Study of Vapour Bubble Dynamics Using a Modified Keller-Kolodner Model

Abbas Z. AL-Asady
Mechanical Engineering Department, Engineering College, Basrah University, Iraq

ISSN–1817–2695
(Received 18/4/2007, Accepted 6/6/2007)

Abstract

Using linear wave equation and Keller-Kolodner method, a new equation is derived for an oscillation of a bubble in an incident sound field. It includes the effects of acoustic radiation, the effects of viscosity, surface tension, compressibility of the liquid, and evaporation and condensation at bubble wall. Where, the liquid temperature at bubble wall is assumed to be constant and that the gas inside the bubble is only vapour.

The classical approach to the analysis of the pulsations of a gas bubble is to assume that the pressure within the bubble follows a polytropic relation. In this study, a new formulation of the dynamics of a bubble is presented in which the internal pressure is obtained numerically.

Numerical results for vapour bubble in water are presented. A good agreement is found between the theoretical result and the experimental data of the bubble radius. The new equation of the bubble radius can be used to study the bubble oscillation in acoustic fields.

Keywords: Cavitation, Ultrasound, Water Vapour

1. Introduction

During the last years acoustic cavitation has become a field of tremendous interest. Considering various applications of acoustic cavitation in industrial practice, the formation of bubble fields and the strength of the cavitation process are important for the development of suitable ultrasonic reactors with regard to effectiveness and economy. The bubble behavior in a fluid considerably varies with the fluid properties, so the cavitation properties as a function of the fluid properties are very important. The thermal behavior is primarily attributed to temperature dependent fluid parameters like density, vapour pressure, surface tension, viscosity and others [1].

The effects of ultrasound (physical or chemical) are linked to acoustic cavitation: locally, the rarefaction phases of the acoustic pressure wave give rise to gas and vapour microbubbles. Under the influence of the sound wave, the bubbles oscillate with a variety of behavior patterns according to their sizes and to the acoustic pressure amplitude (Lauterborn, [2]; Neppiras, [3]; Young, [4]). Some bubbles can expand to many times their initial size and collapse violently. Near a solid surface, these collapses produce violent microjets (and even shock waves when the bubbles rebound) which impinge on the surface causing either erosion or fracture.

A central problem in the study of acoustic cavitation is that of understanding the dynamics of small bubbles set in motion in a liquid by a sound field. The complicated, nonlinear nature of such motions has served in the past to limit investigations to the study of very simple models of such bubbles. In this paper, a mathematical formulation is developed to study the motion of vapour bubble in acoustic field and the effects of mass transfer, heat conduction, shear viscosity, compressibility, and surface tension on the dynamical behavior. The formulation is a “large amplitude” one in that it is specifically designed to describe the motion of a bubble that expands to some maximum radius and then contracts violently. For such a motion, we accept approximations that considerably simplify the mathematical formulation to place upper or lower bounds on physical quantities, such as the pressure or temperature within the bubble. This formulation consists of a set of nonlinear equations that are solved numerically.

It should be noted that the heat conduction equation in the bubble should be solved numerically without assuming a profile of bubble temperature. The comparison study between the calculated results and the experimental data of radius-time cure have done and discussed.

2. The Model Analysis

2.1 Equations of Motion for the Cavity Interface

The aim of this part is to derive a pair of DE's that constitutes the equations of motion for the cavity interface. There is a bubble (cavity) of initial radius R_o containing only vapour in a viscous compressible liquid. At time zero, the ambient pressure is increased instantaneously to $P_\infty(t)$ and then the bubble begins to oscillate accompanied with phase change and heat conduction through the bubble wall. The problem is to investigate these physical effects on the bubble oscillation.

In writing the basic equations, the following assumptions are made. (a) The center of the cavity is at rest at the origin of coordinates and the motions of the interface and the liquid are always spherically symmetrical. (b) The liquid is infinite in extent. (c) The viscosity of the liquid enters the problem only as a boundary condition. (d) The effects of gravity and diffusion are negligible. (e) The pressure inside the bubble is uniform throughout. (f) The vapour is inviscid and obeys the perfect-gas law. (g) The physical properties of liquid are constant. (h) The bubble contains only vapour.

The followings is the description of derivations of equation of bubble radius including effect of mass transfer (evaporation and condensation of vapour) at bubble wall. The derivations start from the linear wave equation [5].

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial r^2} = 0 \qquad \ldots(1)$$

with the boundary conditions [6]

$$\rho_L\left[\left(\frac{\partial\phi}{\partial r}\right)_{r=R} - \dot{R}\right] = -\dot{m} \qquad \ldots(2)$$

and

$$\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2_{r=R}\right] = -H \qquad \ldots(3)$$

where ϕ is the velocity potential of the liquid, and the other symbols have appeared in nomenclature. The general solution of eq. (1) is given by eq. (4) under the spherical symmetry. Following the method of Keller-Kolodner [7] the mass transfer at bubble wall is taken into account,

$$\phi(r,t) = \frac{1}{r}F(t - \frac{r}{c}) \qquad \ldots(4)$$

where F is arbitrary function.

$$y = t - \frac{r}{c} \qquad \ldots(5)$$

$$\bar{y} = t - \frac{R}{c} \qquad \ldots(6)$$

$$F' = \frac{dF}{dy} \qquad \dots (7)$$

So, from eqs. (2) and (4),

$$-\frac{1}{Rc} F'(\bar{y}) - \frac{1}{R^2} F(\bar{y}) = \dot{R} - \frac{\dot{m}}{\rho_L} \qquad \dots (8)$$

and from eqs. (3) and (4)

$$\frac{1}{R} F'(\bar{y}) + \frac{1}{2}\left(\dot{R} - \frac{\dot{m}}{\rho_L}\right)^2 = -H \qquad \dots (9)$$

Consequently, equations (8) and (9) can be described as follows:

$$\frac{F'(\bar{y})}{R} = -H - \frac{1}{2}\left(\dot{R} - \frac{\dot{m}}{\rho_L}\right)^2 \qquad \dots (10)$$

$$\frac{F(\bar{y})}{R^2} = -\dot{R} + \frac{\dot{m}}{\rho_L} + \frac{H}{c} + \frac{1}{2c}\left(\dot{R} - \frac{\dot{m}}{\rho_L}\right)^2 \qquad \dots (11)$$

Differentiating eq. (11) w.r.t. t and multiplying by R yields eq. (12)

$$\left(1 - \frac{\dot{R}}{c}\right) \frac{F'(y)}{R} - 2\dot{R} \frac{F(\bar{y})}{R^2} = -R\ddot{R} + \frac{R\ddot{m}}{\rho_L} + \frac{R}{c}\left(\dot{R} - \frac{\dot{m}}{\rho_L}\right)\left(\ddot{R} - \frac{\ddot{m}}{\rho_L}\right) \qquad \dots (12)$$

The equation of the bubble radius can be concluded by inserting equations (10) and (11) into equation (12) as follows:

$$R\ddot{R}\left(1 - \frac{\dot{R}}{c} + \frac{\dot{m}}{c\rho_L}\right) + \frac{3}{2}\dot{R}^2\left(1 - \frac{\dot{R}}{3c} + \frac{2\dot{m}}{3c\rho_L}\right) - \frac{R\ddot{m}}{\rho_L}\left(1 - \frac{\dot{R}}{c} + \frac{\dot{m}}{c\rho_L}\right) - \frac{\dot{m}}{\rho_L}\left(\dot{R} + \frac{\dot{m}}{2\rho_L} + \frac{\dot{R}\dot{m}}{2c\rho_L}\right) = \left(1 + \frac{\dot{R}}{c}\right)H + \frac{R}{c}\frac{dH}{dt} \qquad \dots (13)$$

where the liquid enthalpy difference between the bubble wall and the infinity (H) is given by the following equation [8];

$$H = \frac{1}{\rho_L}\left(P_{L,R} - P_{L,\infty}\right) \qquad \dots (14)$$

Eq. (13) is reduced to Keller-Kolodner equation when $\dot{m} = \ddot{m} = 0$. $P_{L,R}$ is retreated to $P_v(t)$ by the author [9].

$$P_{L,R} = P_V(t) - \frac{2\sigma}{R} - \frac{4\mu}{R}\left(\dot{R} - \frac{\dot{m}}{\rho_L}\right) - \dot{m}^2\left(\frac{1}{\rho_L} - \frac{1}{\rho_{V,R}}\right) \qquad \dots (15)$$

where $\dot{m}$ is the net rate of evaporation per unit area and unit time (when $\dot{m}=0$ condensation takes place). It is changed with time by evaporation or condensation at bubble wall as it's described by ref. [10] as follows

$$\dot{m}=\beta\,\frac{P_v^*(To)-P_v(t)}{\sqrt{2\pi R_v\,T_o}} \qquad \ldots(16)$$

and

$$P_{L,\infty}=P_o-P_m\,\sin\omega t \qquad \ldots(17)$$

2.2 Pressure Formulation for a Vapour Bubble

Although the polytropic approximation has been generally though to be a fairly accurate model, an experimental study by Crum and Prosperetti; [11,12] has shown significant discrepancies between theory and experiment for oscillations of the bubble. Consequently, Prosperetti *et al.* [13] proposed a more accurate calculation of the internal pressure no longer requiring the polytropic approximation but still neglecting the effects of the vapour-phase change as well as the gas diffusion in the liquid. The model is based on two main assumptions: the internal pressure is uniform and the vapour behaves like a perfect gas. The first one implies that the bubble radius is much smaller than the sound wavelength in the gas and that the gas Mach number of the bubble wall motion is very small. These assumptions were very useful since for a perfect gas with spatial uniform pressure, the energy equation can be combined to obtain an exact expression for the velocity field in the bubble in terms of the temperature gradient, for a perfect gas, the equation of state can be represented as follow [8]:-

$$P_v=\rho_v\,R_v\,T \qquad \ldots(18)$$

The analytic expression for the vapour velocity in the bubble (u(r,t)) have described by [14] as follows:

$$u(r,t)=\frac{1}{\gamma P_v}\left[(\gamma-1)\,k_v\,\frac{\partial T}{\partial r}-\frac{1}{3}r\,\frac{dP_v}{dt}\right] \qquad \ldots(19)$$

So, by using the velocity boundary condition (at r=R) lead to

$$\frac{dP_v}{dt}=\frac{3}{R}\left[(\gamma-1)\left(k_v\,\frac{\partial T}{\partial r}\right)_{r=R}-\gamma\,P_v\,\dot{R}\right] \qquad \ldots(20)$$

Equation (19) and (20) replace the continuity and the momentum equations, respectively. The bubble interior is then described by an ordinary differential equation for the pressure and a partial differential equation (the energy balance) which is written in the following form [8].

$$\frac{\gamma}{\gamma-1}\,\frac{P_v}{T}\left[\frac{\partial T}{\partial t}+u(r,t)\frac{\partial T}{\partial r}\right]-\frac{d\,P_v}{d\,t}=\frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\,k_v\,\frac{\partial T}{\partial t}\right) \qquad \ldots(21)$$

where u(r,t) is expressed by eq. (19). The author has completed the set of equations with the modified Keller and Kolodner equation (eq. (13)) and assumed that the temperature gradient in the liquid is negligible. Then, the energy balance in the liquid is not considered, and the interface temperature is maintained constant equal to its initial value. Eqs. (13), (20) and (21) are solved using a finite-difference, second-order method [15].

3. Results and Discussion

We proceed now to discuss some numerical results for large-amplitude oscillations with our new model (modified Keller-Kolodner model). The following quantities or conditions are used through the calculations as follows: T_o= 20 ^oC, P_o=1 bar, R_o=8 μm, f=16.8 kHz and P_m=1.25 bar. The accommodation coefficient for evaporation or condensation is assumed to be $\beta = 0.4$ [6]. Therefore, the physical properties $(\rho, \sigma, \mu, c \text{ and } P^*_v)$ are chosen at T_o= 20 ^oC, and P_o=1 bar. The results under assume conditions are shown in Figs. 1-6.

The figures revealed that, the physical quantities of the bubble change with time periodically due to change the frequency of the acoustic field applied on the bubble. Figs. 1-4 show the relation of R, P_v, T and $\dot{R}$ as a function of time. In Fig. 1, the radius changes with time due to change the ambient pressure component (sound field). In Figs. 2-3, both the pressure and the temperature of the bubble become higher when the bubble is collapsing (the radius becomes minimum). In the other words, at the collapse of the bubble the pressure and temperature increase suddenly, followed by small oscillations due to the small bounces of the radius. In Fig. 4, the bubble wall velocity ($\dot{R}$) increases suddenly due to the collapsing the bubble. In Fig. 5, the temperature distribution inside the bubble is shown as a function of the bubble radius (minimum radius). High temperature is occurred at the center of the bubble. While at the bubble wall, the temperature is almost equal to the liquid temperature (T_o).

In Fig. 6, direct comparison is given between the radius-time curve calculated by the present model and the experimental data by ref. [16]. The calculated result by the present model gives good agreement with the experimental data.

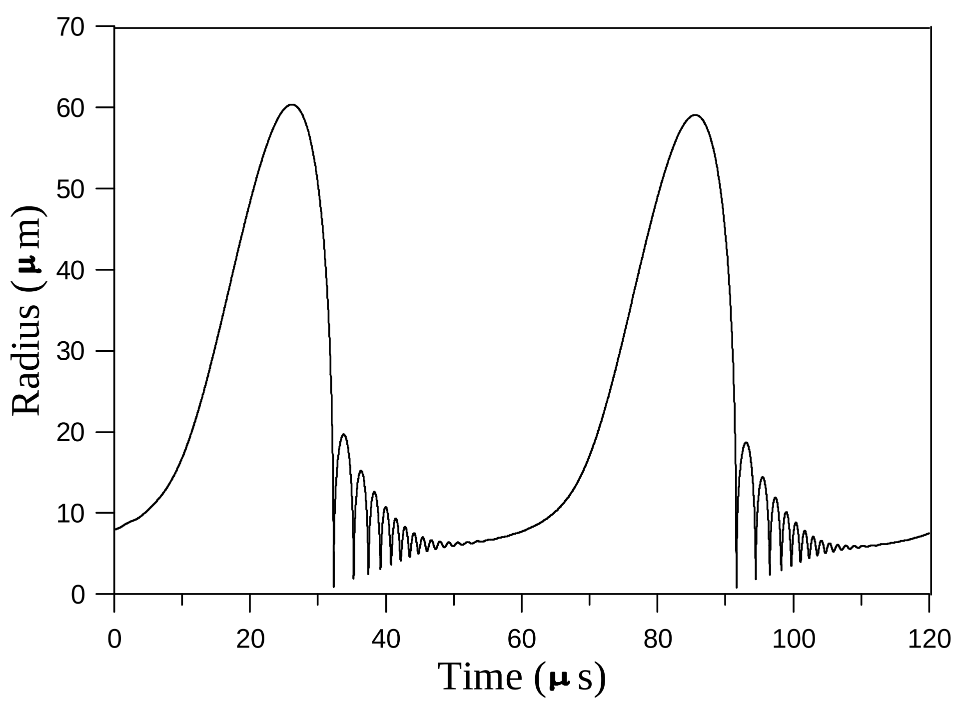

Fig. 1. The bubble radius (R) as a function of time.

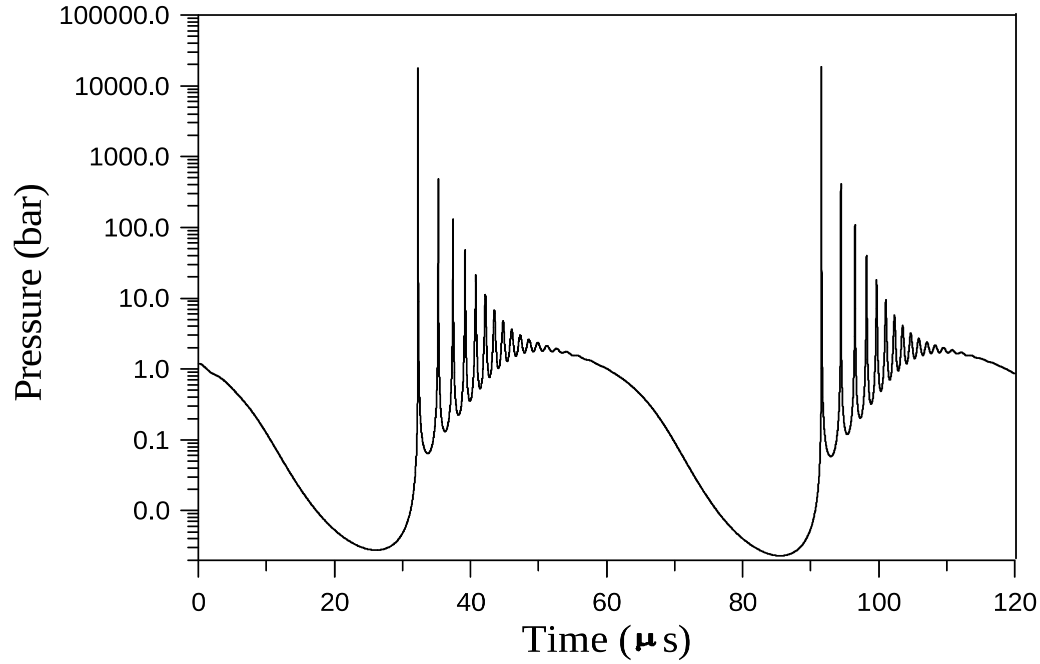

Fig. 2. The pressure inside the bubble (P_V) as a function of time with logarithmic vertical axis.

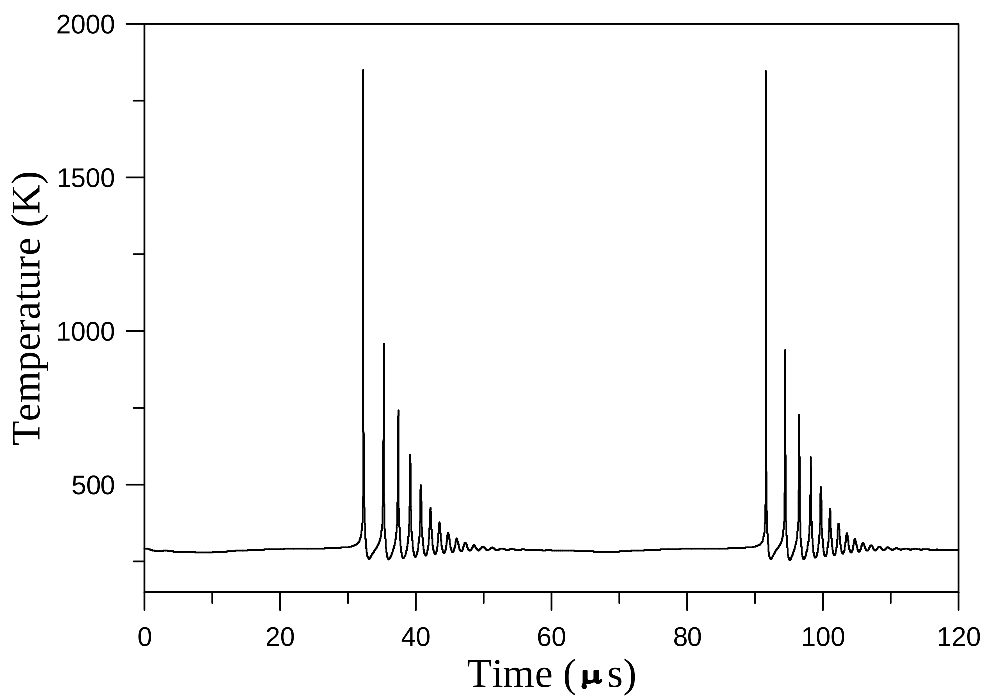

Fig. 3. The temperature at the bubble center as a function of time.

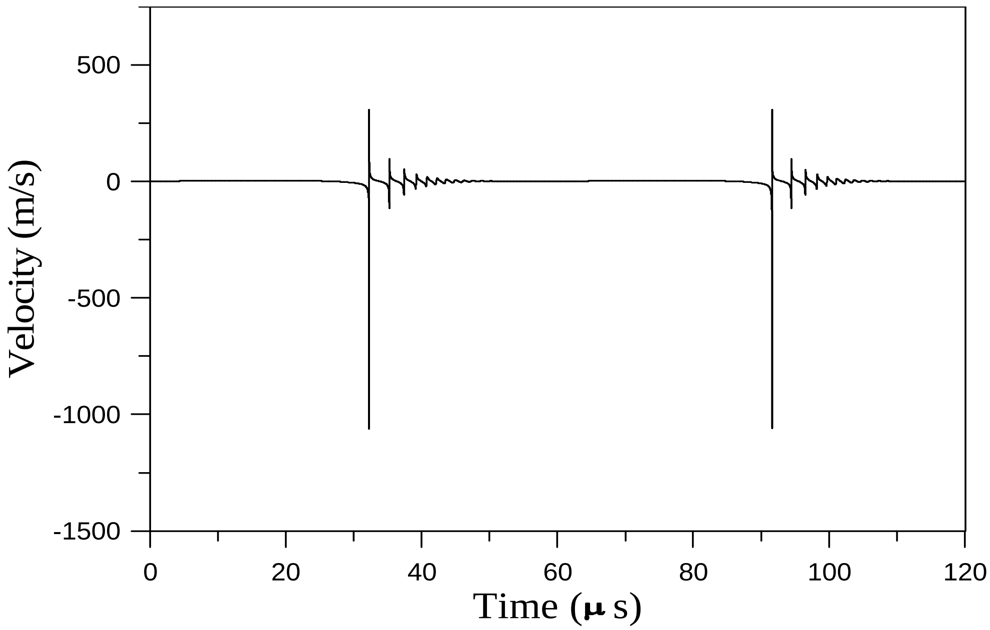

Fig. 4. The bubble wall velocity ($\dot{R}$) as a function of time.

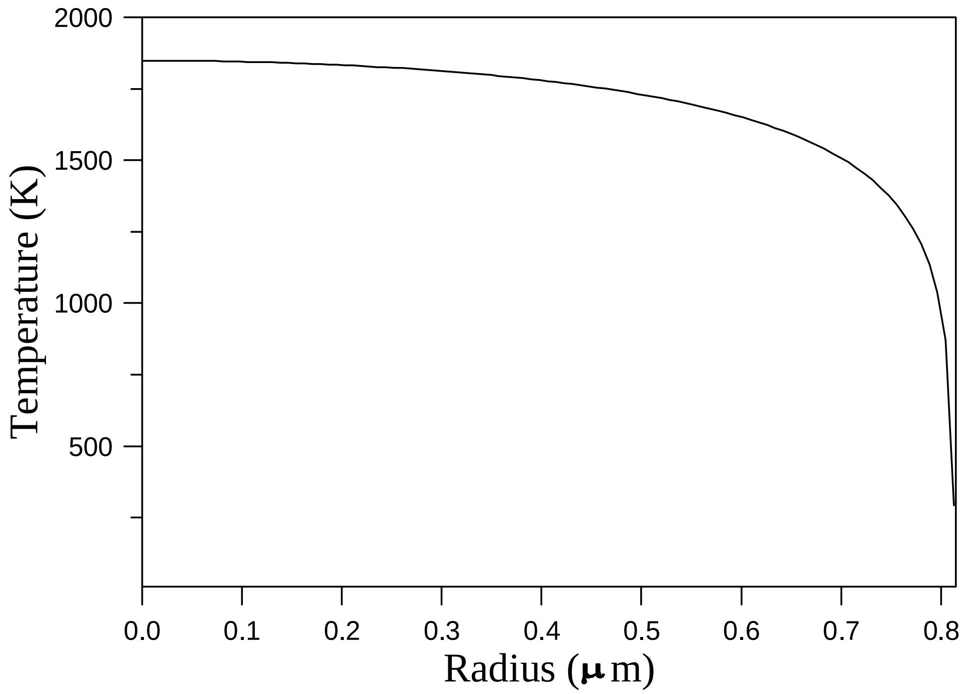

Fig. 5. The temperature distribution inside the bubble at the minimum radius.

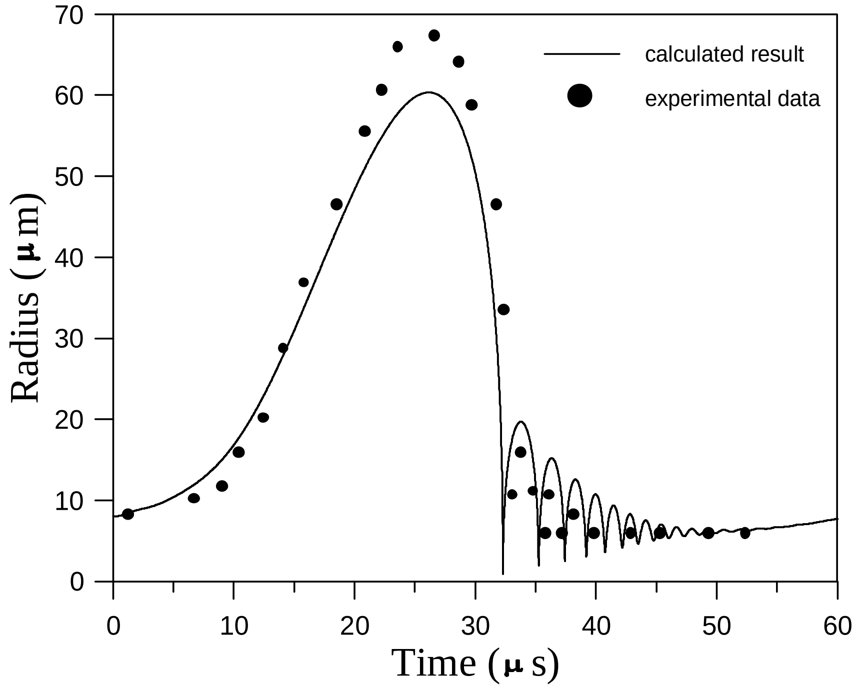

Fig. 6. Comparison between the calculated result and the experimental data[16] of radius-time curve for acoustic cycle.

4. Conclusion

The results obtained in the present study may be summarized as follows. The new theoretical equation of the radius of a bubble oscillating in a viscous compressible liquid is given. It has been shown that a new formulation of bubble dynamics based upon an evaluation (numerically) of the internal pressure within the bubble can give a considerable different form from the standard (classical) form in which the internal pressure is approximated by a polytropic relation.

We have used a bubble, consisting of only vapor surrounded by a viscous compressible liquid (water) with surface tension at water-vapour interface. Our estimated values of R, obtained by the numerical solution of the motion equation and the internal pressure equation is in good agreement with the experimental values obtained by others. The new model can be used to study a bubble oscillating in liquids.

5. Nomenclature

Subscripts

v: Refers to the bubble content (water vapor)
L: Refers to liquid
L,R: Refers to liquid at bubble wall
o: Refers to the equilibrium state
∞: Refers to conditions at a great distance from the bubble

Symbol	Definition	Unit
c	Sound speed in liquid at infinity	m/s
cp	Heat capacity at constant pressure	J/kg. K
f	Acoustic field frequency	Hz
k	Thermal conductivity	W/m.K
$\dot{m}$	Net rate of evaporation and condensation	$kg/m^2.s$
$\dot{m}_{eva}$	Actual rate of evaporation	$kg/m^2.s$
$\dot{m}_{con}$	Actual rate of condensation	$kg/m^2.s$
P	Pressure	Pa
P_m	Acoustic pressure amplitude	Pa
P^*_v	Saturated liquid pressure	Pa
r	Radial distance from bubble center	m
R	Bubble radius	m
$\dot{R}$	Bubble wall velocity	m/s
R_v	Gas constant of water vapor	J/kg. K
t	Time	s
T	Temperature	K
β	Evaporation or condensation accommodation coefficient	
μ	Liquid viscosity	$N.s/m^2$
γ	Ratio of specific heats	
ρ	Density	kg/m^3
σ	Surface tension	N/m
ω	Angular frequency ($\omega=2\pi f$)	rad/s

References

[1] T. Niederdrank and B. Wiesand, ACUSTICA, Vol. 84, pp 425-431, 1998.
[2] W. Lauterborn, J. Acoust. Soc. Am. 59, pp 283-293, 1976.
[3] E. A. Neppiras, Phys. Rep. 61, pp 159-251, 1980.
[4] F. R. Young in Cavitation, McGraw Hill, London, U.K., (1989).
[5] A. Prosperetti and A. Lezzi, J. Fluid Mech. 168, pp 457-478, 1986.
[6] S. Fujikawa and T. Akamatsu, J. Fluid Mech. 97, pp 481-512, 1980.
[7] J. B. Keller and I. I. Kolodner, J. Appl. Phys. 27, pp 1152-1161, 1956.
[8] S. Sochard, A. M. Wilhelm and H. Delmas, Chemical Engineering, Vol. 53, 1997.
[9] A. Z. AL-Asady, Basrah Journal of Eng. Sciences, College of Eng. , Basrah University, Iraq, 2006.
[10] R. I. Nigmatulin, N. S. Khabeev and F. B. Nagiev, Int. J. Heat Mass Transfer, Vol. 24, No. 6, pp 1033-1044, 1981.
[11] L. A. Crum and A. Prosperetti, J. Acoust. Soc. Am. 73, pp 121-127, 1983.
[12] L. A. Crum and A. Prosperetti, J. Acoust. Soc. Am. 75, pp 1910-1912, 1984.
[13] A. Prosperetti, L. A. Crum, and K. W. Commander, J. Acoust. Soc. Am. 83, pp 502-514, 1988.
[14] A. Prosperetti and Y. Hao, Phil. Trans. R. Soc. Lond. A 357, pp 203-223, 1999.
[15] B. Carnahan, H. A. Luther and J. O. Wilkes in Applied Numerical Methods, John Wiley and Sons, N.Y. (1969).
[16] K. Yasui, Physical Review E, Vol. 58, No. 4, pp4560-4567, 1998.

دراسة حركة فقاعة بخارية باستخدام نموذج Keller – Kolodner المطور

عباس زكي الأسدي

قسم الهندسة الميكانيكية – كلية الهندسة – جامعة البصرة – العراق

الخلاصة

تم اشتقاق معادلة جديدة للتذبذب القطري لفقاعة في وسط صوتي باستخدام المعادلة الموجية الخطية وطريقة Keller-Kolodner. تتضمن هذه المعادلة تأثيرات الإشعاع الصوتي وتأثيرات اللزوجة والشد السطحي وانضغاطية السائل والتبخير والتكثيف عند جدار الفقاعة. افترض ان درجة حرارة السائل عند جدار الفقاعة ثابتة والغاز داخل الفقاعة بخار فقط.

يفترض في الطرق الكلاسيكية لتحليل نبضات الفقاعة الغازية أن يكون الضغط في الفقاعة يتبع علاقة بولتروبية. تم تقديم صيغة جديدة لحركة الفقاعة في هذه الدراسة حيث يحصل على الضغط الداخلي عددياً.

تم تقديم نتائج عددية لفقاعة بخارية في الماء. هناك تطابق جيد بين النتيجة النظرية والبيانات التجريبية لنصف قطر الفقاعة. يمكن استخدام المعادلة الجديدة لنصف قطر الفقاعة لدراسة تذبذب الفقاعة في الأوساط الصوتية.