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An Analysis of Double well and Harmonic oscillator trapping potential used to produce BEC

Samer K Ghalib and Noori.H.N. Al-hashimi[1]
Department of Physics; College of Education,
University of Basra; Basra; Iraq
[1] e-mail nalhashmiy@yahoo.com
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Abstract

This paper describes the influences of the centres in (1+1) dimensional external trapping potential which are usually used in the experiment that leads to produce Bose-Einstein condensation BEC in ultra cold gases. Two mixed types of trapping potential are used in this analysis, the first one is a harmonic oscillator potential HOP which is applied along the Y-axis, and the second is a double well potential DWP which is applied along the X-axis. The results show that the value of a, in DWP formula have a major effects on the potential in terms of shape and value.

Introduction

In early BEC experiments, quadratic harmonic oscillator well was used to trap the atoms. Recently more advanced and complicated traps have been applied for studying BECs in laboratories [1, 2, 3, 4]. Magnetic traps for atoms play an essential role in the road to BEC. In these verifications, theoretical exploration of characteristic of trapped potential needs a mathematical model describing those potentials which are used experimentally to produce BEC at very low temperatures. Many different shapes of Bose-Einstein condensation have been achieved by using different type of trapping potential. In this paper, we analyze the roles that the centre of DWP plays on the overall views of the external trapping potential.

Theory

The Hamiltonian for interacting particles in an external trapping potential V_{trap} in second quantization is:

$$\hat{H} = \int dr\, \hat{\psi}^{\dagger}_{(r)} \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}\right)\hat{\psi}_{(r)} + \frac{1}{2}\int dr\, dr' \hat{\psi}^{\dagger}_{(r)}\hat{\psi}^{\dagger}_{(r')} V(r-r')\hat{\psi}_{(r')}\hat{\psi}_{(r)} \quad (1)$$

Here $\hat{\psi}_{(r)}$ and $\hat{\psi}^{\dagger}_{(r)}$ are the bosonic creation and annihilation operators while $V(r-r')$ is the interaction potential for two atoms located at position r and r'. At temperatures T much smaller than the critical temperature Tc [5], the BEC is well described by the macroscopic wave function $\psi = \psi(r,t)$

whose evolution is governed by a self-consistent, mean field nonlinear Schrodinger equation (NLSE) known as the Gross-Pitaevskii equation [6, 7, 8], after independent derivations by Gross [9] and Pitaevskii [10]. If a harmonic trap potential is considered, the single particle equation becomes:

$$i\hbar\frac{\partial}{\partial t}\Phi_{(r,t)} = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{trap}(r) + NU_0|\Phi_{(r,t)}|^2\right)\Phi_{(r,t)} \quad (2)$$

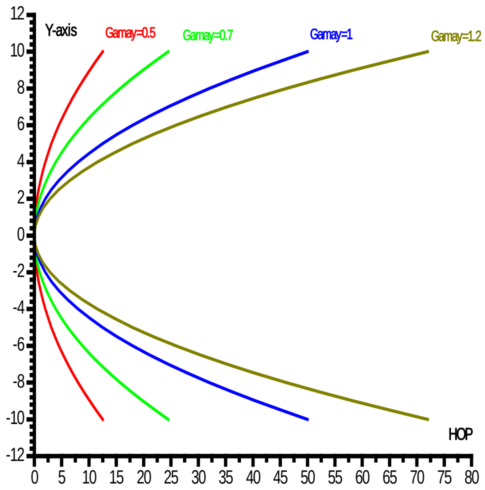
where t is time, $r = (x, y, z)^\tau$ is the spatial coordinate vector, m is the atomic mass, $\hbar$ is the Plank constant, N is the number of atoms in the condensate. V (x) is a real-valued external trapping potential whose shape is determined by the real-valued external trapping potential whose shape is determined by the type of system under investigation. U_0 describes the interaction between atoms in the condensate and has the form $U_0 = 4\pi\hbar^2 a/m$ with a the s-wave scattering length (positive for repulsive interaction and negative for attractive interaction). The normalization condition of the wave function is

$$\int |\Phi_{(r,t)}|^2 dx = 1 \qquad (3)$$

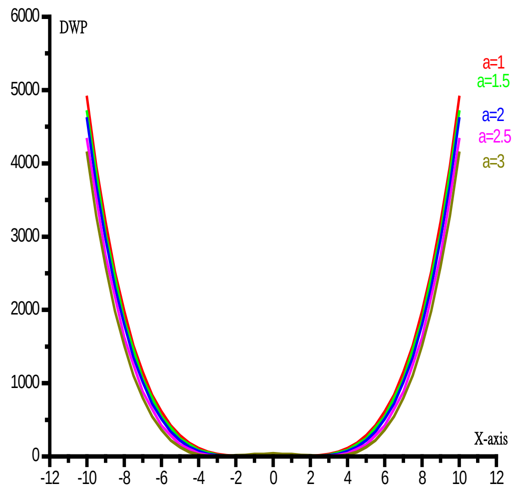
The Gross-Pitaevskii equation incorporates between the atomic interactions and external trapping potential , which is the main concern in this paper .We will discuss the one dimensional harmonic oscillator potential which is widely used in current experiments. Assume the harmonic oscillator potential is applied along the y-axis, in this case the mathematical formula will be read as :$V_{hop}(y) = \frac{m}{2}\omega_y^2 y^2$, where, ω_y, is the trapping frequencies in y-direction. The other trapping used in this analysis is the double well potential dwp [11] (Type I) along the x-axis which is read as: $V_{dwp}(r) = \frac{m}{2}\nu_x^4(x^2 - a^2)^2$, where, $\pm a$ are the double centres along the x-axis, , ν_x is a given constant with physical dimension $1/[\mathrm{m\ s}]^{1/2}$. The choices for the scaling parameters are t_0 and x_0 , the dimensionless potential $V(r)$ with $y_y = t_0\omega_y$ the energy unit $E_0 = \hbar/t_0 = \hbar^2/mr_0^2$, and the interaction parameter $\beta = 4\pi a_s N/r_0$ for harmonic oscillator potential along y-axis is: $t_0 = \frac{1}{\omega_y}$, $a_0 = \sqrt{\frac{\hbar}{m\omega_y}}$, $V(y) = \frac{1}{2}(\gamma_y y)^2$, and for double well potential along x-axis will be: $t_0 = \left(\frac{m}{\hbar v_x^4}\right)^{1/3}$, $x_0 = \left(\frac{\hbar}{m v_x^2}\right)^{1/3}$, $a = \frac{\hat{a}}{x_0}$, $V(x) = \frac{1}{2}\gamma_x^2(x^2 - a^2)^2$.

Result and Discussion:

First we will analyze the two potentials separately. We are concerned with harmonic oscillator in term of value and shape and in more precisely ,the effect of γ_y on this potential, we will fix first the shape and the distribution of double well potential along the x-axis and study the distribution of HOP along y-axis, this potential is shown in figure (1). One can conclude from this figure that γ_y has no effect as long as the shape of the potential concern where the potential HOP preserves the parabola open right like function, however the values of the potential is change with γ_y. Another remarkable point in this figure that is the distribution is symmetric just about the centre of propagation and the point of intersection of these figures is y = 0. The double well trapping potential presented in figure (2). The distribution of this potential along X-axis is shown in this figure for different value of centre of trap (1, 1.5, 2, 2.5, 3). The shape of this distribution for small of a is Parabola open up like function. To make a comprehensive view of the interaction of the two potentials in XY plane a contours level presented in figures (3-6) for difference values of a (1, 2, 4.5, 5) and fixed value γ_y of propagation of harmonic oscillator potential ($\gamma_y = 1$). By a quick view of these figures, one can conclude that the shape of the potential look like harmonic oscillator as long as the values of the centre of propagation stay between one and two as seen in figures (3,4). However the shape of potential is change when the values of the centre of propagation increases to 4.5 or 5 as seen in figures (5,6). One can conclude from these figures that the values of a "centre of propagation" in mathematical formula can be scanned to a value that fits the experimental results.



Figure(1)shown distribution of HOP used in experimental of BEC along the Y-axis



Figure(2)shown distribution of DWP used in experimental of BEC along the x-axis

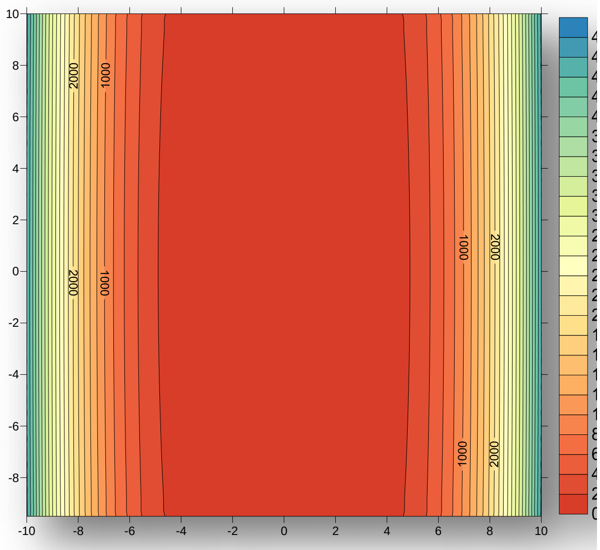


Figure (3) shown the potential in case (1D harmonic oscillator + 1D double well potential (type I)) for value $a = \pm 2$

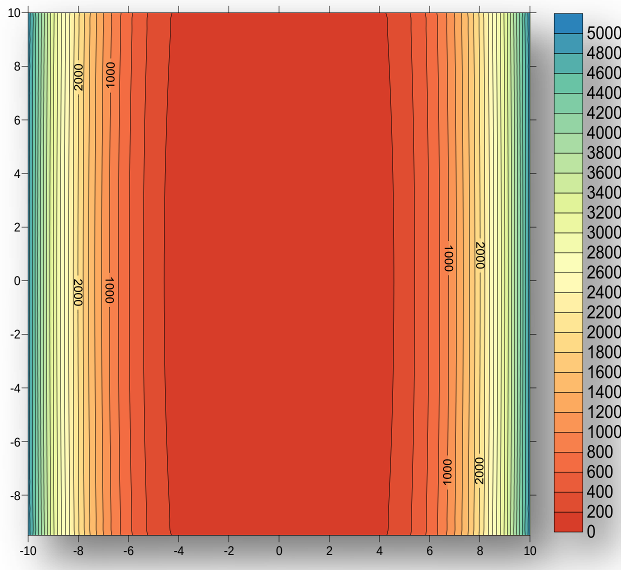


Figure (4) shown the potential in case (1D harmonic oscillator + 1D double well potential (type I)) for value $a = \pm 1$

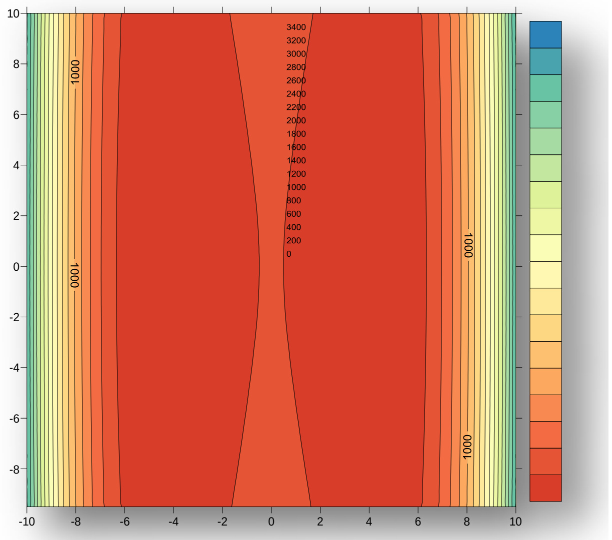


Figure (5) shown the potential in case (1D harmonic oscillator + 1D double well potential (type I)) for value $a = \pm 4.5$

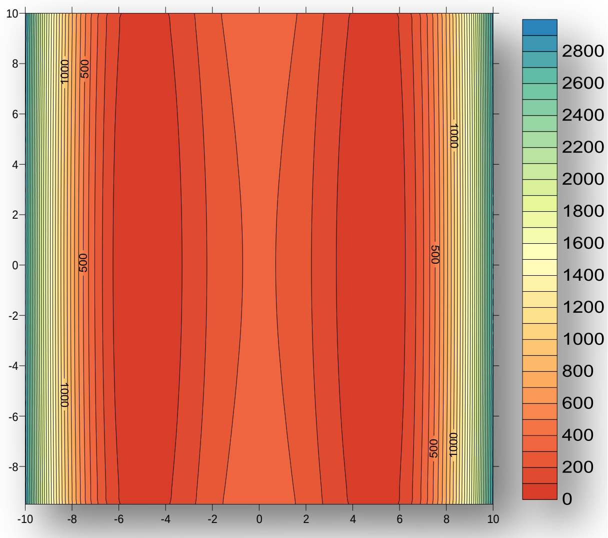


Figure (6) shown the potential in case (1D harmonic oscillator + 1D double well potential (type I)) for value $a = \pm 5$

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تحليل مصائد متذبذبات الجهد التوافقية وبئر الجهد المزدوج والتي تستخدم في التجارب التي تؤدي للحصول على تكثيف بوز–أنشتين

سامر كاظم غالب و نوري حسين نور الهاشمي

قسم الفيزياء/كلية التربية للعلوم الصرفه/جامعة البصرة

بصرة/عراق

الخلاصـــــة

هذه الورقة توصف تاثير مراكز الإنتشار في أبعاد (1+1) في مصائد الجهد الخارجي والتي تستخدم في التجارب للحصول على تكثيف بوز–أنشتين. لقد تم أستخدام و تحليل نوعين من جهود المصيده. النوع الأول هو جهد المتذبذب التوافقي والذي سلط على طول المحور Y, أما الجهد الثاني فهو بئر جهد المصيده المزدوج والذي سلط على إمتداد المحور X . لقد اوضحت النتائج إن معامل مركز الإنتشار في العلاقة الرياضية لحساب بئر الجهد المزدوج تلعب دورا كبيرا في تحديد قيمة وشكل هذا الجهد.