Moment Solution for Scattering Problems from Perfectly Conducting Bodies of Revolution

الحل العدد بطريقة العزوم لمسائل الاستطارة من الأجسام الموصلة المتناظرة محوريا

م.د. احمد هاشم عبود/ كلية التربية الأساسية-جامعة ميسان

Abstract

Moment method has been used to solve the integro-differential equation' fir the electric field (EFIE) for the electromagnetic scattering problem from conducting bodies.

Body under study is the spheroid, the validity of the numerical treatment is done by approximate the body the special case of sphere (i.e. a = b) and the results are good in compression.

The effect of the major and minor radii on the current distribution and the radar cross-section (RCS) pattern are studied. The effect of the major radius on the current distribution and RCS pattern is significant, while the effect of the minor is less significant on the current distribution and RCS pattern with small•

shift in the minimum in RCS pattern

الخلاصة

استخدمت طريقة العزوم لحل المعادلة التفاضلية-التكاملية للمجال الكهربائي عدديا لحل مسائل الاستطارة من الاجسام الموصلة المتناظرة محوريا.موضوع البحث جسم موصل بيضوي الشكل، تم التأكد من صحة المعالجة العددية بتقريب الجسم البيضوي إلى حالة خاصة من الجسم البيضوي وهي الكرة (a=b) وقد وجد إن النتائج متفقة مع الشكل الكروي.تم دراسة تأثير كل من القطر الرئيسي والثانوي على توزيع التيار على السطح الموصل ومساحة المقطع الرادارية للجسم. لوحظ إن تأثير القطر الرئيسي كبير على كل من توزيع التيار مساحة المقطع الرادارية عند ثبوت قيم القطر الثانوي على 0.21. غير ان تغيير القطر الثانوي كان له تأثير على السعة لكل من اليسار ومساحة المقطع الرادارية مع بقاء التوزيع في كلا الحالتين متشابه تقريبا مع حصول إزاحة بمنطقة اقل قيمة لمساحة المقطع الرادارية.

Introduction

Exact solution for the problem of electromagnetic (EM) wave scattering by a perfectly conducting objects is available for limited general shapes, such as elliptic, cylinders, spherical, and spheroid objects.

Unfortunately, some of these solutions are complicated in form and calculations, such as the spheroid objects which are need matrix inverse ofthe same order used with matrix equation as a numerical solution solved by the method of moments MoM [1].In the case of arbitrarily shaped objects the analytical solution is not feasible, so, the approximate or numerical solution must be introduced. The complexity of these numerical solutions depends on the properties of the object under test, such as the objects of imperfectly conducting or good dielectric need an impedance matrix of order two compared with the perfectly conducting objects.The most useful numerical evaluation is that uses the Integral Equation (IE) formulation in the above cases. The IE is the representation of the field vectors for both interior and exterior to the body, these IE may

*be solved by many numerical methods depends on the parameters of the EM problems such as the incident field, the size, and the complexity of geometry.

In this paper the scattering problem by a conducting bodies of revolution BOR using the electric field integral equation EFIE with the MoM as a numerical solution is formulated, which is solved an IE by expanding the unknown surface current distribution in the series of suitable basis functions. Then the IE can be reduced to a set of simultaneous linear equations, whose solution gives the required surface current distribution.

Formulation of the Scattering Problem:

The EFIE for the electric current J induced on surface S on perfectly conducting body of revolution as, shown in Fig.(2.1), by an incident

electric field E satisfying the boundary condition that the tangential electric field must be vanish at the surface, that is,

$$-\hat{n}\times \overline{E}^{s}_{\tan} = \hat{n}\times \overline{E}^{inc}_{\tan} \qquad (1)$$

where $\hat{n}$ is the unit vector normal to the surface S, and e" is the scattered field due to Jon S, the subscript "tan" denotes the tangential components on S. The scattered field that produced by this equivalent current can be expressed in terms of the vector and scalar potentials as,

$$\overline{E}^{s}(r) = -j\omega\overline{A}(r') - \nabla\Phi(r') \qquad (2)$$

The magnetic vector potential is written explicitly as,

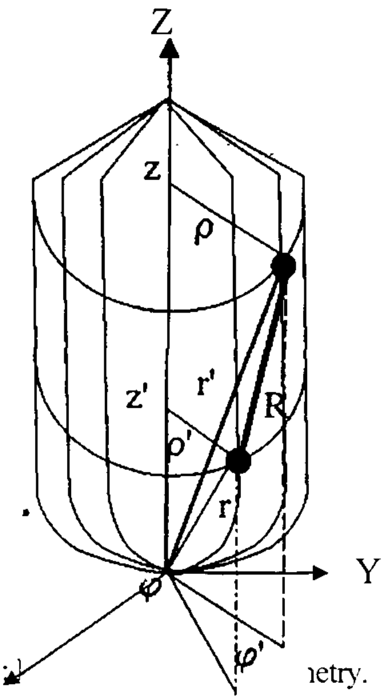
$$\overline{A}(r') = \frac{\mu}{4\pi}\int_{s}\overline{J}(r')G(\bar{\rho},\bar{\rho}')ds \qquad (3)$$

Where

$$G(\bar{\rho},\bar{\rho}') = \frac{\exp(-jk|\bar{\rho}-\bar{\rho}'|)}{|\bar{\rho}-\bar{\rho}'|} \qquad (4)$$

is the Green's function, and for BOR

$$R = |\bar{\rho}-\bar{\rho}'| = \sqrt{(\rho-\rho')^2 + (z-z')^2 + 2\rho\rho'\cos(\phi-\phi')} \qquad (5)$$



Fi 1: ıetry.

R is the distance from the source point representing by the positional vector ($\hat{r}$), to the field point ($\hat{r}'$) as shown in Fig.1. Furthermore, the scalar potential is expressible in the term of equivalent electric charge distribution as,

$$\Phi = \frac{1}{4\pi\varepsilon}\int_{S}\sigma(\hat{\rho}')G(\hat{\rho},\hat{\rho}') \tag{6}$$

where □(r') is the electric charge which related to the electric current by the continuity equation of.

$$\sigma(\hat{\rho}') = \frac{-1}{j\omega}\nabla'_{s}\cdot\bar{J}(r') \tag{7}$$

Therefore, by combining eqs.(1) through (7) the EFIE of eq.(1)can be written as,

$$-\hat{n}\times\bar{E}^{s}_{\tan} = \hat{n}\times\left[\frac{j\omega\varepsilon}{4\pi}\int_{s}\bar{J}(r')G(\hat{\rho},\hat{\rho}')ds + \frac{j\nabla}{4\pi\omega\varepsilon}\int_{s}\nabla'\cdot\bar{J}(r')G(\hat{\rho},\hat{\rho}')ds\right]_{\tan} \tag{8}$$

Eq.(8) ca be represent in term of operator equation as,

$$\left|L(\bar{J}) = \bar{E}^{inc}\right|_{tav} \tag{9}$$

where L is the integro-deferential operator defined as

$$L(\bar{X}) = \int_{s} j\omega\mu\,\bar{J}(r')\frac{e^{-jkR}}{4\pi R}ds + \frac{j\omega\varepsilon}{4\pi}\int_{s}\nabla'\cdot\bar{J}(r')\frac{e^{-jkR}}{4\pi R}ds \tag{10}$$

Moment Solution

The procedure of numerical solution is being with reduced the eq.(8) to the set of linear equation in a matrix form, as we noted earlier the numerical method is MoM with Galerkin's approach[1]. The first step of MoM solution is replacing the perfectly conducting body by equivalent electric surface currents using the equivalence principle [4]. In the equivalence principle, the scattered fields are due to the free space fields of the equivalent currents. Finding these currents is the main task of MoM solution. The next step Ls to obtain a set of coupled IE of the equivalent currents components as in eq.(8). The final step in this method is expanding the equivalent surface current in term of finite set of N basis function and due to the rotational symmetry of the body one can use the Fourior series to represent this function as,

$$\bar{J}(\hat{\rho}') = \sum_{n,j} I^{t}_{nj}J^{t}_{nj}(\hat{\rho}')\hat{u}'_{t} + I^{\varphi}_{nj}J^{\varphi}_{nj}(\hat{\rho}')\hat{u}'_{\varphi} \tag{11}$$

where

$$\bar{J}^{t}_{nj}(\hat{\rho}') = \bar{J}^{\varphi}_{nj}(\hat{\rho}') = \rho' f_{j}(t')e^{jn\varphi} \tag{12}$$

I^{t}_{nj} and I^{φ}_{nj} are the unknown coefficients to be determined, the index n is associated with the summation of Fourior mode, while the index j is associated with the summation of the basis function, when BOR is subdivided into annular, rings, the unknown coefficients obtain by enforcing N weighted (testing) average of the IE

$$\bar{W}(\hat{\rho}) = \sum_{m,j} I^{t}_{mi}W^{t}_{mj}(\hat{\rho})\hat{u}_{t} + I^{\varphi}_{mj}W^{\varphi}_{mj}(\hat{\rho})\hat{u}_{\varphi} \tag{13}$$

Where

$$\bar{W}^{t}_{mj}(\hat{\rho}) = \bar{W}^{\varphi}_{mj}(\hat{\rho}) = \rho f_{j}(t)e^{-jm\varphi} \tag{14}$$

According to the Galerkin's approach ($W=J^*$), W and J are the orthogonal vector to the S. Since the W_{mi} over 0 to □□on □ is orthogonal to the J_{nj} for ($n \neq m$), all inner products are zero except those for which ($n = m$). This fact allowed each mode to be treated completely independently of the other mode[2]. The inner product defined as,

$$\langle \bar{S},\bar{Q}\rangle = \int_s \bar{S}.\bar{Q}ds \tag{15}$$

where S and Q are the tangential vectors to the S, the inner product define for the BOR introduced double integral, as

$$\int_s ds \int_{s'} ds' = \int_0^N dt \int_0^{2\pi} \rho dt \int_0^N dt' \int_0^{2\pi} \rho' dt' \tag{16}$$

After testing each side of eq.(8) .one obtains the generalized "network type" matrix equation,

$$[T_n][I_n] = [V_n] \tag{17}$$

where $[V_n]$ and$[I_n]$ are the excitation and unknown coefficients matrix (voltage matrix) given by

$$[V_n] = \left[\left[W_{ni}^t, E^{inc}\right]\left[W_{ni}^{\varphi}, E^{inc}\right]\right] \tag{18-a}$$

$$[I_n] = \left[\left[I_{ni}^t\right]\left[I_{ni}^{\varphi}\right]\right] \tag{18-b}$$

and $[T_n]$ is the impedance matrix of the body, defined as

$$[T_n] = \begin{bmatrix} T_n^{tt} & T_n^{t\varphi} \\ T_n^{\varphi t} & T_n^{\varphi\varphi} \end{bmatrix} \tag{19}$$

and the T sub-matrices given by explicit form[3]

$$\left(T_n^{\alpha\beta}\right)_{ij} = \int_0^N dt \int_0^{2\pi} \rho dt \int_0^N dt' \int_0^{2\pi} \rho' dt' \left\{ i\mu\omega\left(W_{ni}^{\alpha} \cdot J_{nj}^{\beta}\right) + \frac{1}{j\omega\varepsilon}\left(\nabla \cdot W_{ni}^{\alpha}\right)\left(\nabla' \cdot J_{nl}^{\beta}\right)\right\} G(\bar{\rho},\bar{\rho}') \tag{20}$$

with □ and □ are the combinations of t- and □- directed, n again the mode number, and

$$\nabla' . J = \frac{1}{\rho'}\frac{\partial}{\partial t'}(\rho' J_{nj}^t . \hat{u}_t') + \frac{1}{\rho'}\frac{\partial}{\partial \varphi'}(\rho' J_{nj}^{\varphi} . \hat{u}_{\varphi}') \tag{21}$$

$$\nabla \cdot W = \frac{1}{\rho}\frac{\partial}{\partial t}(\rho W_{ni}^t . \hat{u}_t) + \frac{1}{\rho}\frac{\partial}{\partial \varphi}(\rho W_{ni}^{\varphi} . \hat{u}_{\varphi}) \tag{22}$$

For the complete evaluation of the T elements, one must introduce the tangential unit vectors of the BOR shown in Fig.l), as

$$\begin{aligned} \hat{u}_t &= \sin v \cos\varphi\, \hat{x} + \sin v \sin\varphi\, \hat{y} + \cos v\, \hat{z} \\ \hat{u}_{\varphi} &= -\sin\varphi\, \hat{x} + \cos\varphi\, \hat{y} \end{aligned} \tag{23}$$

for the field point, and for the source point we have,

$$\begin{aligned} \hat{u}_t' &= \sin v' \cos\varphi'\, \hat{x} + \sin v' \sin\varphi'\, \hat{y} + \cos v'\, \hat{z} \\ \hat{u}_{\varphi}' &= -\sin\varphi'\, \hat{x} + \cos\varphi'\, \hat{y} \end{aligned} \tag{24}$$

Since, $\hat{u}_{\varphi}$ or $\hat{u}_{\varphi}'$ is always normal to the z-axis, but $\hat{u}_t$ is at angle v with the z-axis, being positive if $\hat{u}_t$ point away from the z-axis and negative if $\hat{u}_t$ toward it. The □-integral in the eq.(20) may be eliminate by a so-called Green's function defined as[4],

$$g_o = 4\pi \int_s \cos n\varphi \frac{e^{-jkR_o}}{R_o} d\varphi \tag{25}$$

where R^ is given by eq.(5) with (□ = 0)

The remaining integral in eq.(20) for t and t' is approximated by introducing the triangle function for both current expansion function and testing function as follows

$$\rho f_i(t) = F(t - t_i) \tag{26}$$

T is the triangle function given by

$$F(t) = \begin{cases} 1 - |t| & \text{for } |t| < 1 \\ t & \text{for } |t| > 1 \end{cases} \tag{27}$$

The triangle function expansion converges satisfactory and provides accurate solution. For this reason triangle function are used here to represent the current expansion and testing functions.

Substituting Eqs.(21) to (26) into Eq.(20) to obtain the explicit form of Z sub-matrices of eq.(19)

According to the calculation that used by Mautz and Harrington [2] to reduces Eq.(25) to the so-called pulse Green's function with some mathematical manipulation we get.

$$\begin{aligned}
\left(T_n^{tt}\right)_{ij} &= \sum_{p=1}^{4} \sum_{q=1}^{4} j\omega\mu T_p T_q \left(\sin v_p \sin v_q \frac{G_{n+1} + G_{n-1}}{2} + \cos v_p \cos v_q G_n \right) + \frac{1}{j\omega\varepsilon} T_p' T_q' G_n \\
\left(T_n^{t\varphi}\right)_{ij} &= \sum_{p=1}^{4} \sum_{q=1}^{4} - \omega\mu T_p T_q \sin v_p \frac{G_{n+1} - G_{n-1}}{2} + \frac{n}{j\omega\varepsilon\rho_p} T_p' T_q G_n \\
\left(T_n^{\varphi t}\right)_{ij} &= \sum_{p=1}^{4} \sum_{q=1}^{4} \omega\mu T_p T_q \sin v_q \frac{G_{n+1} - G_{n-1}}{2} - \frac{n}{j\omega\varepsilon\rho_q} T_p T_q' G_n \\
\left(T_n^{\varphi\varphi}\right)_{ij} &= \sum_{p=1}^{4} \sum_{q=1}^{4} T_p T_q \left(j\omega\mu \frac{G_{n+1} + G_{n-1}}{2} + \frac{n^2}{j\omega\varepsilon\rho_p\rho_q} G_n \right)
\end{aligned} \tag{28}$$

Also the above Green's functions may be solved numerically using Gauss quadrature[5,6]

Evaluation of Deriving Vector:

To evaluate the deriving vector and the far-scattered fields for conducting BOR as the procedure of measurement matrix or linear measurement formulated by J. R. Mautz and R. F. Harrigton is utilized in this evaluation. A linear measurement is defined as a number which depends linearity on the source. Examples of linear measurements are components of the field at a point, voltage along a given conductor, and current crossing a given surface [4]. It should be noted here, the excitation matrix is matrix result from the induced currents on body surface, while the measurement matrix is the matrix result from the far-scattered fields produced by induced currents. There are tow examples of linear measurements are[7]:

(1) a component of the current at some point on S.

(2) a component of the field \Eor H) at some point in space.

Starting from the definition of the linear measurement

$$[V_n] = \left[\langle \bar{J}, \bar{E}^{inc} \rangle\right]$$

From the reciprocity theorem one can find the radiation field E at a distance r from the origin due to the current J on S.

$$\bar{E}^r . \hat{u}_r = \frac{-j\mu\omega}{4\pi} \exp(-jk_o r) \int_s \bar{E}^r . \bar{J}(r')ds \qquad (29)$$

where $\hat{u}_r$ is the unit vector specifying the polarization of the incident wave, consider

$$\bar{E}^r = \hat{u}_r \exp(-jk.\rho_i)$$

(30)

is an arbitrary plane wave of superposition of the two orthogonal components, say E and E ., where k = k.k0 is wave number unit vector in the direction of propagation and ρ_i' is the vector pointing from origin, as in Fig.(2).

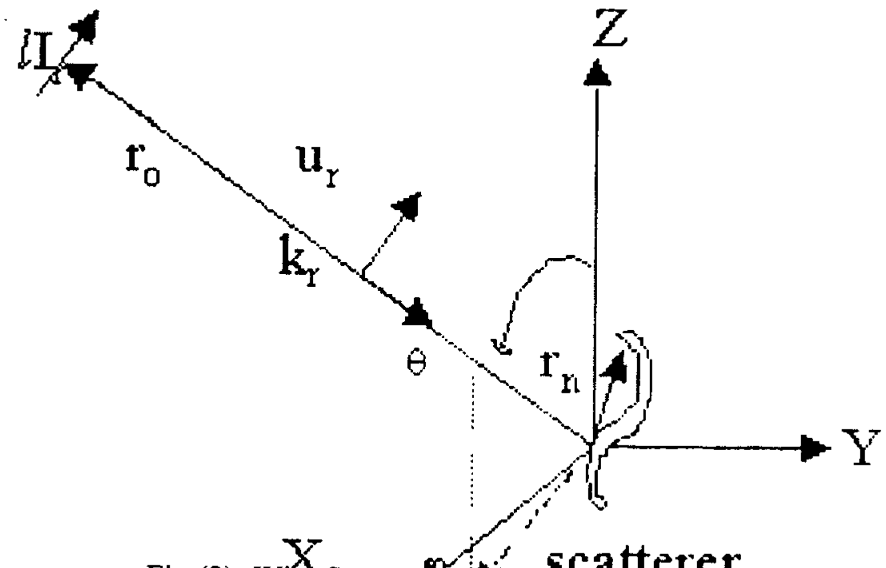


Fig.(2): Wire Scattrer and distance dipole

Substituting eq.(30) into eq.(29) and utilizing from eq.(11) for n_{th} mode, we get,

$$\bar{E}^r . \hat{u}_r = \frac{-j\mu\omega}{4\pi} \exp(-jk_o r)[R_n][I_n]$$

(31)

where $[I_n]$ is the coefficients of the expansion function, and

$$[R_n] = \left[[R_n^t][R_n^\varphi]\right]$$

with

$$[R_n^t] = \left[\langle \bar{E}^r, \bar{J}_{nj}^t \rangle\right]$$

$$[R_n^\varphi] = \left[\langle \bar{E}^r, \bar{J}_{nj}^\varphi \rangle\right]$$

(32)

Now, for the □-polarized plane wave (i.e., $\hat{u}_r = \hat{u}_\theta$) we get

$$\left[R_n^{t\theta}\right]=\left[\left\langle \bar{E}_\theta^r ,\bar{J}_{nj}^t\right\rangle\right]$$
$$\left[R_n^{\varphi\theta}\right]=\left[\left\langle \bar{E}_\theta^r ,\bar{J}_{nj}^\varphi\right\rangle\right]$$
(33)

And

$$\left[R_n^{t\varphi}\right]=\left[\left\langle \bar{E}_\varphi^r ,\bar{J}_{nj}^t\right\rangle\right]$$
$$\left[R_n^{\varphi\varphi}\right]=\left[\left\langle \bar{E}_\varphi^r ,\bar{J}_{nj}^\varphi\right\rangle\right]$$
(34)

for the □-polarized plane wave (i.e., $\hat{u}_r = \hat{u}_\varphi$) One can get, for example,[7]

$$\left(R_n^{t\theta}\right)_j = \pi j^{n+1} e^{-jn\varphi} \sum_{p=1}^{4} e^{jkz_p \cos\theta_r} \left\{\cos\theta_r siv_p J_{n+1}(x) - J_{n-1}(x) + 2j\sin\theta_r \cos vpJ_n(x)\right\}$$
(35)

Where $J_n(x) = J_n(k\rho_p \sin\theta_r)$.

In general, $\left(R_n^{t\theta}\right)_{ij}$ and $\left(R_n^{\varphi\varphi}\right)_{ij}$ are even in n, while $\left(R_n^{t\varphi}\right)_{ij}$ and $\left(R_n^{\varphi\theta}\right)_{ij}$ are odd in n.

The excitation matrix [V_n] differ from the measurement matrix [R_n] by the sign of n. For plane wave excitation with axially incident plane wave only n=±1 modes are excited, and

$$\left(V_n^{\alpha\beta}\right)_j = \left(R_{-n}^{\alpha\beta}\right)_j$$
(36)

where □, and □ represent t□, □□, t□ and □□

The mode symmetry can also be used to calculate the deriving vector and the far-scattered field components. The solution of eq.(12) is obtained from

$$[I_n]=[Y_n][V_n]$$
(37)

where [Y_n] is the admittance matrix, its sub-matrices are obtained by inverting the entire Z matrix not the corresponding sub-matrices, given by

$$[Y_n]=\begin{bmatrix} Y_n^{tt} & Y_n^{t\varphi} \\ Y_n^{\varphi t} & Y_n^{\varphi\varphi} \end{bmatrix}$$
(38)

Finally, the far-scattered field components E□ and E□., are given in the form

$$\begin{bmatrix} E_\theta^s \\ E_\varphi^s \end{bmatrix} = \frac{-j\omega\mu}{4\pi r_o} e^{-jk_o r} \sum_{j=1}^{N-1} \begin{bmatrix} R_{nj}^{t\theta} & R_{nj}^{\varphi\theta} \\ R_{nj}^{t\varphi} & R_{nj}^{\varphi\varphi} \end{bmatrix} \begin{bmatrix} I_{nj}^t \\ I_{nj}^\varphi \end{bmatrix}$$
(39)

Radar Cross Section:

Radar cross section RCS defined as the width (area in three-dimensional problems) for which the incident wave carries sufficient power to produce by omnidirectional radiation, the same scattered power density in a given direction[8]. The other definition is the fraction area property of the target like an antenna, which is often regarded as having an effective area (A_e) used to extract energy from a passing radio wave. The product of incident power density and an effective area can represent the available power at terminal of a receiving antenna. In the same way, the power scattered by the target can be expressed as the product of an effective area and an incident power density. Generally, this factitious area is called the scattering cross section[9].

The RCS are effected by many parameters, such as:

1- The frequency of operation.

2- The polarization of the transmitting antenna.

3- The polarization of the receiving antenna.

4- The orientation of the object relative to the antenna (the aspect).

5- The material of which object is made and the object shape.

Formally the RCS defined as

$$\sigma = 4\pi R^2 \left|\frac{E^s}{E^i}\right|^2$$

(40)

where E^s is the scattered field of the two components related by the scattering matrix of body according to[13].

$$\begin{bmatrix} E_\theta^s \\ E_\varphi^s \end{bmatrix} = \frac{e^{-jkr}}{r} \begin{bmatrix} R_{nj}^{t\theta} & R_{nj}^{\varphi\theta} \\ R_{nj}^{t\varphi} & R_{nj}^{\varphi\varphi} \end{bmatrix} \begin{bmatrix} E_\theta^i \\ E_\varphi^i \end{bmatrix}$$

(41)

Substituting eq. (41) into eq. (40), we get

$$\sigma^{pq} = 4\pi \left|S^{pq}\right|^2$$

(42)

So

$$\sigma^{pq} = \frac{-j\omega\mu}{4\pi r} \left[R_{nj}^{tp}\right] \left[R_{nj}^{\varphi p}\right] \sum_{j=1}^{N-1} \begin{bmatrix} Y_{nj}^{tt} & Y_{nj}^{t\varphi} \\ Y_{nj}^{\varphi t} & Y_{nj}^{\varphi\varphi} \end{bmatrix} \begin{bmatrix} V_{nj}^{tq} \\ V_{nj}^{\varphi q} \end{bmatrix} \qquad (43)$$

where p and q represent □□, □□, □□ and □□̃

For example □□□ denote that the RCS measured in the □-polarized receiver with □ =0 plane, while □□□ represent the plane of □ =□/2 and RCS measured by □-polarized receiver. For the axially incident plane only n = ±1 modes are excited, therefore the RCS components in the horizontal polarization (HP) and the vertical polarization (VP) are given by

$$\sigma^{\theta\theta} = 16\pi \left|S_1^{\theta\theta}\right|^2 \cos^2 \varphi_r$$

(44)

for (HP) with □ = 0 and n=1,

$$\sigma^{\varphi\theta} = 16\pi \left|S_1^{\varphi\theta}\right|^2 \sin^2 \varphi_r$$

(45)

for (VP) with □ = □/2 and n=1

Numerical Results

A computer program has been written to compute the previous relations to find the current distribution and radar cross section from the conducting BOR.

A conducting spheroid of major axis of 0.2X and minor axis of 0.2X has been studied to cheek the validity of formulation discussed in the previous sections and compared with that of the special case of sphere. Fig.(3) shows the general body of revolution in this study. RCS of BOR is shown in Fig.4.

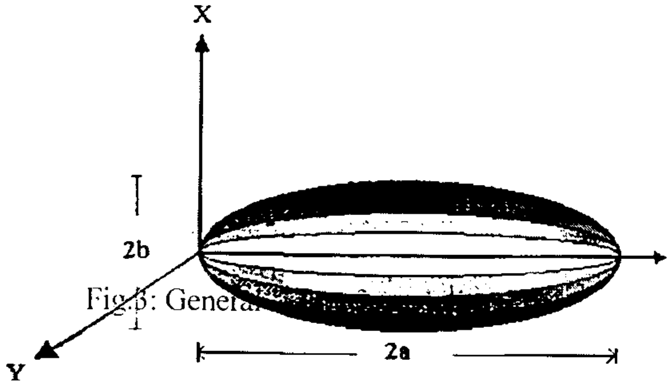


Fig.3: Genera

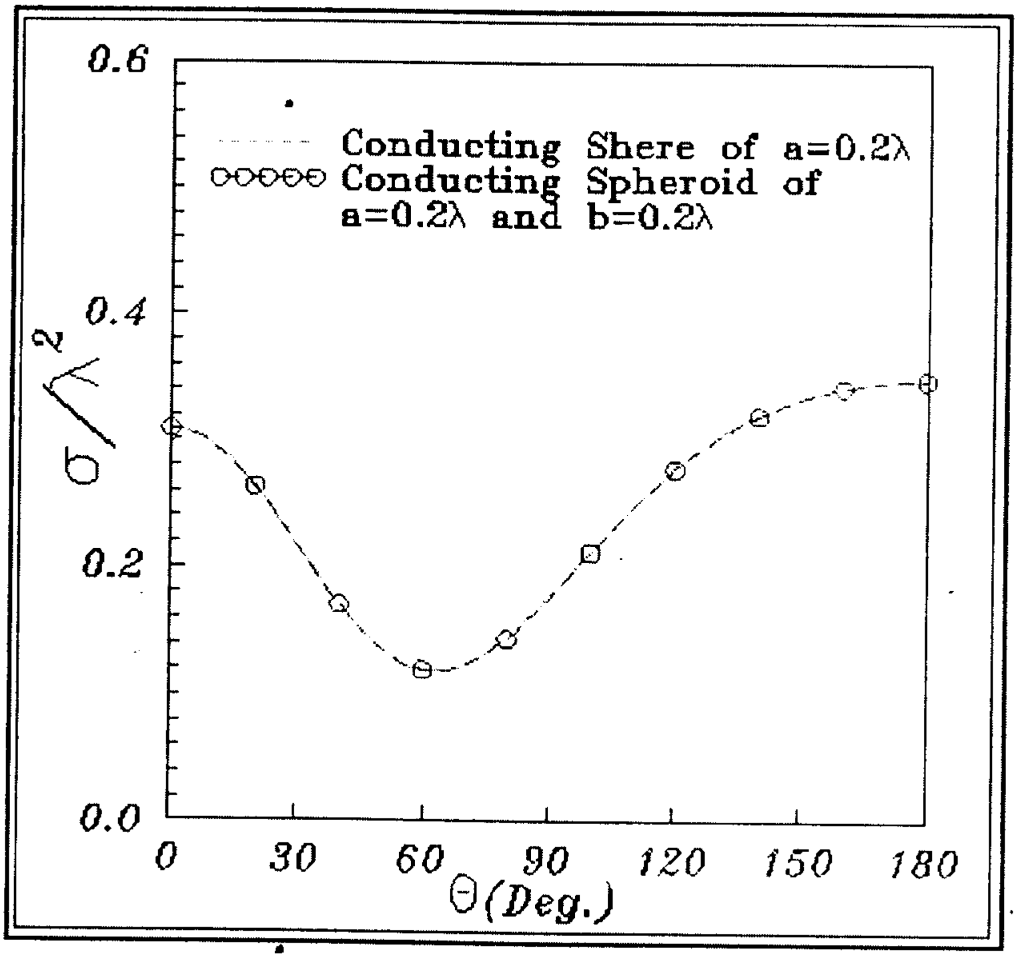


Fig.4: Radar cross-section of spheroid compared with the conducting sphere.

As shown in Fig.4 a good agreement of our result with that of the special case of sphere of a=0.2□[9].

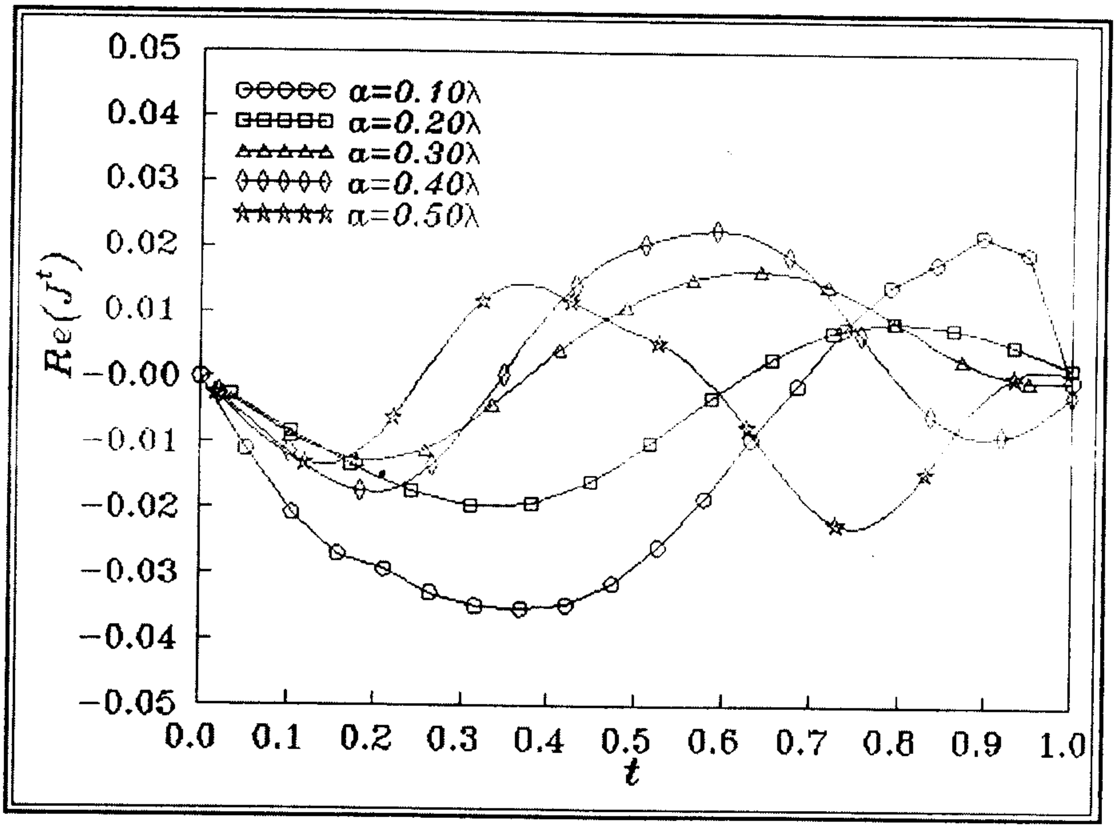


Fig.5: t-directed current distribution on BOR body as a function of major axis (a) (Real part).

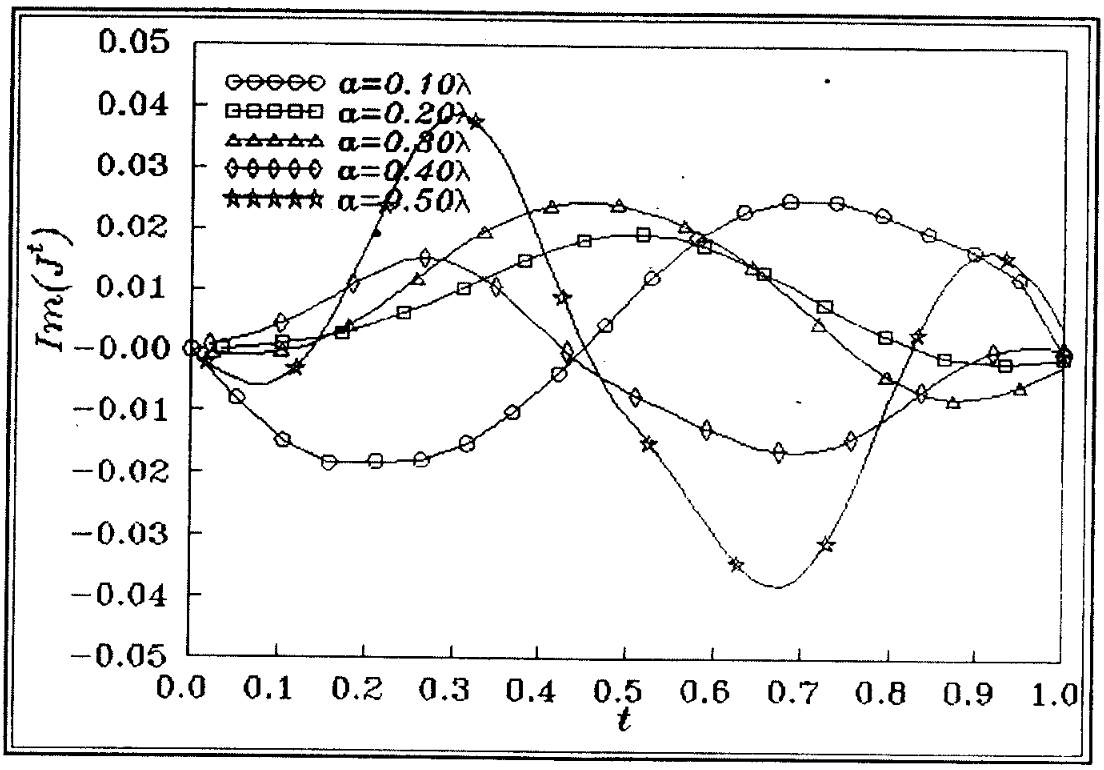


Fig.6: t-directed current distribution on BOR body as a function of major axis (a) (Imaginary part).

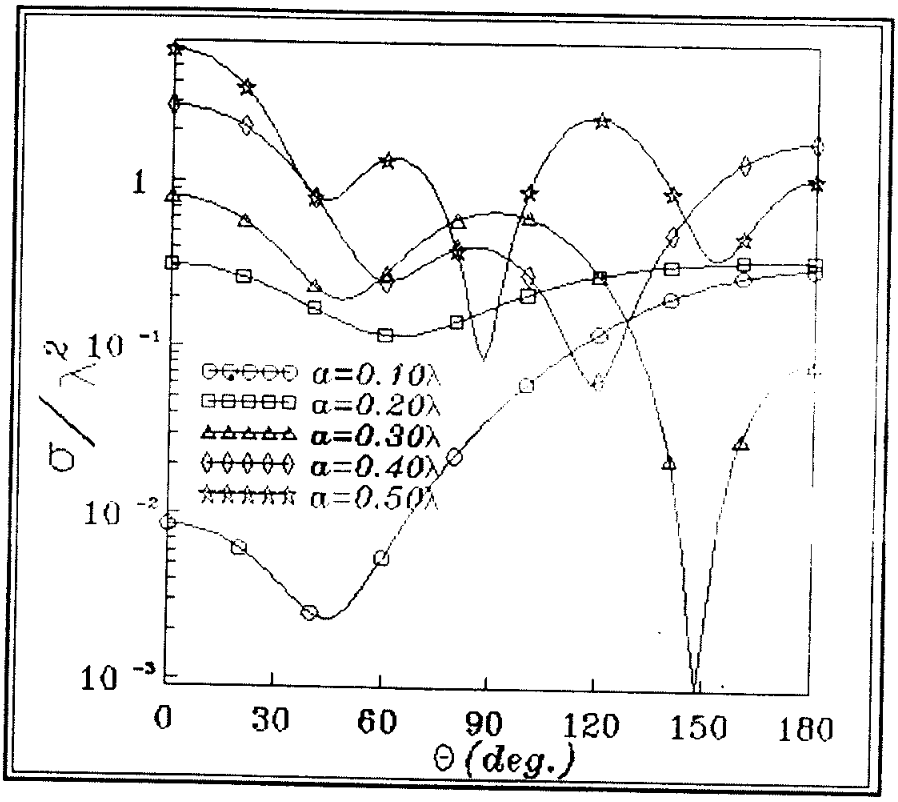


Fig.7: Radar cross-section of spheroid as a function of a major axis (a).

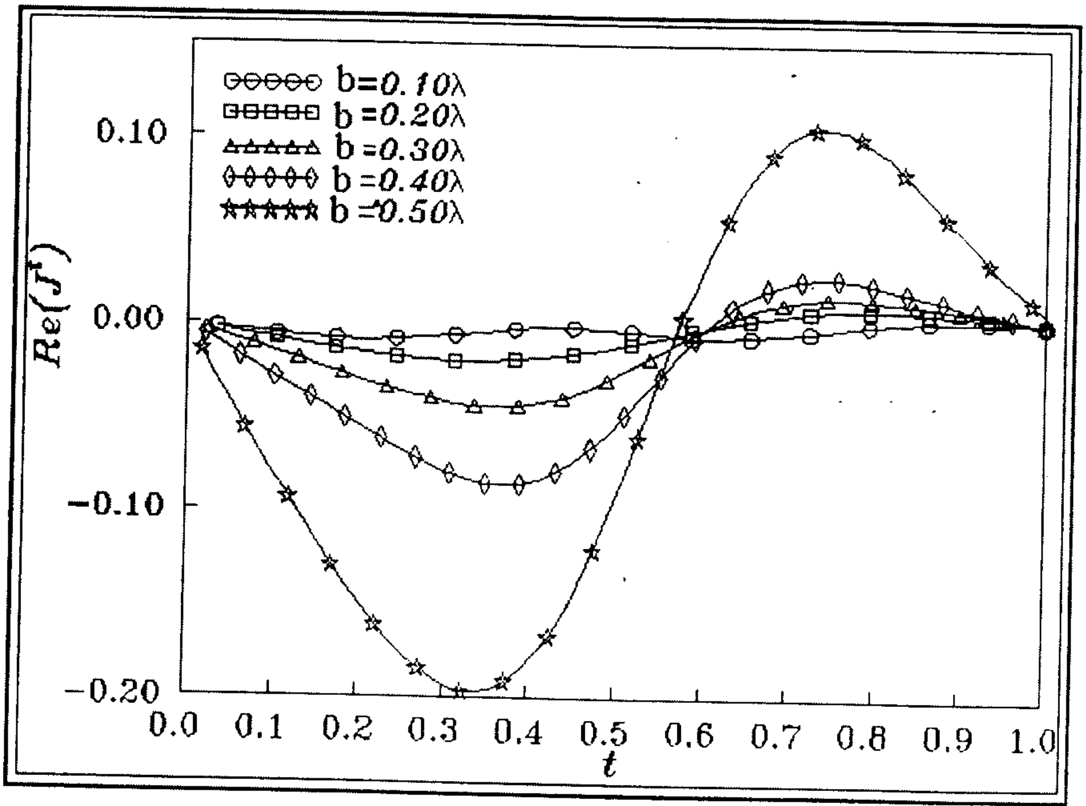


Fig.8: t-directed current distribution on BOR body as a function of minor axis (b) (Real part).

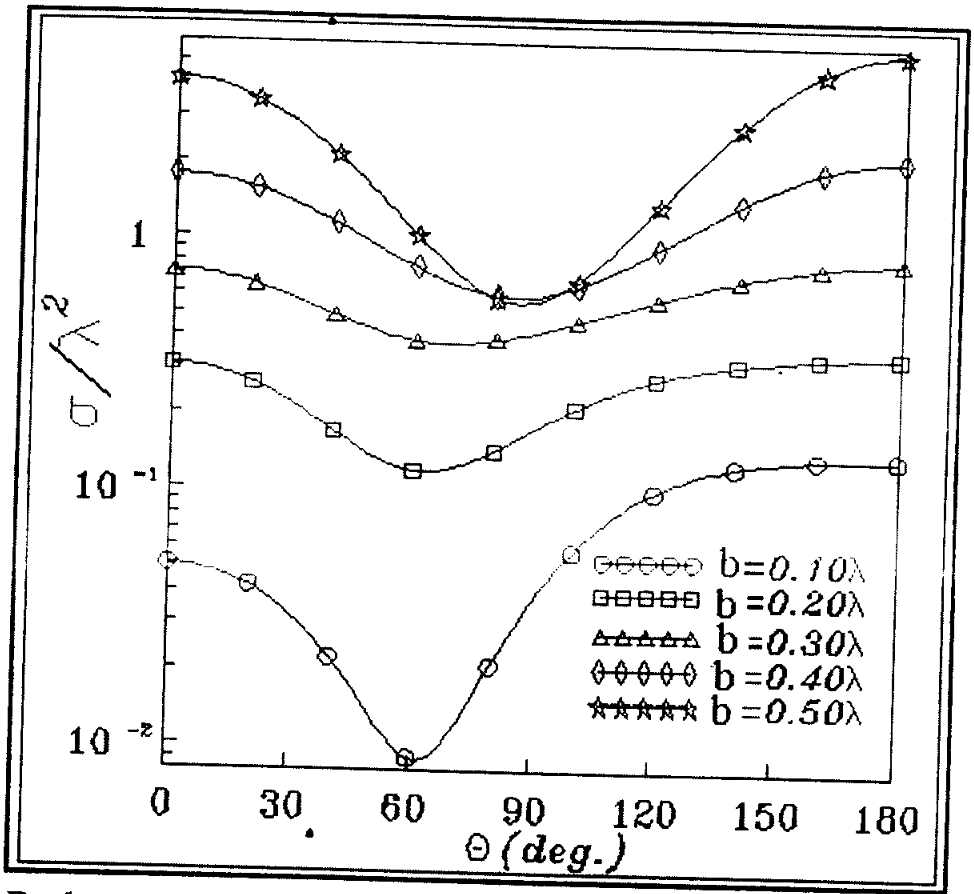


Fig.9: Radar cross-section of spheroid as a function of a minor axis (b).

Radar Cross Section:

1- Current distribution:

The effect of the major axis of the spheroid is shown in Fig. 5 and Fig.6 from these figures we show the oscillating occur in real and imaginary part oft-directed current when body major axis is become large. While in the other hand if the minor axis is become large the amplitude increased while the oscillations dose not occur as shown in Fig.8.

2- Radar Cross Section:

Figure (7) and (9) shows the radar cross section pattern in 00-directed (i.e. E-plan). The amplitude is increased rabidly with the oscillation occur when the value of the major axis increased, when the minor axis increased the amplitude increased without oscillation occur in the RCS pattern with shift in the minimum.

References

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