordingly adopt the weak atom-field coupling scheme. In this scheme the Rabi frequencies of the two atomic transitions are to be taken much smaller than any other frequency involved in the problem.

Define the quantities;

$$w_{21} = w_2 - w_1\ ,\ w_{32} = w_3 - w_2$$

$$\Delta_1 = w_{21} - w_a\ ,\ \Delta_2 = w_{32} - w_b$$

$$\Delta_3 = \Delta_1 + \Delta_2\ ,\ \Delta w = w_a - w_b$$

$$\tau = t - t' \quad , \quad \delta\tau = \frac{r_2}{c} - \frac{r_1}{c} \,, \qquad ..(8)$$

let $\boldsymbol{r}$ and $\boldsymbol{t}$ be respectively the reflection and transmission coefficients of the beam-splitter and let $E_0(\vec{r})$ be the function that describes the spatial distribution of the field emitted by the atomic system, then the results for the mean number of photon coincidences within the same detector or between the two detectors are given as follows,

For an entry state through the same input arm;

$$\langle m_3(T) m_4(T) \rangle = \eta_3 \eta_4 |\boldsymbol{rt}|^2 C_{In}$$
$$\langle m_3^2(T) \rangle = \eta_3^2 |\boldsymbol{r}|^4 C_{In} \qquad .....(9)$$

and for an entry state through the two different input arms;

$$\langle m_3(T) m_4(T) \rangle = \eta_3 \eta_4 |\boldsymbol{rt}|^2 (C_{In} - 4 \operatorname{Re} E_{In})$$
$$\langle m_3^2(T) \rangle = \eta_3^2 (|\boldsymbol{r}|^4 C_{In} + 4 |\boldsymbol{rt}|^2 \operatorname{Re} E_{In}) \;. \quad ...(10)$$

In the above equations,

$$C_{In} = \frac{\mathrm{E}_{01}^4(\vec{r}_1)|g_1\alpha_1|^4}{(\gamma_{12}^2 + \Delta_1^2)^2} \left\{ T^2 + \left(\frac{\Delta_1^2 - 3\gamma_{12}^2}{\gamma_{12}(\gamma_{12}^2 + \Delta_1^2)} \right) T + \right.$$
$$+ \frac{4}{(\gamma_{12}^2 + \Delta_1^2)^2} \left([2\gamma_{12}\Delta_1 \sin \Delta_1 T + (\Delta_1^2 - \gamma_{12}^2) \cos \Delta_1 T] e^{-\gamma_{12} T} \right) +$$
$$\left. + \frac{1}{2\gamma_{12}^2} e^{-2\gamma_{12} T} + \frac{7\gamma_{12}^4 - 10\Delta_1^2\gamma_{12}^2 - \Delta_1^4}{2\gamma_{12}^2(\gamma_{12}^2 + \Delta_1^2)^2} \right\} \qquad ...(11)$$

$$E_{In} = -\mathrm{E}_{01}^2(\vec{r}_1)\mathrm{E}_{02}^2(\vec{r}_2) \left[2 \operatorname{Re} \left\{ \frac{(g_1^*\alpha_1^* g_2^*\alpha_2^*)}{(\gamma_{12} - i\Delta_1)(\gamma_{23} - i\Delta_3)} * \right. \right.$$
$$\left. \left. * \sum_{i=1,4} \frac{c_i}{a_i} \left[\left(\frac{1}{a_i} - T + |\delta\tau| \right) e^{-b_i|\delta\tau|} + \left(T - \frac{1}{a_i} \right) e^{-2\gamma_{12}|\delta\tau|} \right] \right\} \right] \qquad ....(12)$$

Where,

$$a_1 = 2\gamma_{23} - \gamma_{12} - i\Delta_1 - i\Delta w,$$
$$a_2 = -2i\Delta_1 - i\Delta w,$$
$$a_3 = \gamma_{23} - i\Delta_1 + i\Delta_2 - i\Delta w,$$
$$a_4 = \gamma_{12} - i\Delta_1 - i\Delta w,$$
$$b_i = a_i + 2\gamma_{12},$$

$$C_1 = \frac{(g_1\alpha_1 g_2\alpha_2)\gamma_{13}}{(\gamma_{12} - \gamma_{23})(\gamma_{12} - 2\gamma_{23} - i\Delta_1)(\gamma_{13} - 2\gamma_{23} + i\Delta_2)}$$
$$C_2 = \frac{(g_1\alpha_1 g_2\alpha_2)(3\gamma_{12} - i\Delta_1)}{(\gamma_{12} + i\Delta_1)(2\gamma_{23} - \gamma_{12} + i\Delta_1)(\gamma_{23} + i\Delta_3)}$$
$$C_3 = \frac{-(g_1\alpha_1 g_2\alpha_2)(2\gamma_{12} + \gamma_{13} + i\Delta_2)}{(2\gamma_{23} - \gamma_{13} - i\Delta_2)(2\gamma_{12} - \gamma_{13} - i\Delta_2)(\gamma_{23} + i\Delta_3)}$$
$$C_4 = \frac{-2(g_1\alpha_1 g_2\alpha_2)\gamma_{12}}{(\gamma_{23} - \gamma_{12})(\gamma_{12} + i\Delta_1)(\gamma_{13} - 2\gamma_{12} + i\Delta_2)} \qquad \ldots (13)$$

The results in the pervious section are too complicated. Hence, a better understanding may be obtained through some graph plottings.For an entry state through the same input arm, the results (equations (9)) do not show any signs of interference as long as they represent constant quantities. The behavior here is merely classical. The theoretical explanation [9] is based on the fact that the particles have only one choice to reach the beam-splitter, and hence the state function will have only one term and thus the probability amplitude will not include any interference term. We plot these quantities in dashed lines as a reference to the depth of any interference effect that can be obtained in the case of an entry state through the two different input arms.

For the sake of plotting, we adopt the very-well justified approximation [15]that in the range of the dimensions $\vec{r}_1$ and $\vec{r}_2$ in the experiment, one can put $\mathrm{E}_{01}^2(\vec{r}_1) \cong \mathrm{E}_{02}^2(\vec{r}_2)$. We will also plot all our results for the case of a (50 / 50 Beam-splitter) that is for $(|\boldsymbol{r}| = |\boldsymbol{t}|)$. In this way the quantities represented on the y-axis of our plots will be the mean number of coincidences multiplied by the quantity $(\gamma_{12}^2 / \eta_3\eta_4 |\boldsymbol{rt}|^2 \mathrm{E}_{01}^4(\vec{r}_1))$ for the case of two detectors and by $(\gamma_{12}^2 / \eta_3^2 |\boldsymbol{rt}|^2 \mathrm{E}_{01}^4(\vec{r}_1))$ for the case of one detector. The x-axis will always contains the quantity $(\gamma_{12}|\delta\tau|)$.

Figures ((2)-(5)) show a selected sample of the results in the previous section for $\langle m_3(T) m_4(T) \rangle$ when the coupling between the atom and the exciting fields is weak ($\frac{|g_1\alpha_1|^2}{\gamma_{12}^2} = \frac{|g_2\alpha_2|^2}{\gamma_{12}^2} = 0.1$). Signs of interference effect are always present. However, the visibility of such effects depends on the parameters involved in the interaction. Of course such visibility would depend on the extent of correlation between the pair of emitted photons. This correlation is very sensitive to the different experimental parameters [3]. It is noted that the visibility of interference effects in these graphs improves rapidly once the detunings ($\Delta w, \Delta_1, \Delta_2$) and the ratio (γ_{23} / γ_{12}) get smaller. For some appropriate values of these parameters, the visibility can exceed 50% and thus the effect can be easily detected in any experiment of this kind. The quantity plotted here is the complete expression in (10). Such expression contains terms that proportional to (T^2). Such terms come from photons produced in different cascade emissions and thus play a negative role in the visibility of any interference effects.Fearn and Loudon (1989) have dropped such terms in their plots for the interference effects in the same atomic system under thermal excitation. Figure (6) shows few samples of the dependence of the mean number of photon coincidences within the same detector on the path difference ($\delta\tau$). The interference effects here are of course complementary to those showed in the previous figures.

In summery, we have found that three-level cascade atomic system that is weakly- driven by two coherent light fields is a very promising source for photons in the experiments of two- photon interference.

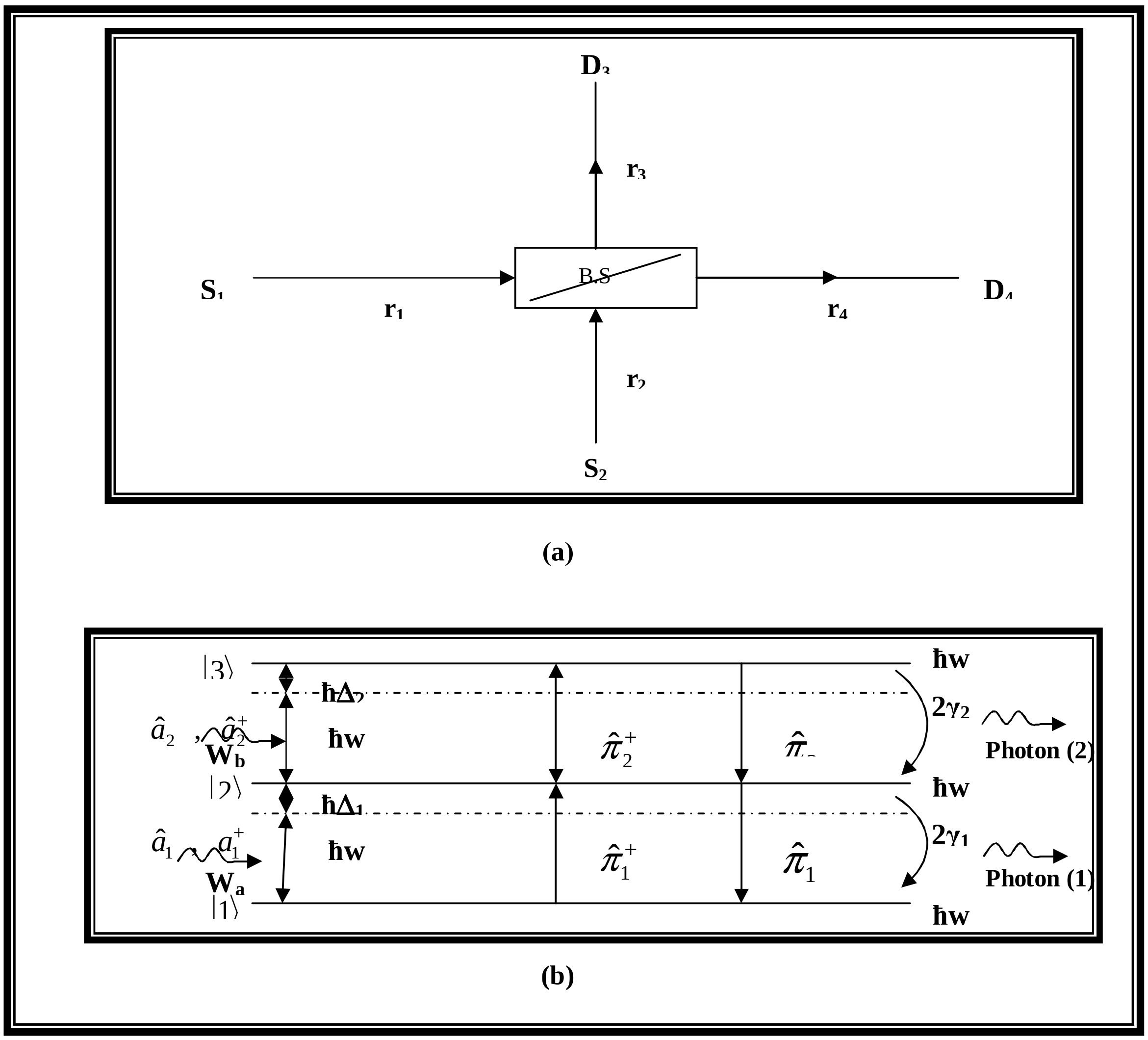

Fig. (1)

(a) Schematic diagram of a two-photon interference experiment with a beam-splitter.

(b) Schematic diagram of the three-level atomic system showing the different atom-field parameters involved.

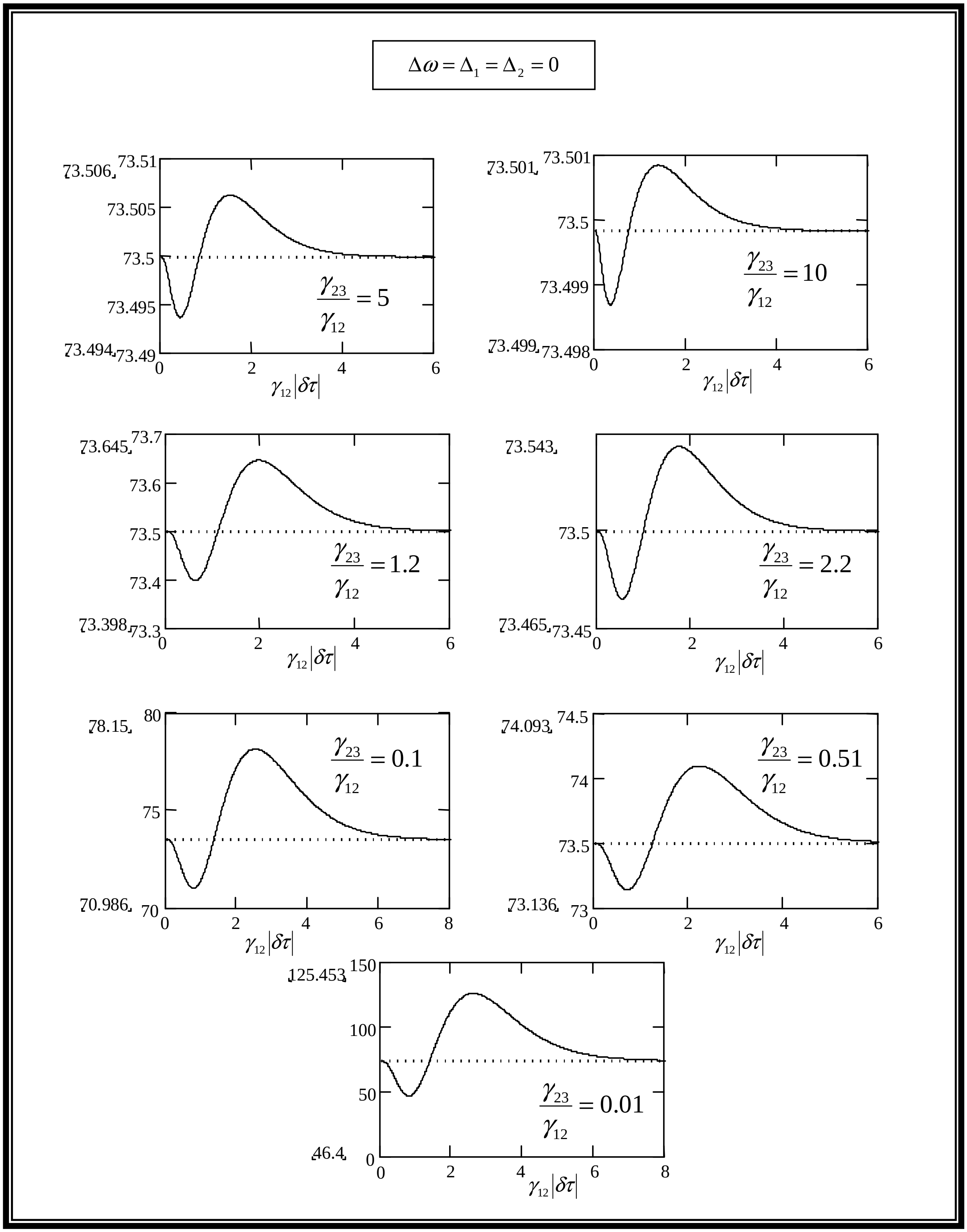

Fig. (2); Graphs for the quantity $(\gamma_{12}^2/\eta_3\eta_4|\mathcal{E}|^2 E_{01}^4(\vec{r}))\ \langle m_3(T)m_4(T)\rangle$ **against** $(\gamma_{12}|\delta\tau|)$ **for** $\gamma_{12}T = 10$, $\Delta w = \Delta_1 = \Delta_2 = 0$.

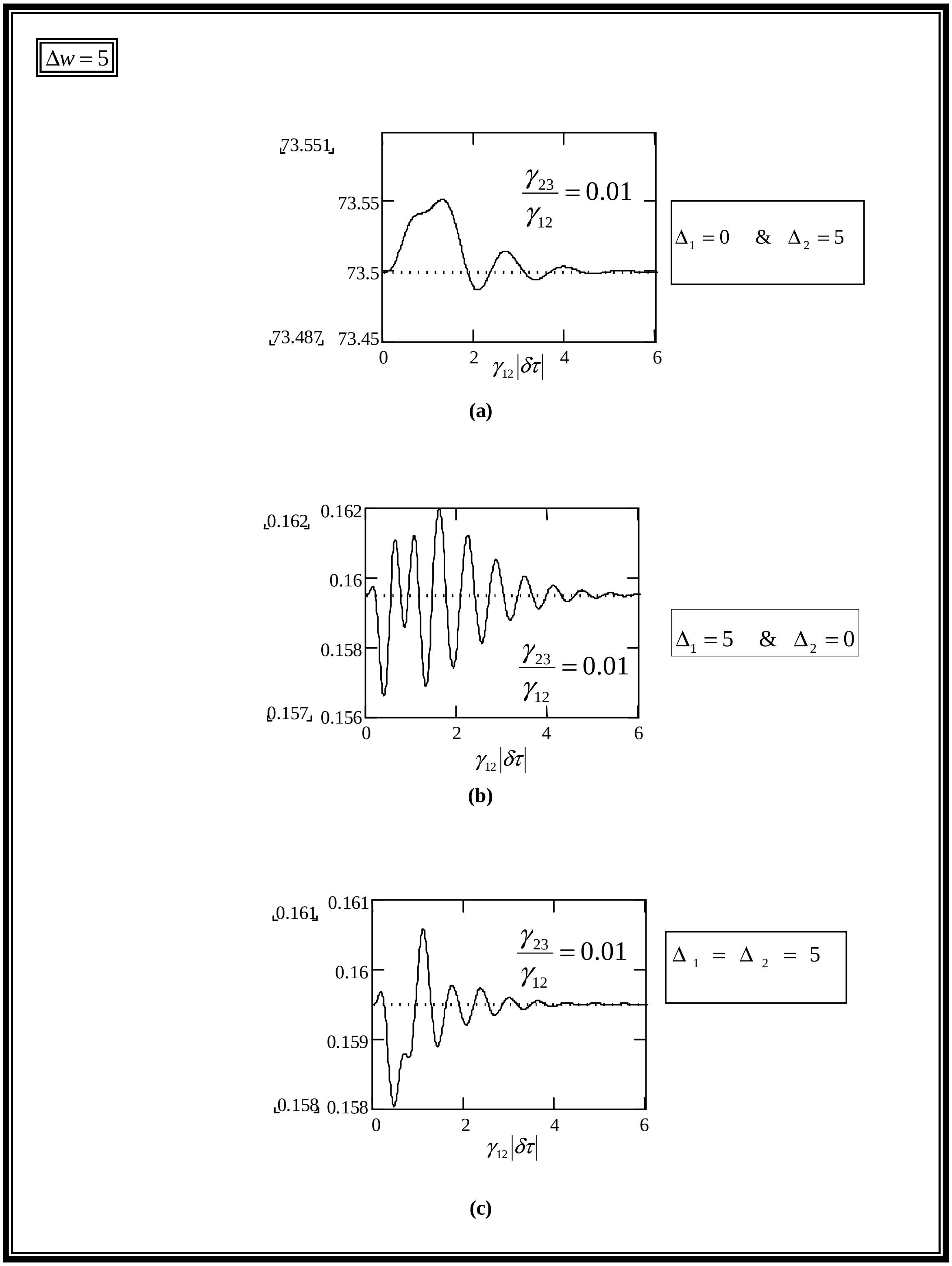

Fig. (3); As in fig. (2) with $\Delta\omega = 5\gamma_{12}$ **.**

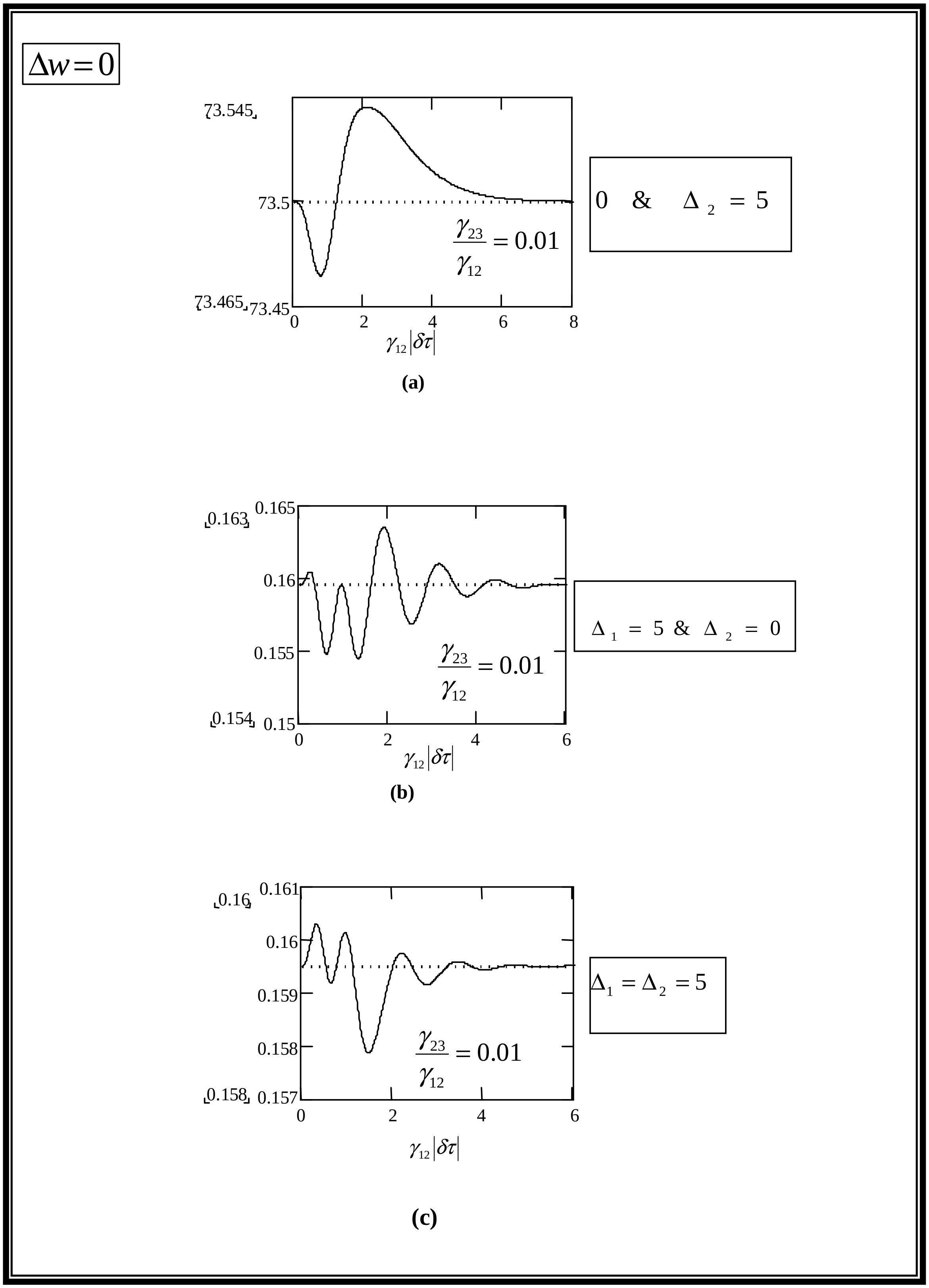

Fig. (4); As in fig. (2) with $\Delta\alpha = 0$.

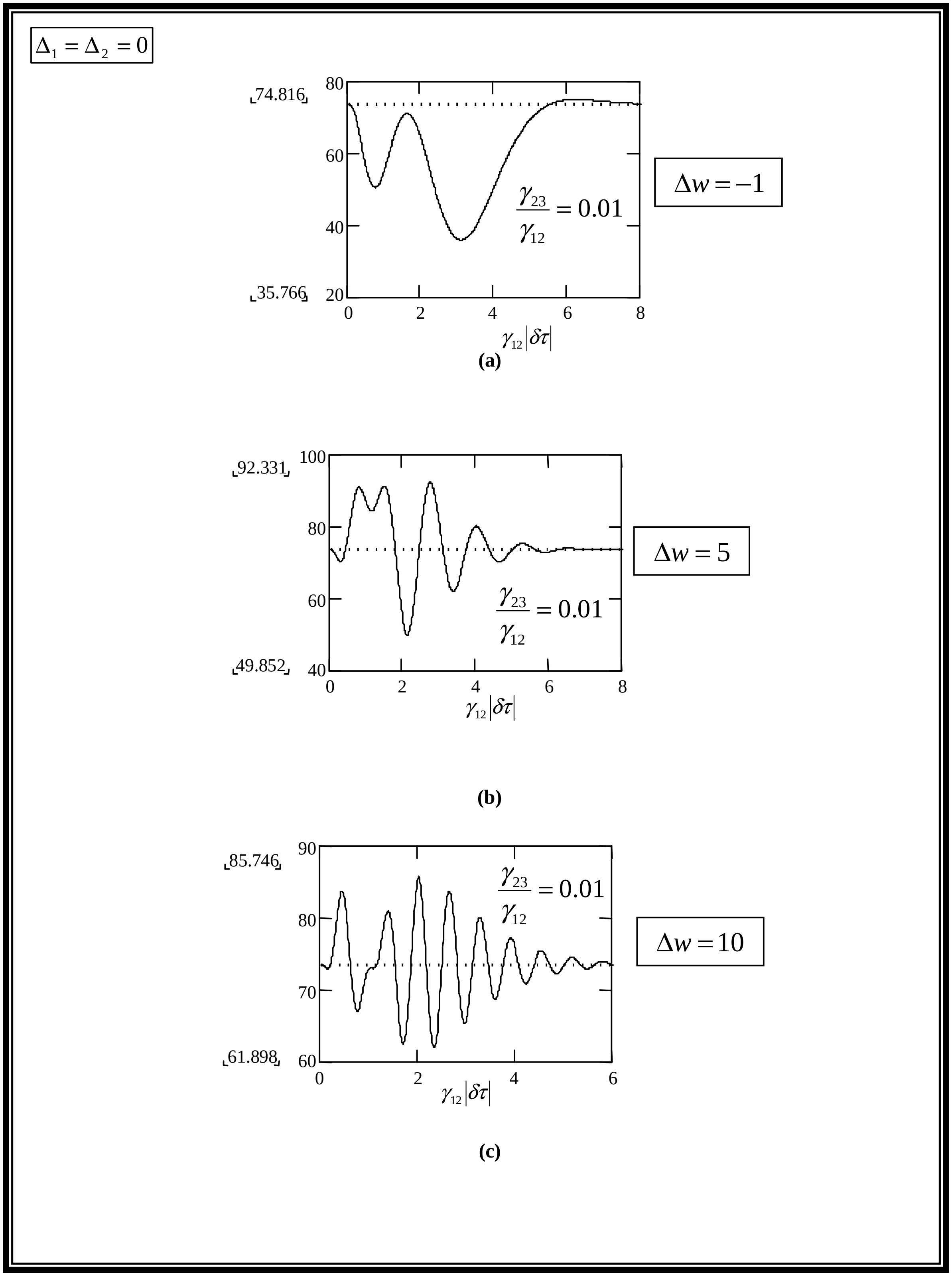

Fig. (5); As in fig. (2) with $\Delta_1 = \Delta_2 = 0$**.**

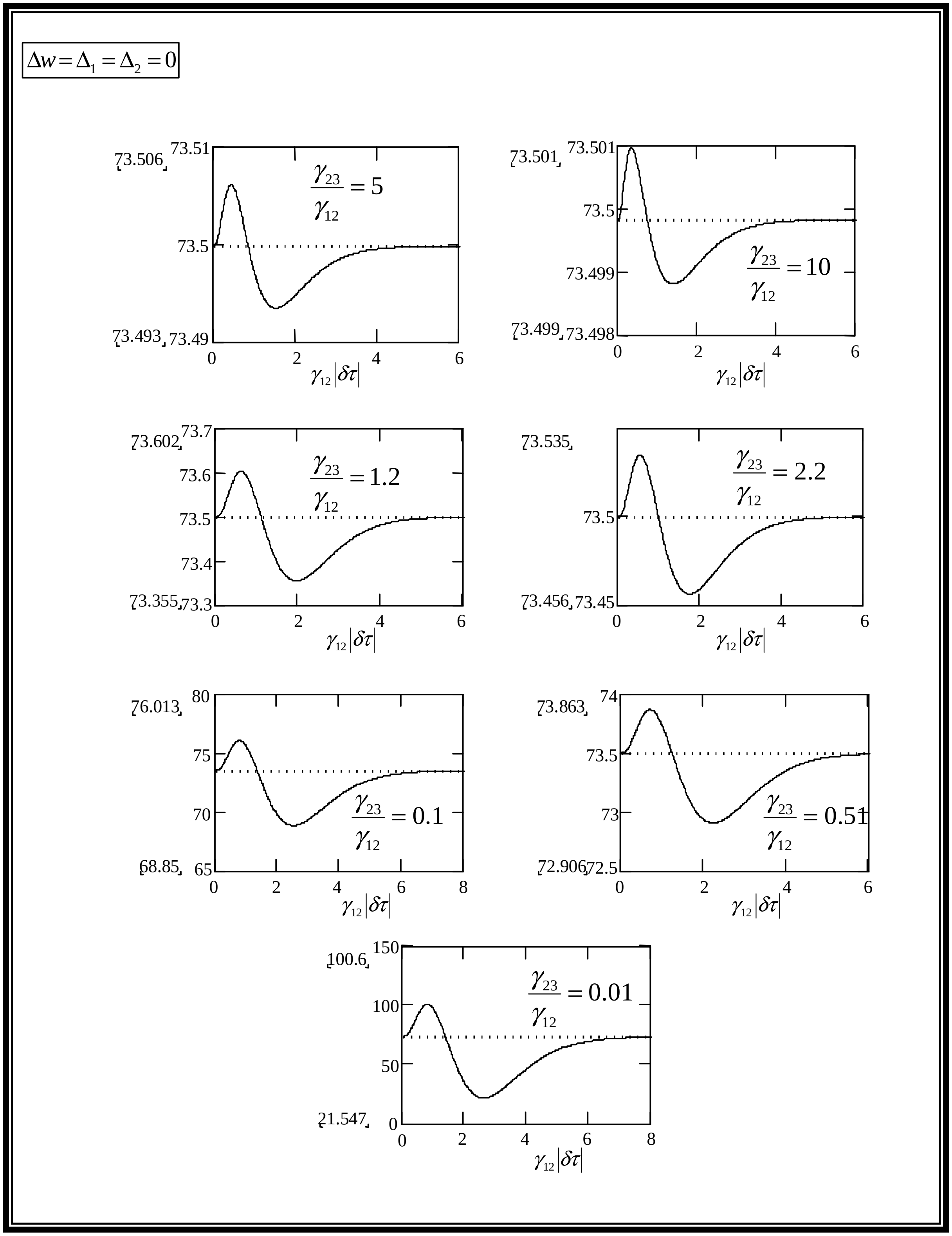

Fig.(6); Graphs for the quantity $(\gamma_{12}^2/\eta_3^2|\varkappa|^2 \mathrm{E}_{01}^4(\vec{r}_1))\ \langle m_3^2(T)\rangle$ **against** $\gamma_{12}|\delta\tau|$ **for** $\gamma_{12}T = 10\ ,\Delta\omega = \Delta_1 = \Delta_2 = 0$.

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تداخل– الفوتونين باستعمال نظام ذري ثلاثي المستوي يتفاعل مع مجالين بصريين متشاكهين

تحسين الخزاعي و عبدالحسن الحلفي

قسم الفيزياء/ كلية التربية/ جامعة البصرة

البصرة – العراق

المستخلص

لقد ارتبطت تجارب تداخل – الفوتونين تاريخيا بحالة الفوتونين الكمية المستحصلة في عملية التحويل البارامتري الخافض التلقائي. في هذه الدراسة يتم اختبار حالة الفوتونين المتشابكين المستحصلة من نظام ذري ثلاثي المستويات متعاقب الانبعاث مهيج بواسطة مجالين بصريين متشاكهين كحالة إدخال في تجربة تداخل – الفوتونين باستعمال مجزيء الشعاع الضوئي. لحالة الارتباط الضعيف وقيم مختارة للعوامل الذرية تم استحصال تأثيرات تداخل بأعماق تتعدى بكثير تلك المستحصلة من نفس النظام الذري المهيج حراريا والمدروسة سابقا.