ANALYTICAL SYNTHESIS OF THE SYMMETRIC MAGNETIC LENS

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Abstract

In the present work, a new analytical function has been proposed to represent the axial magnetic field strength for the symmetrical magnetic lenses. Thereby, the correspondence magnetic flux distribution, that is necessary, for determing the aberration integrals, can be deduced immediately. The effects of the geometrical and physical parameters as an optimization parameters on the objective focal properties and the pole piece configuration have been investigated. The results have shown, that new conclusions concerning comparison between the results of synthesis procedure with that of analysis procedure for the same initial conditions can be set up.

Key words: Electron Optics, Magnetic Lenses, Optimization , Synthesis , Aberrations.

1.Introduction

It is well known that complete information about the optical properties of a certain imaging system can be obtained when its axial potential or field distribution is well defined. Thus, the determination of magnetic field distribution may be considered being an initial step to start the optical investigation of any magnetic lens especially for the analysis procedure. In fact, the problem of determining the field distribution is not an easy and straightforward task especially for the practical studies, where several processes have to be done in order to makethe produced field useable to solve the paraxial ray equation such as the fitting or something. Now, the question here is that what should it do to perform a rapid approximate evaluation of lens properties without actually carrying out a detailed analysis. The conventional answer for this question is that this mission can be accomplished if one has a simple mathematical model for the lens [Szilagyi, 1988]. Accordingly, lens model may be defined as a mathematical expression used to approximating lens field that is reasonably close to the real one and allows a solution in closed form. Since the invations of magnetic lenses, several mathematical models had been introduced to approximate the axial magnetic field distribution. For more details about these points one may consult [Szilagyi, 1988] and [AL-Obaidi and Drigan 2001]. Unfortunately, most of these models describe the lens field in terms of physical parameters, such as field peak and half width. With such models, it is not any easy matter to define the lens geometrical parameters when they are considered as a target function by means of synthesis procedure. Thus, it is important to propose mathematical models for the lenses that mainly depend on the conventional lens geometrical parameters. Such models, however, may give an objective design about the points when the air gap width and the bore diameter can measure at the reconstructed pole piece. The present synthesis is concerning to deal with a model of such characteristics. Where, this model is proposed to be a target function, hence its objective optical properties are investigated for the zero and high magnification conditions.

The Proposed Target Function

According to [Hawkes, 1982], the axial magnetic fields strength H_z along the optical axis z may be represented by the following analytical formula:

$$H_z(z) = \frac{1.318NI/D}{Cosh^2(2.636z/D)}\text{..................}(1)$$

where NI is the imaging field excitation and D is the bore diameter of the lens proposed to produce such field. In order to deduce the correspondence magnetic flux distribution, equation (1) should be multiplied by the free space permeability μ_o, so;

$$B_z(z) = \frac{1.318\mu_o NI/D}{Cosh^2(2.636z/D)}\text{..................}(2)$$

It can be seen that the geometrical parameter D beside the physical one is the most adjustable variables that control the distribution $B_z(z)$ along the optical axis. Furthermore, this equation has a relatively simple mathematical form, i.e. it easy to be handled by means of the conventionally mathematical process. Consequently it is straightforword to find the axial magnetic scalar potential V_z correspond to B_z with the aid of Amper's law [AL-Obaidi, 1995];

$$\int_{Z_1}^{Z_2} B_z dz = \mu_o(V_2 - V_1)\text{..................}(3)$$

As shown in the following equation:

$$V_z(z) = 0.5NI\tanh(2.636z/D)\text{..................}(4)$$

The symbols V_1 and V_2 refer to the potentials at the terminals of the lens axis Z_1 and Z_2 respectively. However, the second derivative of V_z with respect to optical axis z can be found directly as in the following formula:

$$V_z^{''}(z) = \frac{-6.948NI[\sec h^2(2.636(z/D)\tanh(2.636(z/D))}{D^2}\text{..................}(5)$$

The Problem Analysis

Actually, any task in the field of charge particle optic aims at evaluating the aberration integrals required to know several axial functions. Mathematically, these integrals may be abbreviated as in the following integral:

$$C = \int_a^b F[V(z), V'(z), V''(z), B(z), B'(z), B''(z), r_\alpha(z), r_\alpha'(z), r_\alpha(z), r_\alpha'(z),]dz \quad ....(6)$$

Where the symbols ´ and ″ refer to the first and the second derivative respectively, the constant a and b are the limits of the imaging field on the optical axis for the projector case and the object Zo and image Zi coordinates for the objective case V(z) and B(z) are the axial electrostatic potential and magnetic field distribution respectively. $r_\alpha(z)$, $r_\gamma(z)$ are the independent solutions of the following paraxial ray equation;

$$r''(z) + \frac{\eta}{8V_r} B_z^2 r(z) = o \quad ....(7)$$

In last equation η represents the electron to mass quotient and V_r is the applied accelerating voltage. Since, the present work is concerned to deal with the spherical and chromatic aberration coefficients of a magnetic imaging field equation (6) reduced to the following form:

$$C = \int_{Zo}^{Zi} F[B(z), B'(z), r_\alpha(z), r_\alpha'(z)]dz \quad ....(8)$$

Equation (8) reveals that four axial functions have to be determined in order to calculate spherical and chromatic aberration coefficients. In the present work, the $B_z(z)$ distribution is determined by means of equation (2). However, the gradient of $B_z(z)$,i.e $B_z'(z)$ is calculated with the aid of the following expression:

$$B_z'(z) = 6.948\mu o\left(\frac{NI}{D^2}\right).\mathrm{sech}^2\left(\frac{2.636z}{D}\right) \quad ....(9)$$

The axial functions *r(z)* and its slope *r´(z)* can be determined by solving equation (7). In this study, this paraxial ray equation is solved numerically using the forth order Runge-Kutta method. Now, the integral given in equation (8) can be evaluated by using any suitable numerical integration technique. Throughout, present calculations, Simpson's rule has been used to computing spherical C_s and chromatic C_c aberration coefficients through the following expression:

$$C_s = \left[\frac{\eta}{128V_r}\right]\int_{Zo}^{Zi}\left[\frac{3\eta}{V_r}B_z^4 r_\alpha^4 + 8B_z'^2 r_\alpha^4 - 8B_z'^2 r_\alpha^2 r_\alpha'^2\right]dz \quad ....(10)$$

$$C_c = \left[\frac{\eta}{8V_r}\right]\int_{Zo}^{Zi} B_z^2 r_\alpha^2 dz \text{.......... ....}(11)$$

The present synthesis procedure, now, at a stage that one can decide whether the values of C_s and C_c are acceptable or not. For the undesirable case, one has to varying the adjustable parameters of equation (2) and repeated the calculations until a satisfactory values for C_c and C_s are reached, i.e the optimum imaging magnetic field of equation (2) is obtained.

The final step, as usual, is to determine the pole piece configuration that is capable to producing the optimum field obtained previously. The well known technique that was introduced by Zsilagyi [1984] for electrostatic lenses and developed by AL-Obaidi [1995] to deal with the magnetic lenses has been used in this work. Accordingly, the equpotential surface (i.e the pole piece) height R(z) along the optical axis z is given by the following two terms equation:

$$R(z) = 2\left[\frac{V_p - V_z}{V_z''}\right]^{1/2} \text{.......... ....}(12)$$

where V_p is the potential value at the terminal of z. Since V_z and V_z'' can be calculated with equations (4) and (5) respectively, the pole piece profile may be determined along the optical axis with the aid of the last equation.

Discussion and Results:

In this context, the influence of the bore diameter and the lens excitation as an adjustable parameters on the objective properties and the reconstructed pole pieces have been studied for zero and high magnification conditions. It is important to mention that, calculations were executed at constant values of optical axis length L = 10mm and the excitation parameter $NI/V_r = 20$ respectively.

I. The case of zero magnification condition

a. Influence of NI

The axial magnetic flux density distribution B_z and its corresponding axial magnetic scalar potential V_z for five values of NI ranging from 200 A.t to 1000 A.t are plotted in figure (1) and (2) respectively.

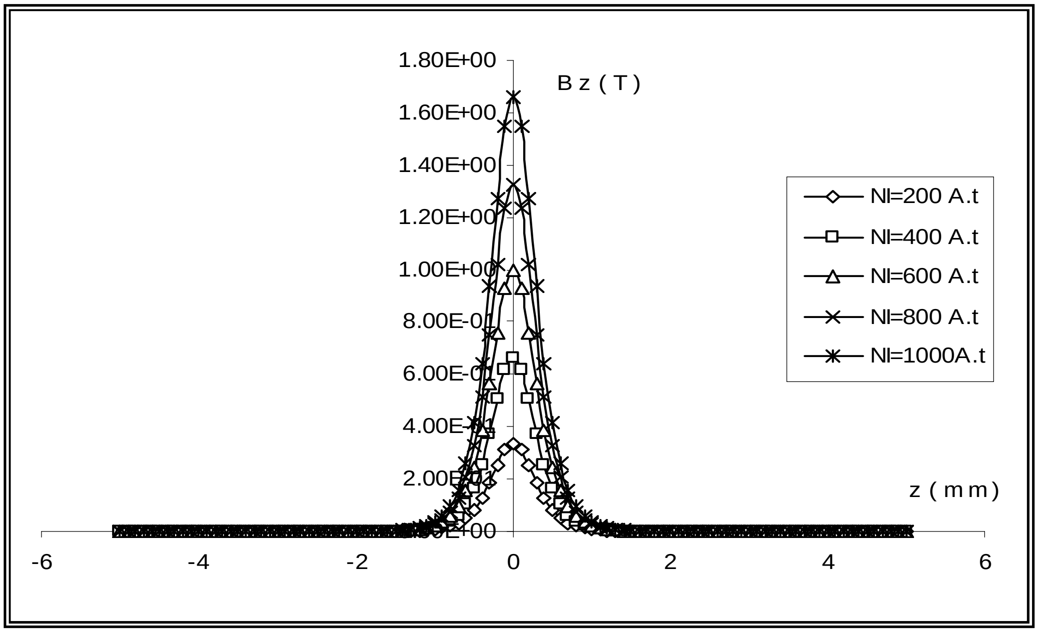

Figure (1): The axial magnetic field distributions for different values of the excitation parameter at D=1mm,for zero magnification condition.

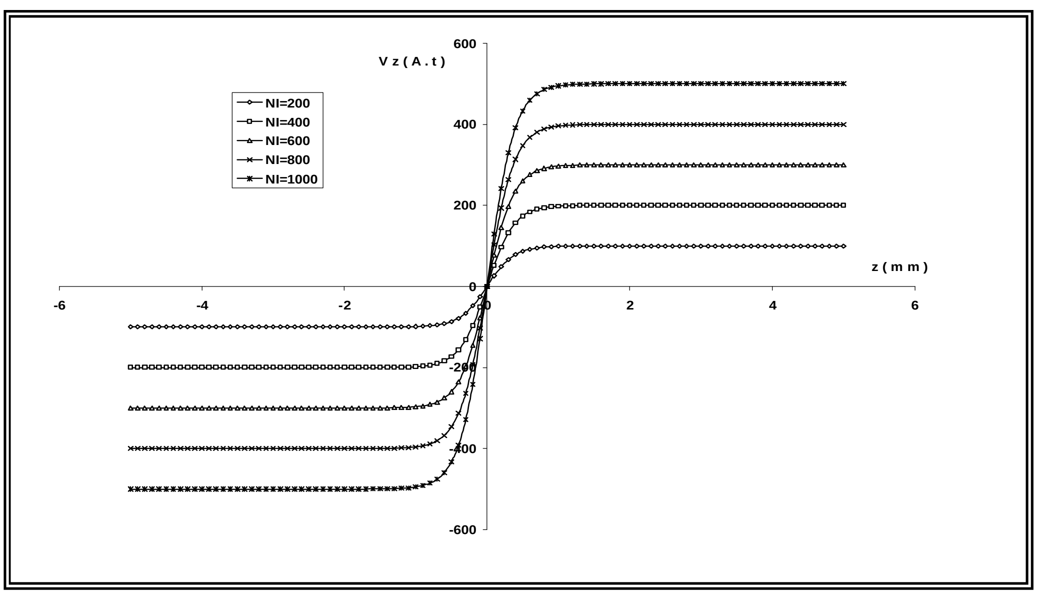

Figure (2): The axial magnetic scalar potential distributions for different values of the excitation parameter at D= 1mm, for zero magnification condition.

where the optical axis length and the bore diameter were kept constant at the values L = 10mm and D = 1mm respectively. Initially, it is apparent from figure (1) that, the B_z distribution has the same value of half width while the peak of the flux density B_{max} increases with the increases of NI. Figure (3) shows the variation of the aberrations coefficients C_s and C_c and the focal length (f_o) as a function of NI.

It is clear that these properties remain constant for each value of NI since the increase of NI gives a constant half-width w. The pole piece configuration for each B_z distribution of the different value of NI is plotted in figure (4). It is seen that the pole piece shape is not affected with the variation of the lens excitation NI.

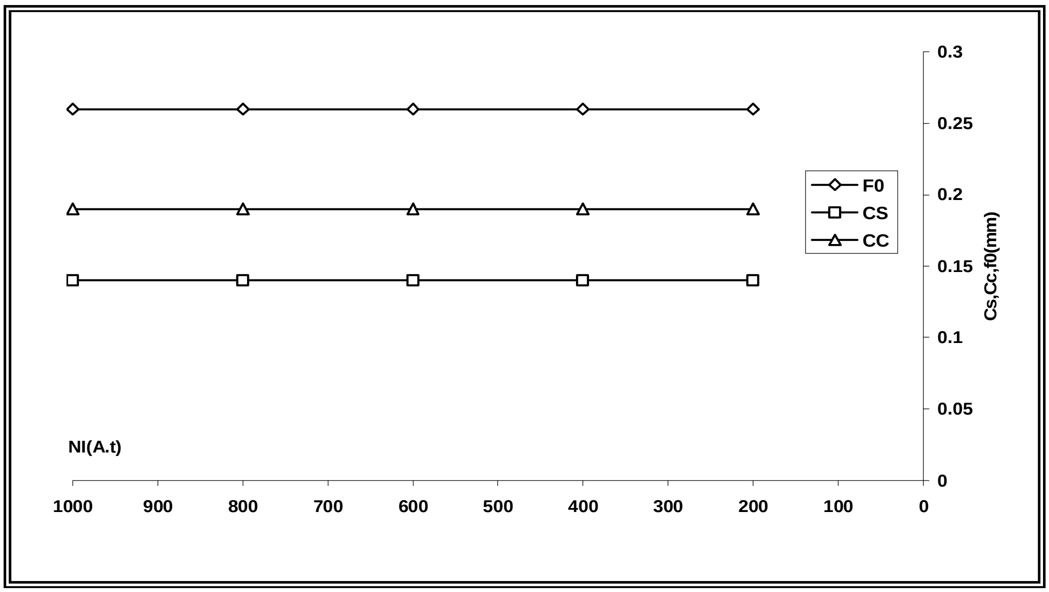

Figure (3): The objective focal properties for different values of the excitation parameter at D= 1mm, for zero magnification condition.

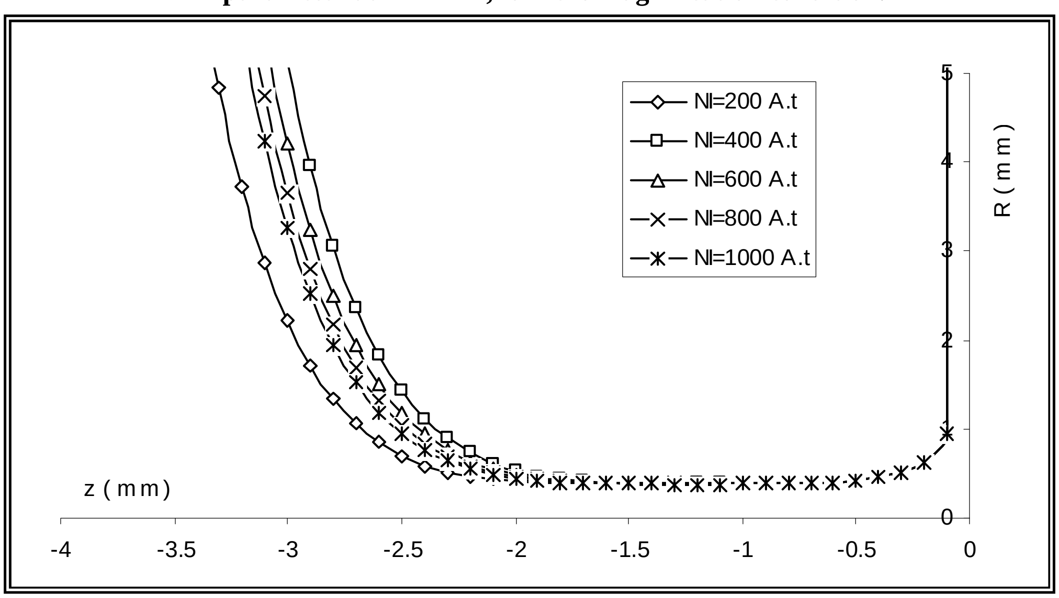

Figure (4): The pole pieces configurations for different values of the excitation parameter at D= 1mm, for zero magnification condition.

b. Influence of D

The axial magnetic flux density B_z and its associated axial magnetic scalar potential along the optical axis z are plotted in figures (5) and (6) respectively for different values of D,where

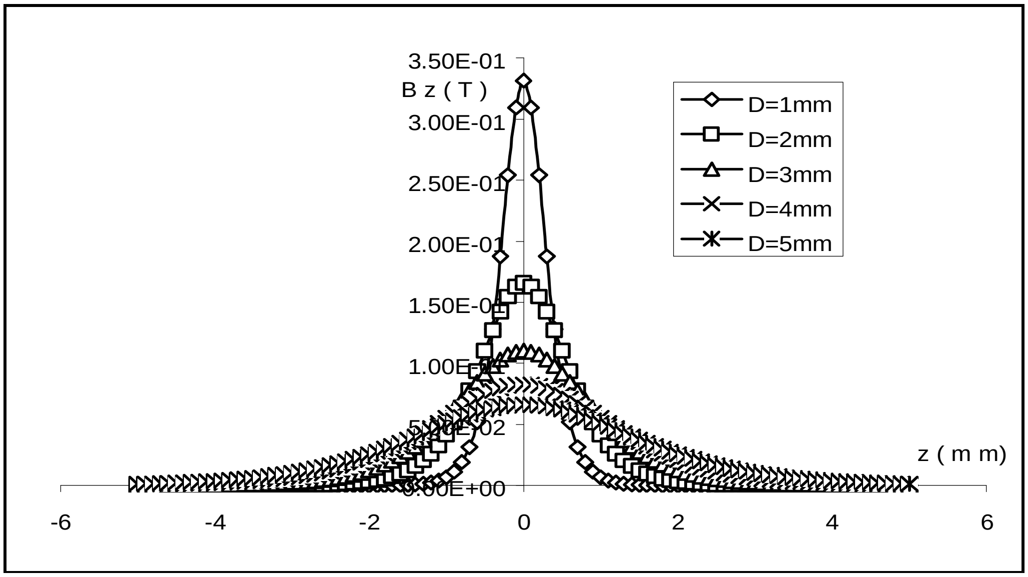

Figure (5): The axial magnetic flux density distributions for different values of the bore diameter at NI= 200(A.t), for zero magnification condition.

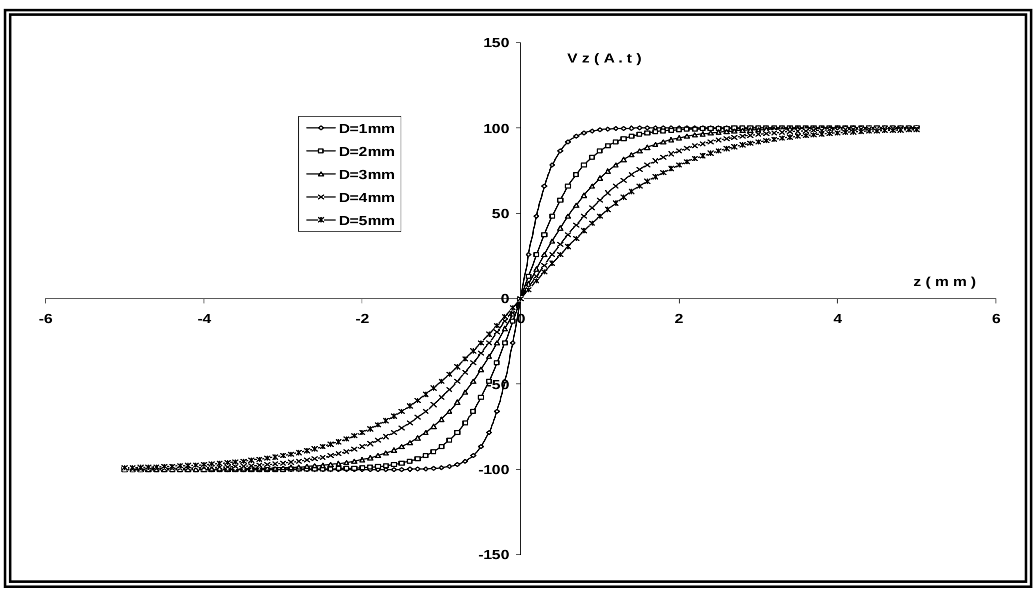

Figure (6): The axial magnetic scalar potential distributions for different values of the bore diameter at NI=200 (A.t), for zero magnification condition.

five values of D have been chosen ranging from 1mm to 5mm, when the optical axis length and the lens excitation were kept constant at L = 10mm and NI = 200 A. t. It is clear that when D increases the maximum value of B_z distribution decreases and the half-width of each distribution increases.

Figure (7) shows the varying of the optical properties as a function of the bore diameter D. It is clear that as long as D decreases the properties become raising better. This means, however, the magnetic lens of low aberrations and low operation power can be designed as the bore diameter chosen to be smaller. Figure (8) shows that the reconstructed pole pieces for the considered values of D.

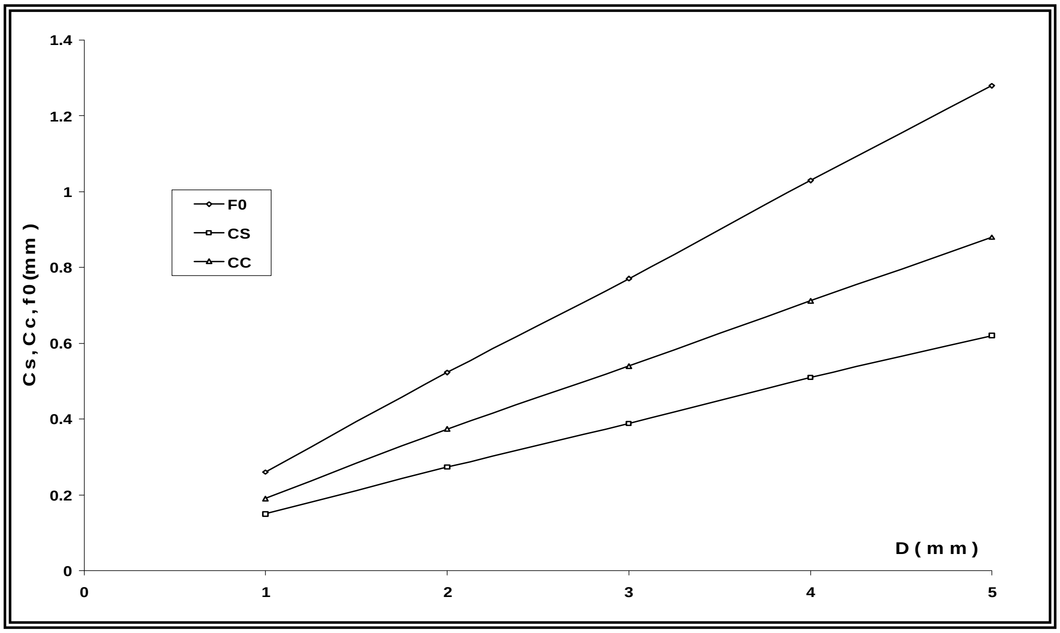

Figure (7): The objective focal properties for different values of the excitation parameter at D= 1mm, for zero magnification condition.

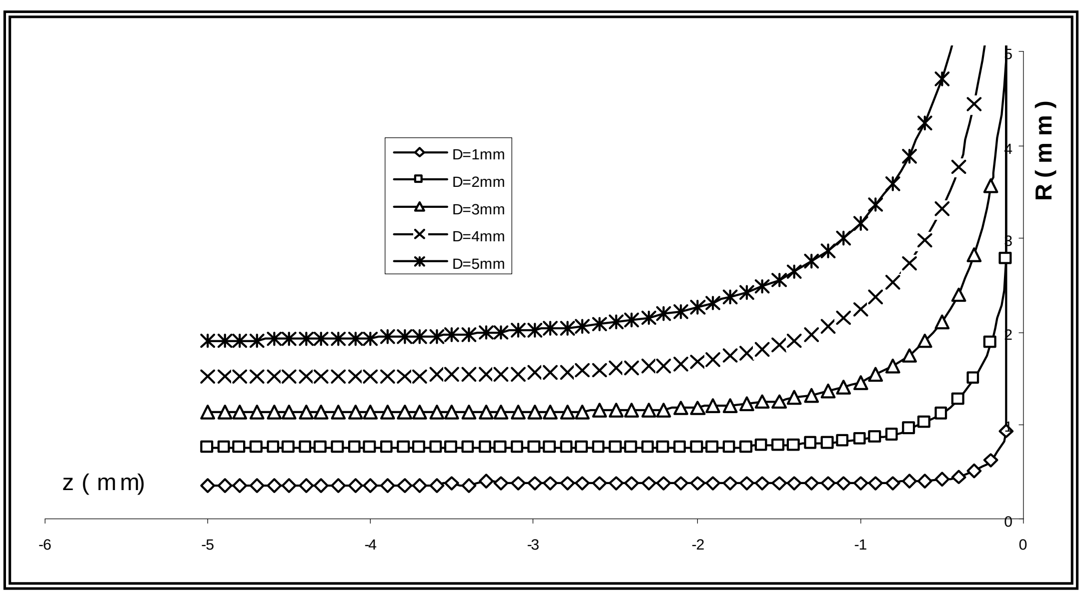

Figure (8): The pole pieces configurations for different values of the bore diameter at NI= 200 (A.t), for zero magnification condition.

II. The case of High magnification condition

a. Influence of NI

The effect at the lens excitation on the axial magnetic field distribution and its corresponding axial magnetic scalar potential has been investigated for five values of NI ranging from 200 to 1000 A.t as shown in figure (9) and (10) respectively. It is apparent that the increases of the lens excitation NI will increase the maximum value of B_z distribution increases, while the half-width w remains constant.

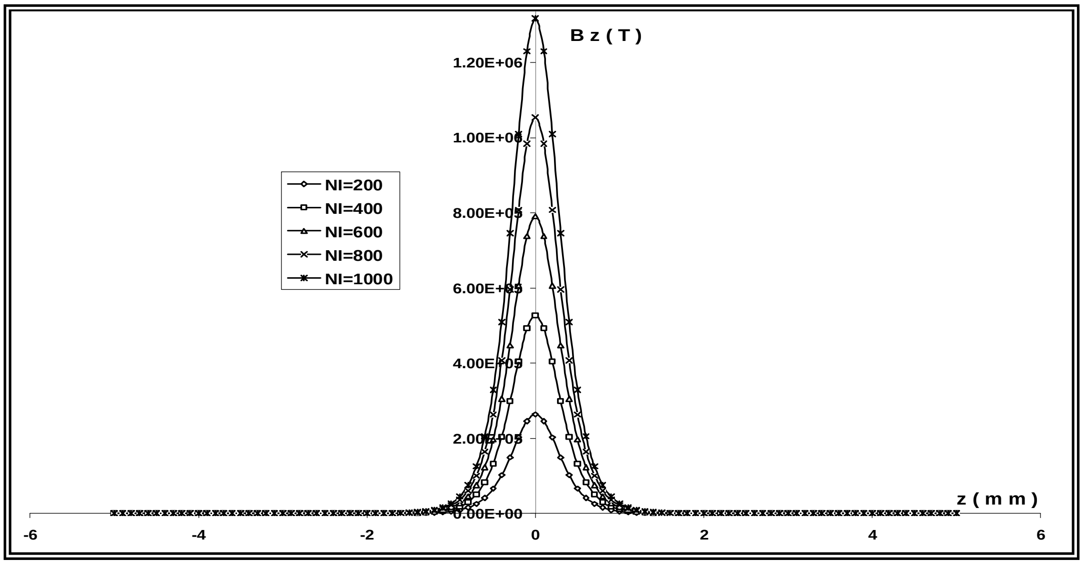

Figure (9): The axial magnetic flux density distributions for different values of the excitation parameter at D= 1mm, for high magnification condition.

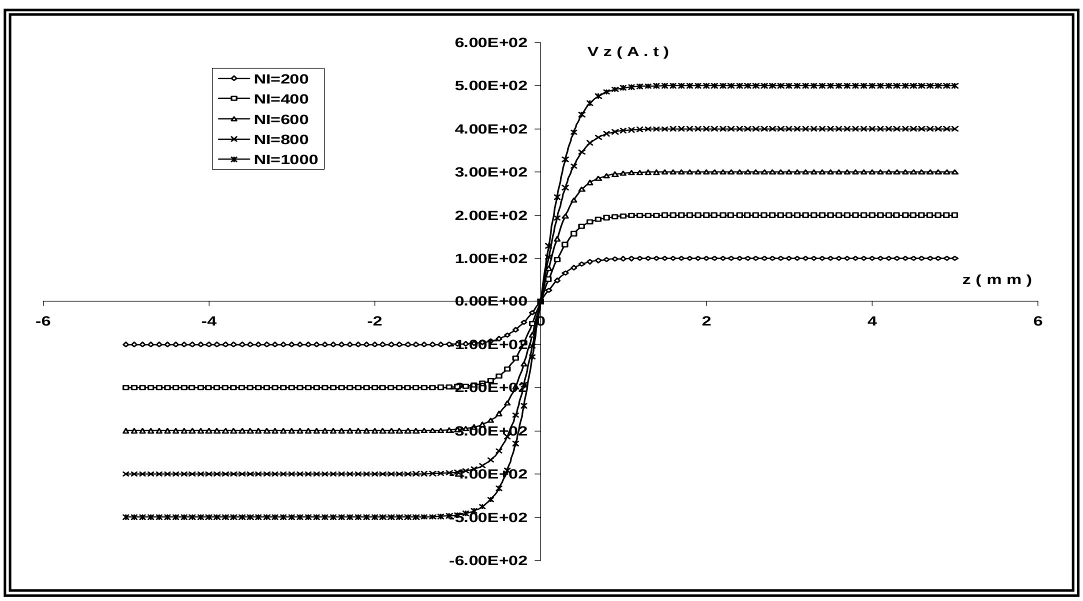

Figure (10): The axial magnetic scalar potential distributions for different values of the excitation parameter at D= 1mm, for high magnification condition.

Figure (11) shows the aberration coefficient C_s and C_c for each value of NI. It is seen that the optical properties are not affected with the variation of the lens excitation NI. On the other hand, the pole piece profile remains constant with the variation of NI as shown in figure (12).

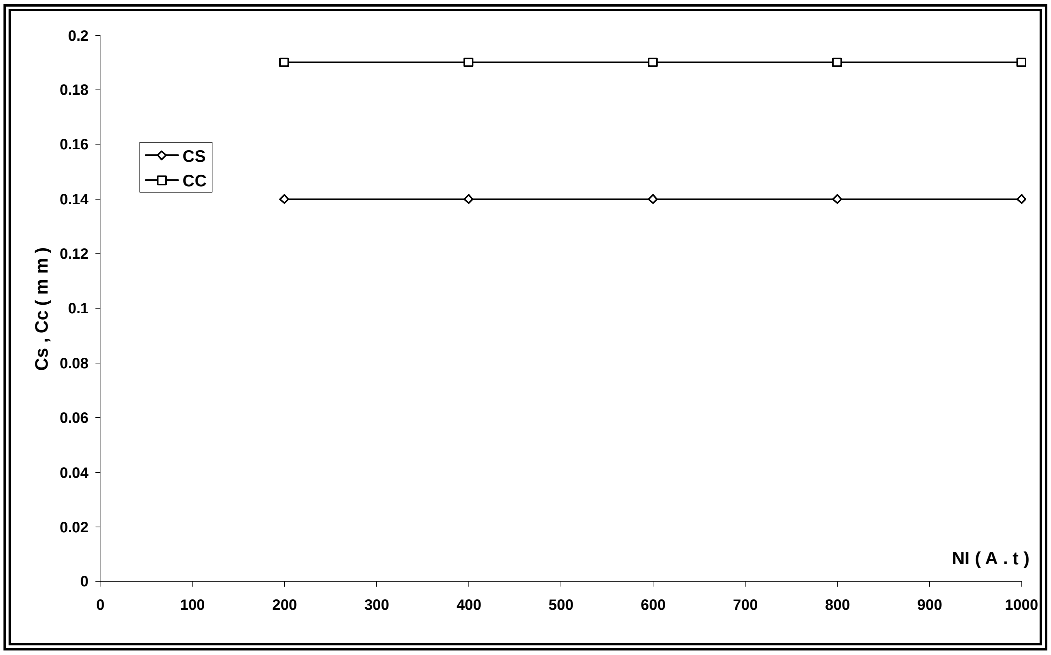

Figure (11): The objective focal properties for different values of the excitation parameter at D= 1mm, for high magnification condition.

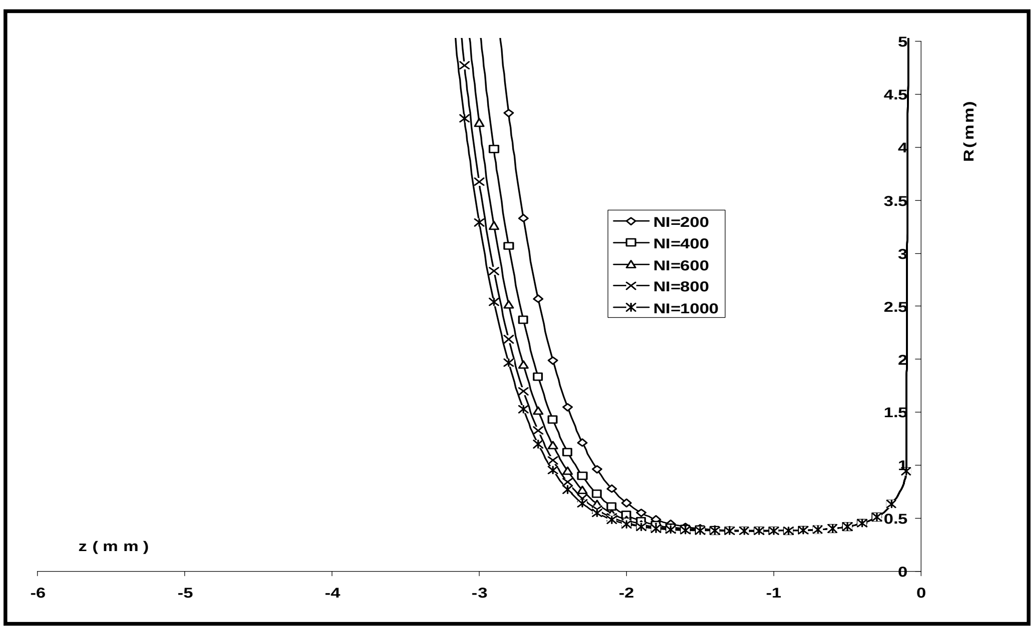

Figure (12): The pole pieces configurations for different values of the excitation parameter at D= 1mm,for high magnification condition.

b.Influence of D

The effect of the bore diameter has been investigated for five values ranging from 1mm to 5mm the axial magnetic field distribution and its corresponding axial magnetic scalar potential distribution V_z are plotted in figures (13) and (14) respectively. It is clear that, as D decreases the B_z distribution become mere localized in a narrow region.

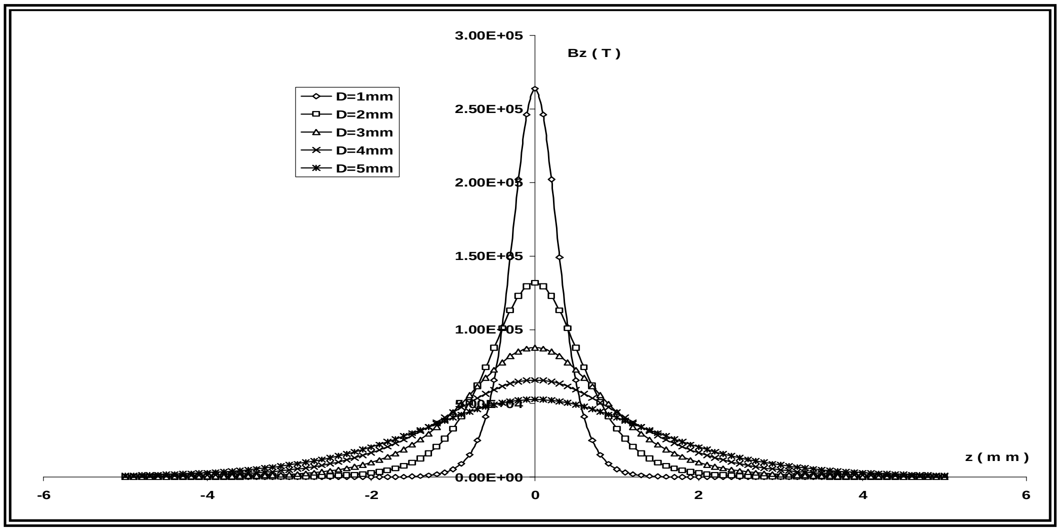

Figure (13): The axial magnetic scalar potential distributions for different values of the bore diameter at NI= 200 (A. t),for high magnification condition.

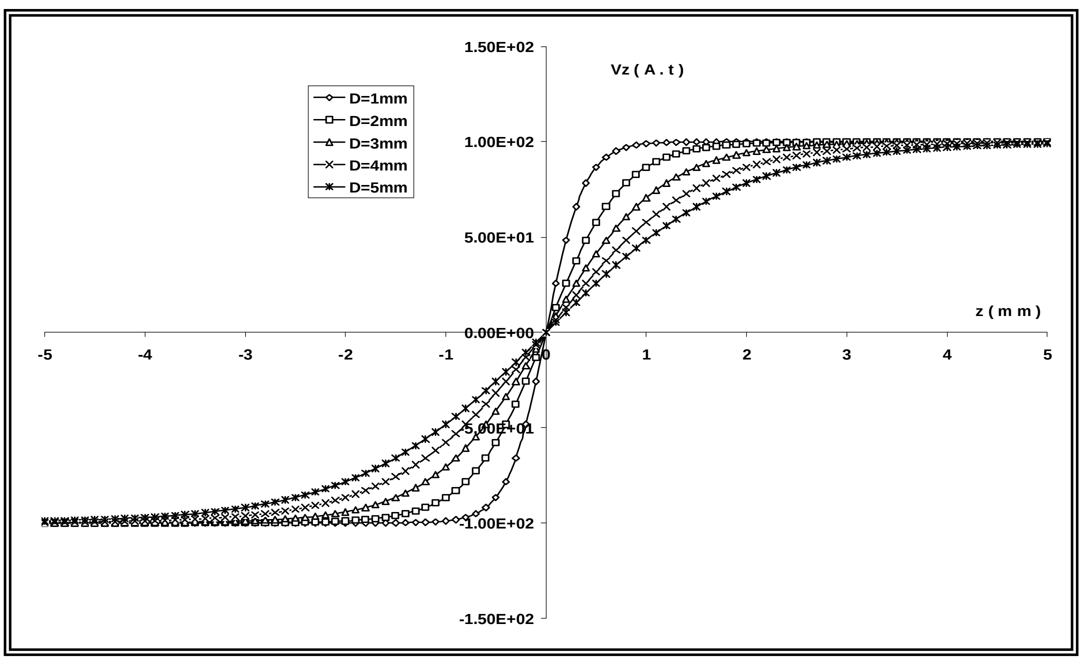

Figure (14): The axial magnetic field distributions for different values of the bore diameter at NI= 200 (A.t), for high magnification condition.

The behaviour of the objective properties as a function of the bore diameter D is shown in figure (15). It is seen that the objective properties get better well as D decreases.

The pole piece profile corresponding to the different B_z distributions are shown in Figure (16).It is seen that the shape becomes more different to each other as long as D will be changed.

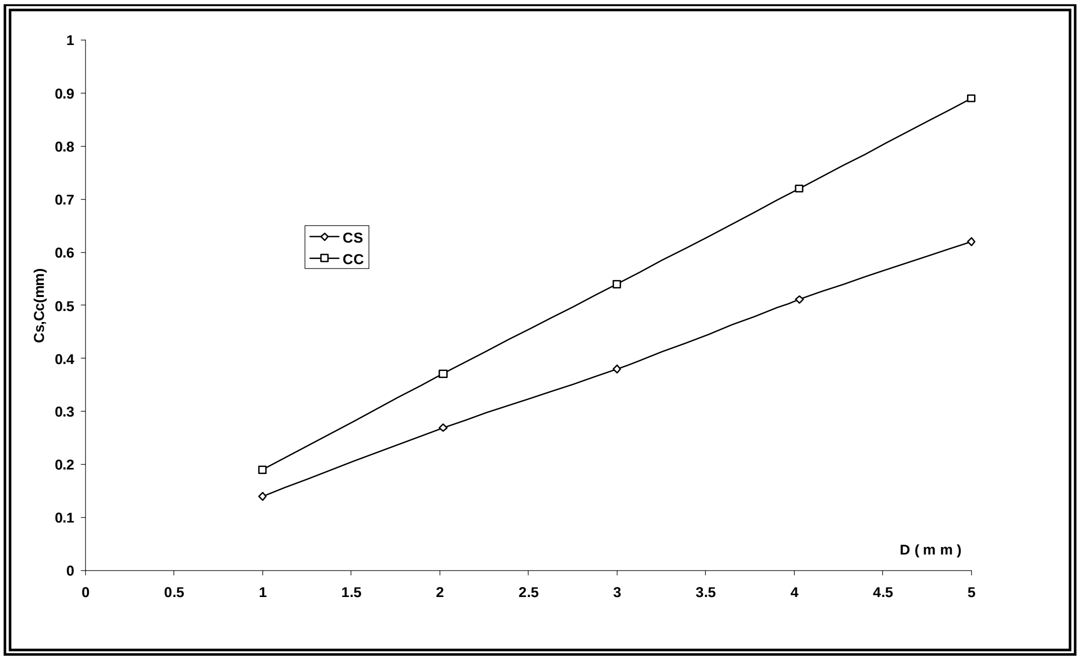

Figure (15): The objective focal properties for different values of the bore diameter at NI= 200 (A. t), High magnification condition.

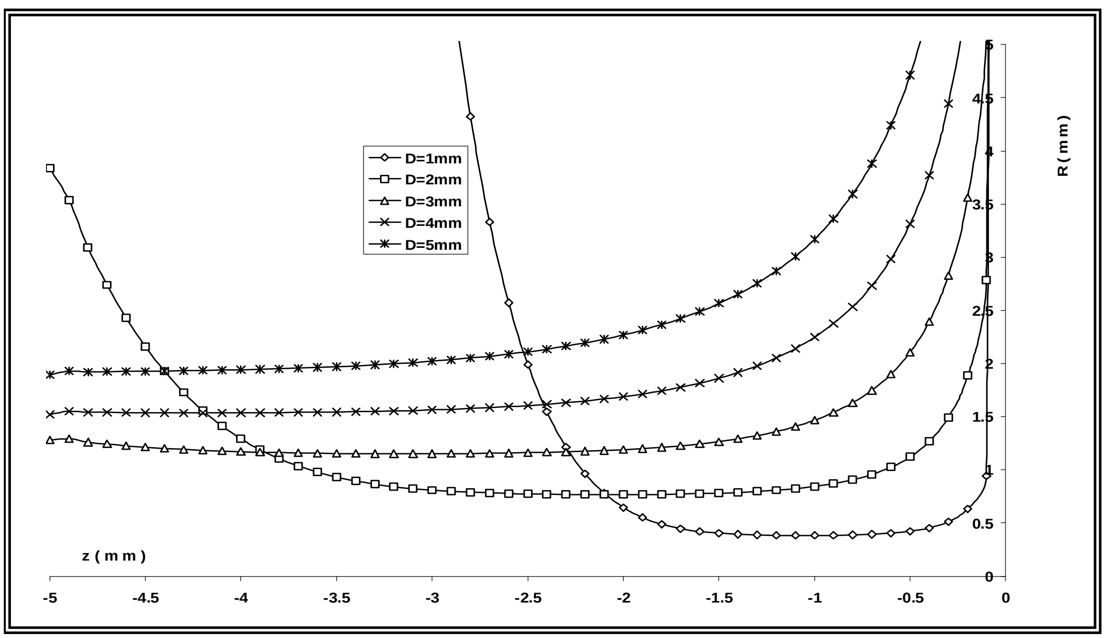

Figure (16): The pole pieces configurations for different values of the bore diameter at NI= 200 (A. t),for high magnification condition.

Conclusions

According to the results of this work it can be said that the introduced model can be widely used with synthesis procedure to investigate magnetic lenses. In addition to that, calculations of the present work reveal a high efficiency and excellent accuracy in reconstructing the magnetic lenses for zero and high magnification conditions that can be made with this approach.Furthermore, the present investigation proves the efficiency of the present target function to the investigation, the first order aberrations of the double pole pieces magnetic lenses.

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الخلاصة

في هذا البحث تم اقتراح دالة تحليلية جديدة لتمثيل شدة المجال المغناطيسي للعدسات المغناطيسية المتناظرة. ومن خلال ذلك يمكن الحصول بشكل مباشر على كثافة الفيض المغناطيسي الضرورية في تحديد تكاملات الزيوغ . تم دراسة تأثيرات المعلمات الهندسية والفيزيائية (كمتغيرات أمثلية)على الخواص البؤرية الشيئية وأشكال الاقطاب. أوضحت الدراسة امكانية الحصول على أستنتاجات جديدة تتعلق بمقارنة النتائج للاجراء التحليلي والاجراء التوليفي لنفس الشروط الابتدائية.